Riassunto della lezione precedente

- e^+e^- inclusivo : formalismo e interpretazione in QPM
  scaling della sezione d'urto totale ; rapporto R → test di $SU_c(3)$ e $SU_f(N_f)$

- e^+e^- semi-inclusivo : formalismo e interpretazione in QPM
  distribuzione angolare dell'adrone rivelato da processo elementare
  funzione di frammentazione incognita da confronto con dati
  scaling della sez. d'urto e violazioni

- Semi-Inclusive DIS (SIDIS) : formalismo e interpretazione in QPM
  ipotesi fattorizzazione → universalità delle funzioni partoniche
  confronto SIDIS – e^+e^- semi-inclusivo → info sulle funz. frammentazione

- e^+e^- semi-inclusivo in due adroni : formalismo e interpretazione in QPM
  jet ≡ fascio di adroni che portano frazione $0 \leq z \leq 1$ dell'energia del
  partone che frammenta
  sezione d'urto di jet ; distribuzione angolare e asse del jet

- DIS polarizzato : proprietà generali di S^μ e del tensore adronico

DIS polarizzato: tensore adronico

$W^{\mu\nu} = W^{\mu\nu}{}_S + W^{\mu\nu}{}_A$

$$W_S^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1 + \frac{\tilde{P}^\mu \tilde{P}^\nu}{M^2} W_2$$

$$\tilde{P}^\mu = P^\mu - \frac{P \cdot q}{q^2} q^\mu$$

$$W_A^{\mu\nu} = \frac{i}{M^2} \epsilon^{\mu\nu\rho\sigma} q_\rho \left[A_1 S_\sigma + A_2 P_\sigma\right]$$

scalare pseudoscalare

$$\begin{aligned} W_A^{\mu\nu} &= \frac{i}{M^2} \epsilon^{\mu\nu\rho\sigma} q_\rho S_\sigma \left[\underline{G_1(\nu, Q^2)} + \frac{P \cdot q}{M^2} \underline{G_2(\nu, Q^2)}\right] \\ & -\frac{i}{M^2} \epsilon^{\mu\nu\rho\sigma} q_\rho P_\sigma \frac{S \cdot q}{M^2} G_2(\nu, Q^2) \\ &= i\epsilon^{\mu\nu\rho\sigma} q_\rho P_\sigma \lambda \frac{G_1(\nu, Q^2)}{M^3} \\ & + i\epsilon^{\mu\nu\rho\sigma} q_\rho S_{\perp\sigma} \frac{1}{M^2} \left[G_1(\nu, Q^2) + \frac{P \cdot q}{M^2} G_2(\nu, Q^2)\right] \end{aligned}$$

Ampiezza di scattering

leptone polarizzato con elicità h=±

tensore leptonico : $L_{\mu\nu} = L_{\mu\nu}^S \pm L_{\mu\nu}^A$

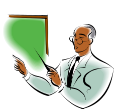
$$
\begin{aligned}
L^S_{\mu\nu} &= 2k_\mu k'_\nu + 2k_\nu k'_\mu - 2k\cdot k' g_{\mu\nu} \\
L^A_{\mu\nu} &= h2i\varepsilon_{\mu\nu\rho\sigma}k^\rho q^\sigma
\end{aligned}
$$

$L_{\mu\nu}^S \; W^{\mu\nu}{}_S \rightarrow$

$$\frac{d\sigma^0}{dE'd\Omega} = \frac{4\alpha^2}{Q^4}E'^2\left(2\sin^2\frac{\theta_e}{2}W_1 + \cos^2\frac{\theta_e}{2}W_2\right)$$

$L_{\mu\nu}^A \; W^{\mu\nu}{}_A \leftarrow$

$$
\begin{aligned}
L^A_{\mu\nu} i\varepsilon^{\mu\nu\rho\sigma}q_\rho S_\sigma &= 8EE'\sin^2\frac{\theta_e}{2}S\cdot(k+k') \\
L^A_{\mu\nu}(-i)\varepsilon^{\mu\nu\rho\sigma}q_\rho P_\sigma &= -8EE'\sin^2\frac{\theta_e}{2}P\cdot(k+k')
\end{aligned}
$$



$$L^A_{\mu\nu}W^{\mu\nu}_A = \frac{8EE'}{M^2}\sin^2\frac{\theta_e}{2}\left[\left(G_1 + \frac{P\cdot q}{M^2}G_2\right)S\cdot(k+k') - \frac{S\cdot q}{M^2}G_2 P\cdot(k+k')\right]$$

Sezione d' urto

$$\left\{\begin{array}{rcl} k & = & (E, 0, 0, E) \\ k' & = & (E', E'\sin\theta_e, 0, E'\cos\theta_e) \\ \hat{S} & = & (0, \sin\alpha\cos\phi, \sin\alpha\sin\phi, \cos\alpha) \end{array}\right.$$

coplanar → $\phi = 0$

$$\begin{array}{rcl} S\cdot(k+k') & = & -E'(\cos\theta_e\cos\alpha + \sin\theta_e\sin\alpha) - E\cos\alpha \\ P\cdot(k+k') & = & M(E+E') \\ S\cdot q & = & E'(\cos\theta_e\cos\alpha + \sin\theta_e\sin\alpha) - E\cos\alpha \end{array}$$

$$\frac{d\Delta\sigma^h}{dE'd\Omega} = -h\,\frac{2\alpha^2}{Q^2}\,\frac{1}{M^2}\,\frac{E'}{E}\left\{\underline{\cos\alpha}\left[(E+E'\cos\theta_e)\,G_1 - \frac{Q^2}{M}G_2\right] + E'\sin\theta_e\,\underline{\sin\alpha}\,(G_1 + \frac{2E}{M}G_2)\right\}$$

$\alpha = 0$ ⇔ S || k $\qquad\qquad$ $\alpha = \pi/2$ ⇔ S ⊥ k

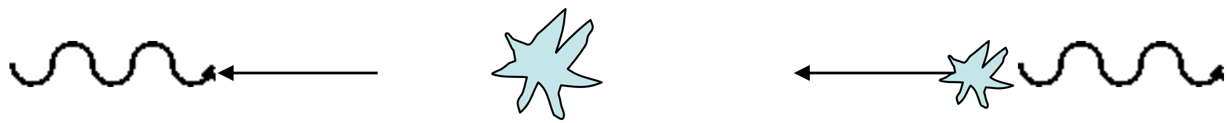
Sezione d' urto (continua)

$$\frac{d\sigma^o}{dE'\,d\Omega},\ \frac{d\Delta\sigma^{h\pm}}{dE'\,d\Omega} \Longrightarrow F_1,\ F_2,\ G_1,\ G_2$$

perchè 4 funzioni di struttura F_1, F_2, G_1, G_2 ?

sezione d' urto totale per assorbimento di γ^* : $\sigma_{tot}\,(\gamma^*\,N)$
teorema ottico : $\sigma_{tot}\,(\gamma^*\,N) \propto \mathrm{Im}\,[\,f(\theta_e=0)\ \text{Compton}\,]$

$f(\theta_e = 0, \vec{q}, \vec{N})$



±1 , 0 ±1/2 ±1/2 ±1 , 0

iniziale intermedio finale

1	+1	+1/2	+3/2	+1	+1/2
2	+1	-1/2	+1/2	+1	-1/2
3	+1	-1/2	+1/2	0	+1/2
4	0	+1/2	+1/2	+1	-1/2
5	0	+1/2	+1/2	0	+1/2

legati da time-reversal
→ 4 strutture indipendenti

Asimmetrie di elicità

riarrangiamento delle 4 combinazioni indipendenti

$$
\begin{array}{rcccccc}
\left[(1,\tfrac{1}{2}) \to (1,\tfrac{1}{2})\right] + \left[(1,-\tfrac{1}{2}) \to (1,-\tfrac{1}{2})\right] & \equiv & W_T & = & W_1 & = & \sigma^T_{3/2}+\sigma^T_{1/2} \\
(0,\tfrac{1}{2}) \to (0,\tfrac{1}{2}) & \equiv & W_L & = & \left(1+\frac{\nu^2}{Q^2}\right) W_2 - W_1 & = & \sigma^L_{1/2} \\
\left[(1,\tfrac{1}{2}) \to (1,\tfrac{1}{2})\right] - \left[(1,-\tfrac{1}{2}) \to (1,-\tfrac{1}{2})\right] & \equiv & W_{TT} & = & \frac{1}{M^3}\left(-\nu M G_1 + Q^2 G_2\right) & = & \sigma^T_{3/2}-\sigma^T_{1/2} \\
(1,-\tfrac{1}{2}) \to (0,\tfrac{1}{2}) & \equiv & W_{LT} & = & \frac{\sqrt{Q^2}}{M^3}\left(M G_1 + \nu G_2\right) & = & \sigma^{LT}_{1/2}
\end{array}
$$

elicità di γ^* $\quad \sigma^{\lambda_{\gamma^*}}_{J_z}$

intermedio

asimmetrie per scattering da γ^*

$$A_1 = \frac{\sigma^T_{1/2} - \sigma^T_{3/2}}{\sigma^T_{1/2} + \sigma^T_{3/2}} = -\frac{W_{TT}}{W_T} = \frac{\nu M G_1 - Q^2 G_2}{M^3 W_1} \qquad 1 \geq |A_1|$$

$$A_2 = \frac{W_{LT}}{W_T} = \frac{\sqrt{Q^2}(M G_1 + \nu G_2)}{M^3 W_1} \qquad R = \frac{\sigma_L}{\sigma_T} \geq |A_2| = \frac{\sigma_{LT}}{\sigma_T}$$

Accesso sperimentale alle asimmetrie

$\boxed{S \parallel k \rightarrow \alpha = 0}$

misura sperimentale accede a

$$A_{\parallel} = \frac{d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}}{d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}} = \frac{E - E'\epsilon}{E(1+\epsilon R)} A_1 + \frac{\epsilon Q}{E(1+\epsilon R)} A_2$$

$$A_{\perp} = \frac{d\sigma^{\uparrow\leftarrow} - d\sigma^{\uparrow\rightarrow}}{d\sigma^{\uparrow\leftarrow} + d\sigma^{\uparrow\rightarrow}} = \frac{E - E'\epsilon}{E(1+\epsilon R)} \sqrt{2\epsilon(1+\epsilon)}\, A_2 - \frac{\epsilon Q}{E(1+\epsilon R)} \sqrt{\frac{(1+\epsilon)^3}{2\epsilon}}\, A_1$$

$\boxed{S \perp k \rightarrow \alpha = \pi/2}$

polarizz. lineare trasversa di γ^* $\epsilon = \left[1 + 2\frac{q^2}{Q^2} \tan^2 \frac{\theta_e}{2}\right]^{-1}$

$$R = \frac{W_L}{W_T} = \left(1 + \frac{\nu^2}{Q^2}\right) \frac{W_2}{W_1} - 1$$

inversione

$$A_1 = \frac{2(E - E'\epsilon)\, E\,(1+\epsilon R)}{2(E - E'\epsilon)^2 + \epsilon(1+\epsilon)\, Q^2} A_{\parallel} - \sqrt{\frac{2\epsilon}{1+\epsilon}} \frac{QE(1+\epsilon R)}{2(E - E'\epsilon)^2 + \epsilon(1+\epsilon)\, Q^2} A_{\perp}$$

$$A_2 = \frac{E\,(1+\epsilon R)\, Q\,(1+\epsilon)}{2(E - E'\epsilon)^2 + \epsilon(1+\epsilon)\, Q^2} A_{\parallel} + \sqrt{\frac{2}{\epsilon(1+\epsilon)}} \frac{(E - E'\,\epsilon)\, E(1+\epsilon R)}{2(E - E'\epsilon)^2 + \epsilon(1+\epsilon)\, Q^2} A_{\perp}$$

misura di Q^2, ε, R, $A_{\parallel}$, $A_{\perp}$ $\rightarrow$ A_1, A_2

Limite DIS

$\nu, Q^2 \to \infty$ con x_B fisso; se $Q^2\, \sigma_{Jz}^{\lambda}$ scala allora

scaling :
$$
\begin{aligned}
M W_1(\nu, Q^2) &\to F_1(x_B) \qquad & \frac{\nu}{M} G_1(\nu, Q^2) &\to \tilde{G}_1(x_B) \\
\nu W_2(\nu, Q^2) &\to F_2(x_B) \qquad & \frac{\nu^2}{M^2} G_2(\nu, Q^2) &\to \tilde{G}_2(x_B)
\end{aligned}
$$

(vedi espressioni di A_1 e A_2)

scaling delle asimmetrie di elicità :

$$
A_1 = \frac{\nu M G_1(\nu, Q^2) - Q^2\, G_2(\nu, Q^2)}{M^3\, W_1(\nu, Q^2)} \to \frac{\tilde{G}_1(x_B)}{F_1(x_B)} - \frac{Q^2}{\nu^2} \frac{\tilde{G}_2(x_B)}{F_1(x_B)} \to \frac{\tilde{G}_1(x_B)}{F_1(x_B)}
$$

$$
A_2 = Q \frac{M G_1(\nu, Q^2) + \nu\, G_2(\nu, Q^2)}{M^3\, W_1(\nu, Q^2)} \to \sqrt{\frac{2 M x_B}{\nu}} \frac{\tilde{G}_1(x_B) + \tilde{G}_2(x_B)}{F_1(x_B)} \to 0
$$

QPM picture

$$\frac{d\Delta\sigma^h}{dx_B\, dy} = \frac{2M\nu\pi}{E'}\,\frac{d\Delta\sigma^h}{dE'\, d\Omega} = \frac{2M\nu\pi}{E'}\, h\, \frac{\alpha^2}{Q^4}\,\frac{E'}{E}\, L^A_{\mu\nu}\, W^{\mu\nu}_A$$

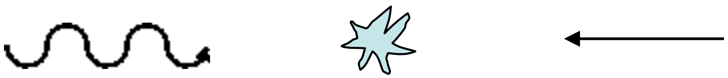
$$= h\,\frac{4\pi\alpha^2}{Q^2}\left[\lambda\,(2-y)\,\tilde{G}_1 - |\mathbf{S}_\perp|\,\sqrt{1-y}\,\frac{Q}{E}\,(\tilde{G}_1+\tilde{G}_2)\right]$$

Poi :

- scrivere sez. d'urto elementare per processo $\vec{e}\,\vec{q} \to e'\,q$
- scrivere convoluzione in ipotesi QPM di fattorizzazione
  $\to$ dedurre funzioni di struttura in termini di
  densità partoniche

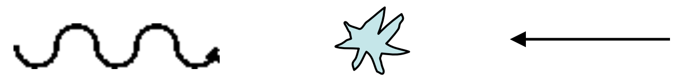
oppure

Metodo alternativo



γ^* **+1** **J_z=3/2** $\sigma^T_{3/2}$ **+1/2** **$P^\uparrow$**

-1 **J_z=1/2** $\sigma^T_{1/2}$ **+1/2**



γ^* **± 1** **J_z=1/2** **±1/2** **$q^{\uparrow\downarrow}$**

perché L_z = 0 (processo collineare)

→ conservazione del momento angolare

quindi $\gamma^{*\uparrow}\, q^\downarrow \rightarrow q^\uparrow$ $\cancel{\gamma^{*\uparrow}\, q^\uparrow}$

$\gamma^{*\downarrow}\, q^\uparrow \rightarrow q^\downarrow$ $\cancel{\gamma^{*\downarrow}\, q^\downarrow}$

$$\left.\begin{array}{l} \sigma^T_{3/2} \leftrightarrow \gamma^{*\uparrow} P^\uparrow \propto \sum_f \, e_f^2 \, q_f^\downarrow \\ \sigma^T_{1/2} \leftrightarrow \gamma^{*\downarrow} P^\uparrow \propto \sum_f \, e_f^2 \, q_f^\uparrow \end{array}\right\} \longrightarrow A_1 = \frac{\sigma^T_{1/2} - \sigma^T_{3/2}}{\sigma^T_{1/2} + \sigma^T_{3/2}} = \frac{\sum_{f,\bar f} e_f^2\,(q_f^\uparrow - q_f^\downarrow)}{\sum_{f,\bar f} e_f^2\,(q_f^\uparrow + q_f^\downarrow)}$$

$$= \frac{\tilde G_1(x_B)}{F_1(x_B)} \equiv \frac{g_1(x_B)}{f_1(x_B)}$$

distribuzione di elicità $$g_1(x_B) = \frac{1}{2}\sum_{f,\bar f} e_f^2\,[q_f^\uparrow(x_B) - q_f^\downarrow(x_B)]$$

Distribuzione di polarizzazione trasversa

procedura simile

$$\tilde{G}_1(x_B)+\tilde{G}_2(x_B) \equiv g_1(x_B)+g_2(x_B) = \frac{1}{2Mx_B}\sum_{f,\bar{f}} e_f^2\, m_f\, [q_f^{\rightarrow}(x_B)-q_f^{\leftarrow}(x_B)]$$

risulta $$-\frac{g_1(x)}{x} = \frac{\partial}{\partial x}[g_1(x)+g_2(x)]$$

relazione di Wandzura−Wilczek $$g_2(x) = \int_x^1 \frac{dy}{y} g_1(y) - g_1(x)$$

regola di somma Burkhardt−Cottingham $$\int_0^1 dx g_2(x) = 0$$

e in generale $$\int_0^1 dx\; x^{J-1}\left[\frac{J-1}{J} g_1(x) + g_2(x)\right] = 0$$

Distribuzione di polarizzazione trasversa

se $p_T \neq 0$ $\quad \gamma^{*\uparrow} q^{\uparrow}$, $\gamma^{*\downarrow} q^{\downarrow}$ permesse

ad esempio per 1 flavor solo con $q^{\uparrow}$ in $\sigma_{Jz}^{\lambda}(\gamma^* q^{\uparrow})$

$p_T = 0$

$$A_1 = \frac{\sigma^T_{1/2} - \sigma^T_{3/2}}{\sigma^T_{1/2} + \sigma^T_{3/2}} = \frac{\sum_{f,\bar{f}} e_f^2 \, (q_f^{\uparrow} - q_f^{\downarrow})}{\sum_{f,\bar{f}} e_f^2 \, (q_f^{\uparrow} + q_f^{\downarrow})} \sim \frac{q_f^{\uparrow}}{q_f^{\uparrow}} = 1$$

$p_T \neq 0$

$$A_1 = \frac{\sigma^T_{1/2} - \sigma^T_{3/2}}{\sigma^T_{1/2} + \sigma^T_{3/2}} = 1 - \frac{p_T^2}{E(E+m)} \sim 1$$

Distribuzione di elicità e misura dello spin

In generale $g_1(x_B,Q^2)$: dipendenza da Q^2 (= violazione dello scaling)
calcolabile in QCD perturbativa

interesse in $g_1(x_B\,,Q^2)$ è dovuto al fatto che il suo 1° momento di Mellin fornisce informazioni sull'elicità dei quark ed inoltre è calcolabile su reticolo

1° momento di Mellin di g_1

$$\Gamma_1(Q^2) = \int_0^1 dx\, g_1(x,Q^2) = \frac{1}{2}\sum_{f,\bar{f}} e_f^2 \int_0^1 dx\,(q_f^{\uparrow}(x,Q^2) - q_f^{\downarrow}(x,Q^2)) = \frac{1}{2}\sum_{f\bar{f}} e_f^2\, \Delta q_f$$

$$\Delta q_f = \int_0^1 dx\,(q_f^{\uparrow}(x,Q^2) - q_f^{\downarrow}(x,Q^2))$$

exp. $\rightarrow A_{||} \rightarrow A_1\ (A_2 \sim 0) \rightarrow g_1\ (x_B,Q^2) \rightarrow \Gamma_1(Q^2) \rightarrow \Delta q_f$

1 relazione per $f \geq 3$ incognite !

(continua)

in QPM per protone : $\Gamma_1^p = \frac{1}{2}\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right)$

QPM : funz. d' onda del q in P↑ "ispirata" a $SU_f(3) \otimes SU(2)$

$$|P^\uparrow\rangle \approx \frac{1}{\sqrt{6}}\left(2u^\uparrow u^\uparrow d^\downarrow - u^\uparrow u^\downarrow d^\uparrow - u^\downarrow u^\uparrow d^\uparrow\right)$$

$\rightarrow \Gamma_1^p = 5/18 \sim 0.28$

$\Delta\Sigma = 1$

3 incognite → info da corrente assiale $A_\mu^a \sim \gamma_\mu \gamma_5 T^a$ in decadimenti semi-leptonici (ex. β decay) nell' ottetto barionico

Risulta

$$\Gamma_1^p = \int_0^1 dx\, g_1^p(x) \sim \frac{1}{12}\langle A_\mu^3\rangle \left[1 + \frac{5}{3}\frac{\langle A_\mu^8\rangle}{\langle A_\mu^3\rangle}\right] = \frac{1}{12}\left|\frac{g_A}{g_V}\right|_{np}\left[1 + \frac{5}{3}\frac{3F-D}{F+D}\right]$$

$$= 0.17 \pm 0.01$$

$$\Delta\Sigma = 3F - D = 0.60 \pm 0.12$$

da fit a decadimenti semi-leptonici → F= 0.47 ± 0.004 ; D=0.81± 0.003

regola di somma di Ellis-Jaffe (' 73)
(hp.= perfetta simmetria $SU_f(3)$ + $\Delta s = 0$)

correzioni complicate

Esperimento EMC (CERN, '87)

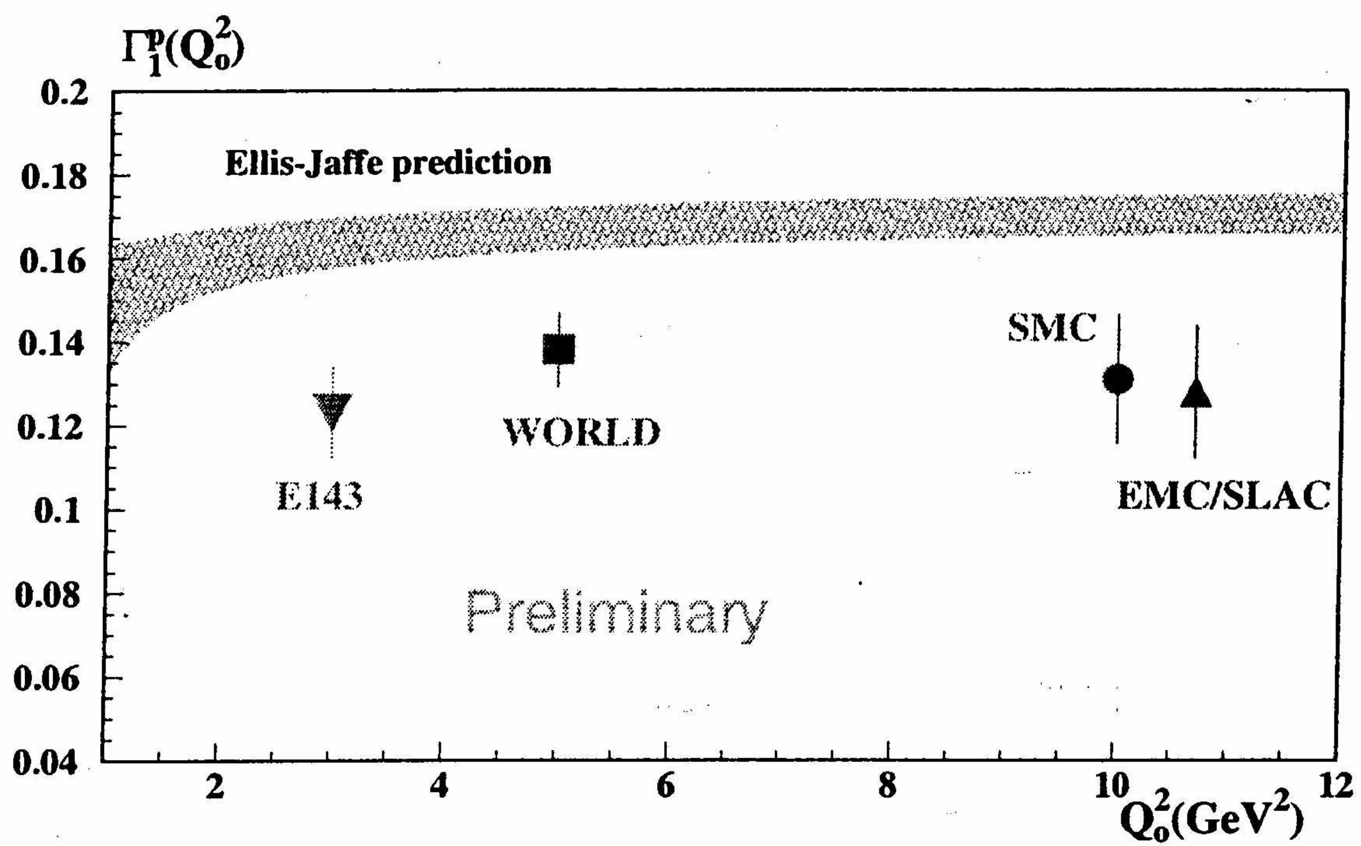
$\mu^{\uparrow}p^{\uparrow} \to \mu p$ at $Q^2 = 10.7$ GeV^2

$$A_{\parallel} = \frac{d\sigma^{\uparrow\downarrow} - d\sigma^{\uparrow\uparrow}}{d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}} \sim \frac{E - E'\epsilon}{E(1+\epsilon R)} A_1 \sim \frac{E - E'\epsilon}{E(1+\epsilon R)} \frac{g_1(x_B, Q^2)}{F_1(x_B, Q^2)}$$

$$\Gamma_1^p(10.7) = \int_{x_{min}}^{x^{max}} dx\, g_1(x, 10.7) = 0.126 \pm 0.010 \pm 0.015$$

R = σ_L/σ_T
da sez. d'urto non polarizzata

confermato da altri esperimenti:
SMC (Cern),
E142 e E143
(SLAC)



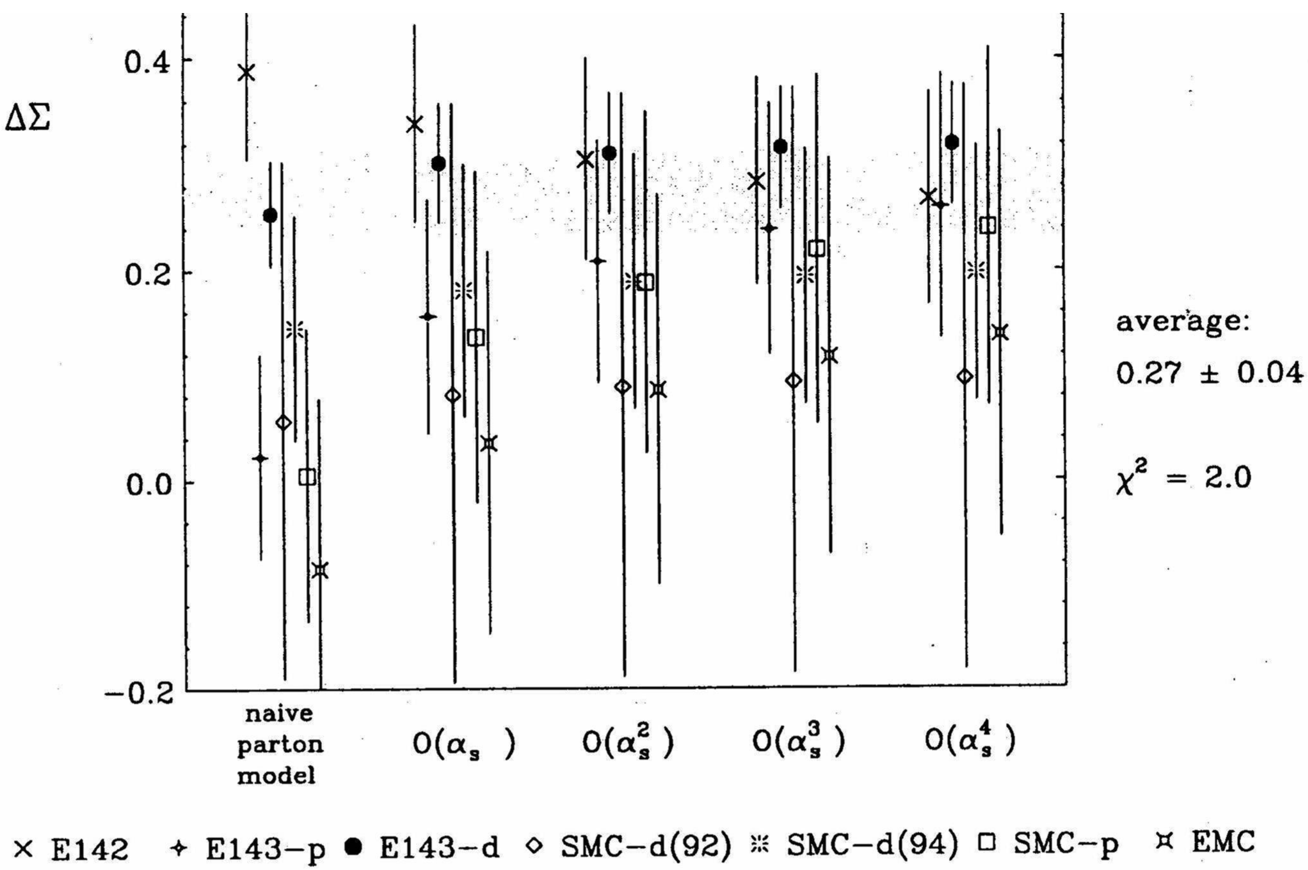
Spin crisis

F,D, $\Gamma_1^p(Q^2) \rightarrow \Delta\Sigma(Q^2) \rightarrow \Delta u, \Delta d, \Delta s$

$Q^2 = 10.7\ \mathrm{GeV}^2$
$\Delta\Sigma = 0.13 \pm 0.19$

$\Delta u = 0.78 \pm 0.10$
$\Delta d = 0.50 \pm 0.10$
$\Delta s = -0.20 \pm 0.11$

polarizzazione negativa del mare



$Q^2 = 3\ \mathrm{GeV}^2$
$\Delta\Sigma = 0.27 \pm 0.04$

(spin crisis continua)

QPM	Ellis – Jaffe sum rule	exp.
	$SU_f(3)$ + $\Delta s = 0$	$Q^2 = 10.7\ GeV^2$
$\Gamma_1^p \sim 0.28$	$\Gamma_1^p = 0.17 \pm 0.01$	$\Gamma_1^p = 0.126 \pm 0.010 \pm 0.015$
$\Delta\Sigma = 1$	$\Delta\Sigma = 0.60 \pm 0.12$	$\Delta\Sigma = 0.13 \pm 0.19$
		$Q^2 = 3\ GeV^2$
	discrepanza > 2σ	$\Delta\Sigma = 0.27 \pm 0.04$

violazione di $SU_f(3)$

estrapolazione $g_1(x)$ per $x \to 0$

anomalia assiale $\partial^\mu A^0_\mu = \frac{n_f \alpha_s}{2\pi} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$

$\to$ contributo di gluoni

$$\Delta q = \Delta q' - \frac{\alpha_s}{2\pi} \Delta g$$

nessuna ipotesi spiega quantitativamente la discrepanza osservata