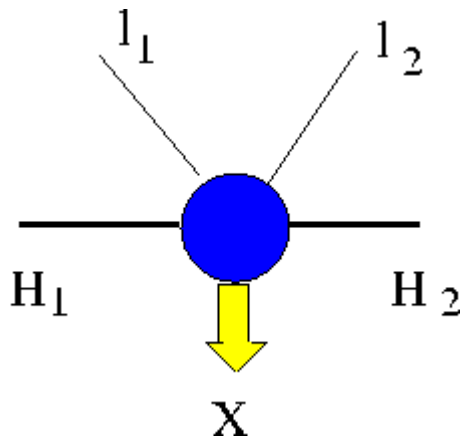


# Riassunto della lezione precedente

- verifica sperimentale di QPM in reazioni elettrodeboli :
  1. DIS con (anti)neutrini su nuclei isoscalari
    - sez. d'urto di neutrino domina quella per antineutrino
    - antipartoni soppressi in mezzo nucleare
  2. DIS con (anti)neutrini su protone
    - dominanza di quark valenza / mare per  $x_B \rightarrow 1/0$
  3. rapporto di carica per nuclei isoscalari
- regole di somma :
  1. Gottfried sum rule →  $SU_f(3)$  rotta per i quark del mare
  2. Momentum sum rule → gluoni portano metà del momento dell'adrone
- verifica fattorizzazione → esplorare nuovi processi fondamentali

## Drell - Yan

(Drell & Yan, P.R.L. **25** (70) 316)



adroni in annichilazione  $H_{1/2}$  con momento  $P_{1/2}$   
leptoni prodotti  $l_{1/2}$  con momento  $k_{1/2}$

energia disponibile nel c.m. degli adroni  
 $s = (P_1 + P_2)^2$

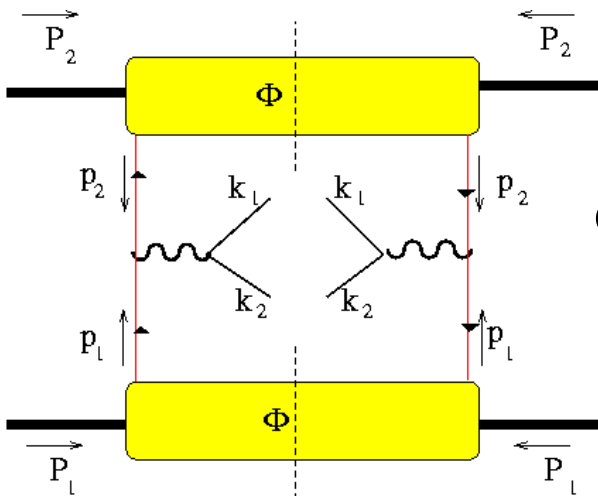
massa invariante della coppia di leptoni  
 $M^2 \equiv q^2 = (k_1 + k_2)^2$

$q^2 = Q^2 \geq 0$  time-like

regime DIS :  $q^2, s \rightarrow \infty$  con  $\tau = q^2 / s$  fissato  $1 \geq \tau \geq 0$

la coppia di leptoni non interagisce con la coppia di adroni iniziali  
 $\rightarrow$  è manifestazione del decadimento dei bosoni di gauge intermedi  
prodotti dalla annichilazione adronica

bosoni di gauge a spin 1 con  $Q^2 \geq 0 \rightarrow$  risonanze mesoniche vettoriali  
 $\rightarrow$  decadimento  $\rightarrow$  produzione di coppie leptoniche con  $p_T$



$$p_1 = x_1 P_1 \quad \text{con} \quad x_1 = \frac{Q^2}{2P_1 \cdot q} \quad 1 \geq x_{1/2} \geq 0$$

$$p_2 = x_2 P_2 \quad x_2 = \frac{Q^2}{2P_2 \cdot q}$$

energia disponibile della reazione elementare :

$$(p_1 + p_2)^2 \sim 2 p_1 \cdot p_2 = x_1 x_2 (P_1 + P_2)^2 = x_1 x_2 s$$

$$\tau = \frac{q^2}{s} = \frac{M^2}{s} = x_1 x_2 \quad 1 \geq \tau \geq 0$$

$$x_F = x_1 - x_2 \quad 1 \geq x_F \geq -1$$

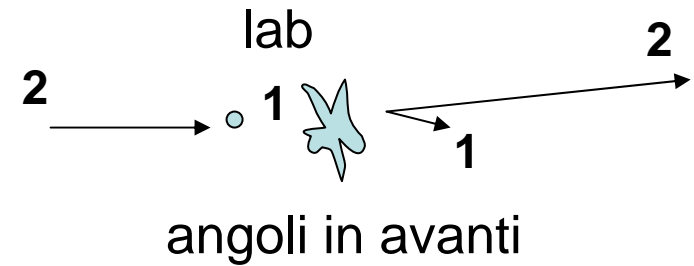
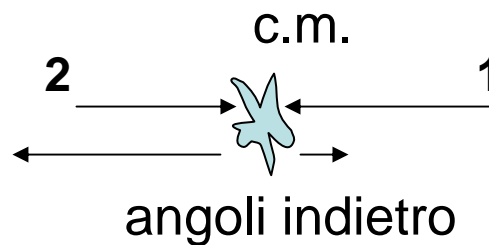
$$x_{1/2} = \frac{1}{2} \left( \pm x_F + \sqrt{x_F^2 + \frac{4M^2}{s}} \right)$$

$x_{1/2}$  = frazione del momento longitudinale  $\rightarrow$

$x_F$  = momento longitudinale della coppia nel c.m. rispetto al momento longitudinale massimo possibile

supponiamo  $H_2$  = fascio e  $H_1$  = target

$$x_F \rightarrow -1 \Leftrightarrow \begin{aligned} x_2 &\rightarrow 1 \\ x_1 &\rightarrow 0 \end{aligned}$$



$$x_F \rightarrow 1 \Leftrightarrow \begin{aligned} x_2 &\rightarrow 0 \\ x_1 &\rightarrow 1 \end{aligned}$$

situazione rovesciata

“solite” formule, applicate al processo DY

$$d\sigma = \frac{1}{F} |\mathcal{M}|^2 dR$$



$$F = 4\sqrt{(P_1 \cdot P_2)^2 - M_1^2 M_2^2} \sim 2s$$

$$dR = (2\pi)^4 \delta(P_1 + P_2 - P_X - k_1 - k_2) \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} \frac{d\mathbf{k}_1}{(2\pi)^3 2E_1} \frac{d\mathbf{k}_2}{(2\pi)^3 2E_2}$$

$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} H^{\mu\nu} \quad H^{\mu\nu} = \sum_X \langle P_1 S_1, P_2 S_2 | J^\mu | P_X \rangle \langle P_X | J^\nu | P_1 S_1, P_2 S_2 \rangle$$

$$(2\pi)^3 2E_1 (2\pi)^3 2E_2 \frac{d\sigma}{d\mathbf{k}_1 d\mathbf{k}_2} = 2 \frac{1}{2s} (2\pi)^4 \frac{e^4}{Q^4} L_{\mu\nu} \times \frac{1}{(2\pi)^4} \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta(P_1 + P_2 - P_X - q) H^{\mu\nu}$$

somma sugli stati  
di polarizzazione dei  
leptoni finali

$$= (2\pi)^4 32\pi^2 \frac{1}{2s} \frac{\alpha^2}{Q^4} L_{\mu\nu} W^{\mu\nu}$$

$$\begin{aligned} \frac{d\mathbf{k}_1}{(2\pi)^3 2E_1} \frac{d\mathbf{k}_2}{(2\pi)^3 2E_2} &= d^4 k_1 d^4 k_2 \\ &= d^4 q d^4(k_1 - k_2) \stackrel{c.m.}{=} \frac{d^4 q d\Omega}{(2\pi)^4 32\pi^2} \end{aligned}$$

$$\frac{d\sigma}{d^4 q d\Omega} = \frac{\alpha^2}{2s Q^4} L_{\mu\nu} W^{\mu\nu}$$

# QPM picture

approssimazione:  $Q^2$  non elevato  $\rightarrow$  bosone di gauge  $\gamma$

energia disponibile nel c.m. della reazione elementare :

$$(p_1 + p_2)^2 \sim 2 p_1 \cdot p_2 = x_1 x_2 2 P_1 \cdot P_2 \sim x_1 x_2 (P_1 + P_2)^2 = x_1 x_2 s$$

processo elementare:  $(q\bar{q} \rightarrow l\bar{l}) \equiv (e^+e^- \rightarrow l\bar{l})$  elastico

esempio: produzione di  $\mu^+\mu^-$   $\frac{d\sigma^{el}}{dQ^2} = \frac{4\pi\alpha^2}{3Q^2} e_f^2 \delta(x_1 x_2 s - Q^2)$

$$\frac{d\sigma}{dQ^2} = \left( \frac{1}{N_c} \sum_f \right) \int_0^1 dx_1 dx_2 \phi_f(x_1) \frac{d\sigma^{el}}{dQ^2} \phi_{\bar{f}}(x_2)$$

$N_c$  modi di creare  
la coppia  
conservando il colore  
nel vertice;

ciascuna  $\phi_f$  porta  $N_c$   
 $\rightarrow (N_c \times N_c) / N_c = N_c$   
 $\rightarrow$  test di  $SU_c$  (3)

$$= \frac{4\pi\alpha^2}{9Q^4} \sum_f e_f^2 \int_0^1 dx_1 dx_2 \phi_f(x_1) \phi_{\bar{f}}(x_2) \delta \left( x_1 x_2 \frac{s}{Q^2} - 1 \right)$$

$$\begin{aligned} & \sum_f \dots \phi_f(x_1) \phi_{\bar{f}}(x_2) \\ & \equiv \sum_f \dots \left[ \phi_f(x_1) \phi_{\bar{f}}(x_2) + \phi_{\bar{f}}(x_1) \phi_f(x_2) \right] \end{aligned}$$

## QPM: test sperimentali

a) scaling della sez. d'urto

$$Q^4 \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{9} \sum_f e_f^2 \int_0^1 dx_1 dx_2 \phi_f(x_1) \phi_{\bar{f}}(x_2) \delta\left(\frac{x_1 x_2}{\tau} - 1\right)$$



$$Q^3 \frac{d\sigma}{dQ dx_1 dx_2} = \frac{8\pi\alpha^2}{9} \sum_f e_f^2 \phi_f(x_1) \phi_{\bar{f}}(x_2) \delta\left(\frac{x_1 x_2}{\tau} - 1\right)$$

oppure, con  $\begin{cases} x_F = x_1 - x_2 \\ \tau = x_1 x_2 \end{cases} \quad J = \begin{vmatrix} \frac{\partial x_F}{\partial x_1} & \frac{\partial x_F}{\partial x_2} \\ \frac{\partial \tau}{\partial x_1} & \frac{\partial \tau}{\partial x_2} \end{vmatrix} = x_1 + x_2$



$$M^3 \frac{d\sigma}{dM dx_F} = \int d\tau \frac{d\sigma}{dM dx_F d\tau} = \frac{8\pi\alpha^2}{9} \frac{x_1 x_2}{x_1 + x_2} \sum_f e_f^2 \phi_f(x_1) \phi_{\bar{f}}(x_2)$$

scaling  $\forall s \rightarrow$  interazione elementare puntiforme !

$$x_F = (1 - \tau) x'_F$$

$$M^3 \frac{d\sigma}{dM dx'_F} \sim a |1 - \tau|$$



piccole deviazioni perche`  
pQCD  $\rightarrow \phi_f(x, \log Q^2)$

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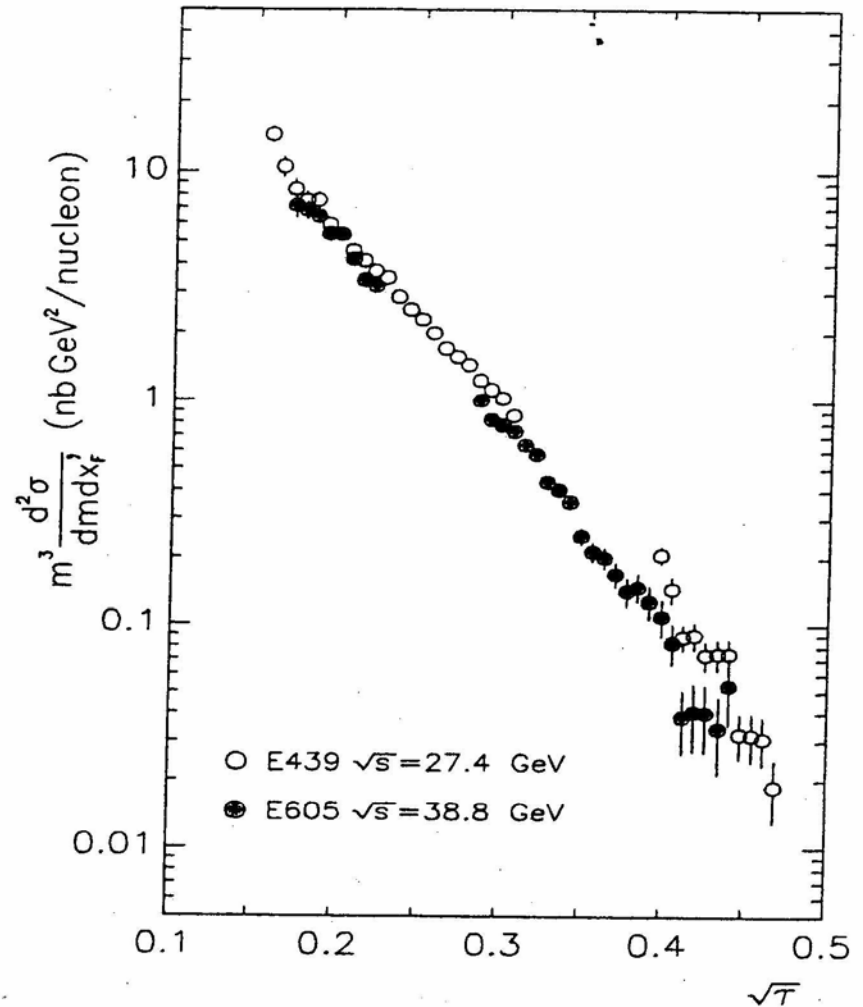


FIG. 12. Scaling form of the dimuon yield versus  $\sqrt{\tau}$  comparing this experiment with experiment E439 (Smith *et al.*, Ref. 31,  $\sqrt{s} = 27.4$  GeV) for the interval  $0 < x'_F < .2$ .

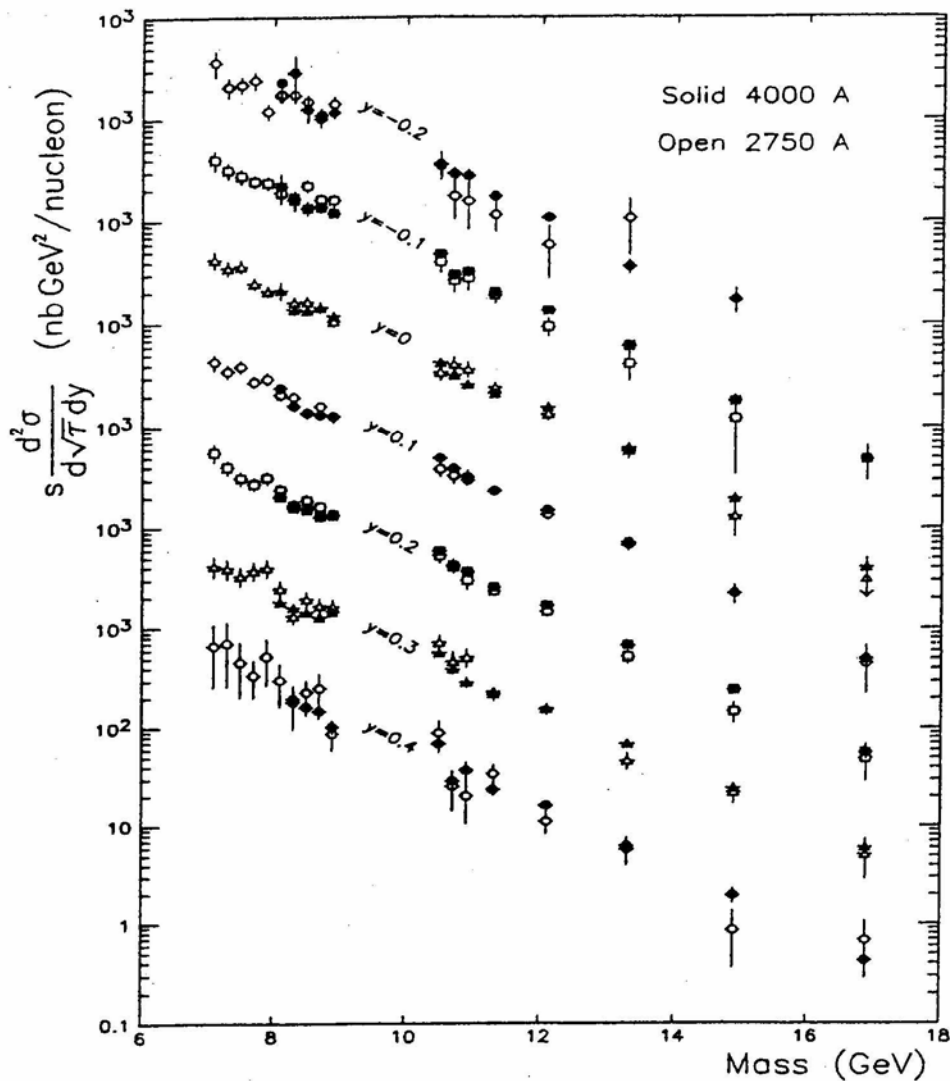


FIG. 10. Scaling form of the dimuon yield, excluding the  $\Upsilon$ 's, showing the agreement of the two sets in the region of overlap. The average of both sets is given in Table VIII.

$$\tau = \frac{M^2}{s} \rightarrow \frac{d}{d\sqrt{\tau}} = \sqrt{s} \frac{d}{dM}$$

$$\text{rapidity } y = \frac{1}{2} \log \frac{x_1}{x_2} \rightarrow \frac{d}{dy} = 2x_1 \frac{d}{dx_F}$$

$$M^3 \frac{d\sigma}{dM dx_F} = \sqrt{x_1 x_2} \frac{x_2}{2} s \frac{d\sigma}{d\sqrt{\tau} dy}$$

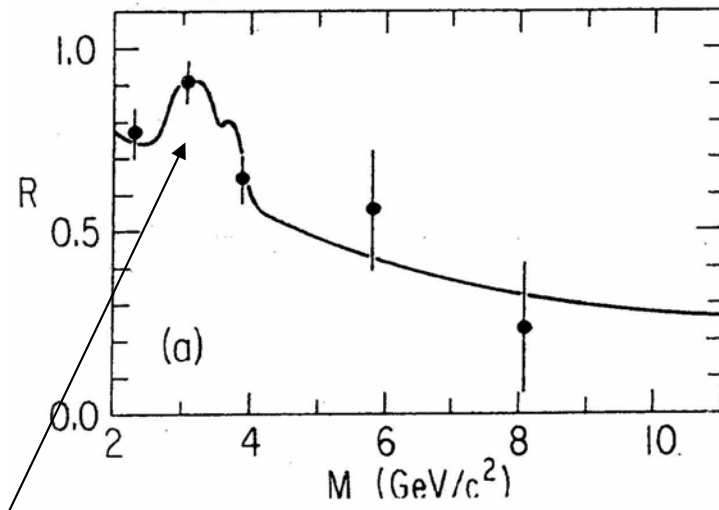


$$s \frac{d\sigma}{d\sqrt{\tau} dy} \sim \frac{1}{\sqrt{\tau}} \propto \frac{\sqrt{s}}{M}$$

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b) rapporto di carica di DY su nuclei isoscalari



$M \sim J/\psi \rightarrow$  meccanismo differente

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nuclei isoscalari  $\rightarrow n_u = n_d$  (ex.  $^{12}\text{C}$ )

$$1 \quad \frac{\tau \rightarrow 0}{\tau \rightarrow 1} \quad \frac{\pi^+ {}^{12}\text{C} \rightarrow \mu^+ \mu^- X}{\pi^- {}^{12}\text{C} \rightarrow \mu^+ \mu^- X} \quad \frac{\tau \rightarrow 1}{\tau \rightarrow 0} \quad \frac{1}{4}$$

perche'?

$\tau = x_1 x_2 \rightarrow 1$  valence area

$$\frac{\pi^+ (u\bar{d}) C(u_1..u_n d_1..d_m)}{\pi^- (d\bar{u}) C(u_1..u_n d_1..d_m)}$$

$$\sim \frac{e_d^2}{e_u^2} = \frac{1}{4}$$

$\tau = x_1 x_2 \rightarrow 0$  sea area

$$\frac{\pi^+ C(..\text{sea quarks}..)}{\pi^- C(..\text{sea quarks}..)}$$

$$\sim 1$$

meccanismo elementare

$$q\bar{q} \rightarrow l\bar{l}$$

c) rapporto DY tra  $\pi$  e N

N non ha antiquark di valenza  
per annichilazione

$$\rightarrow \frac{\pi^- N \rightarrow \mu^+ \mu^- X}{N N \rightarrow \mu^+ \mu^- X}$$



cresce con  $M = Q$

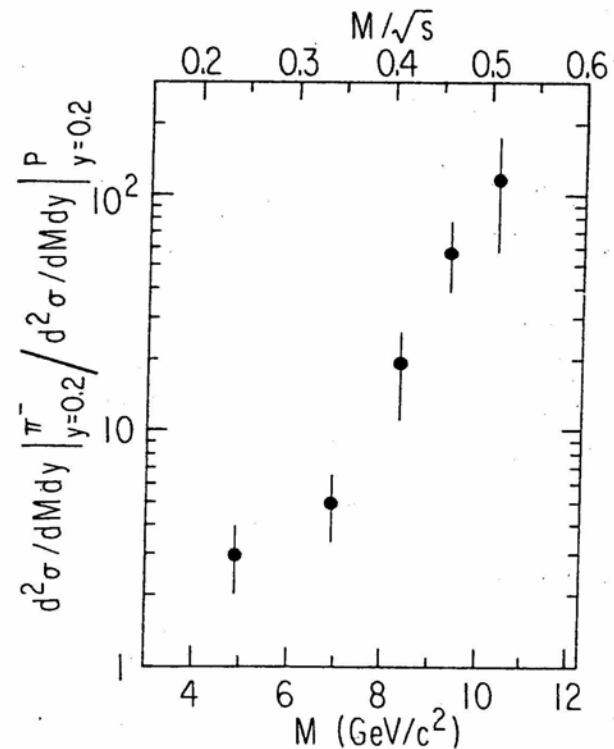


FIG. 1. The ratio of  $\pi^-$ -induced to proton-induced  $\mu$ -pair cross section at  $y_{c.m.} = 0.2$  as a function of mass. Proton data at 225 GeV/c has been calculated from the scaling observed in 200-, 300-, and 400-GeV/c data of Ref. 4.

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## QPM picture : discrepanze

$$\gamma^* \rightarrow \mu^+ \mu^- X$$

al crescere di  $Q^2 \equiv M^2$  si eccitano  
altre risonanze mesoniche vettoriali :

$$M \sim 3 \text{ GeV} \quad J/\psi \rightarrow \mu^+ \mu^- X$$

$$M \sim 9 \div 10.5 \text{ GeV} \quad Y \rightarrow \mu^+ \mu^- X$$

....

$$M \gtrsim 70 \text{ GeV} \quad Z^0, W^\pm$$



meccanismo elementare  
diverso da QPM picture (QED)

$$\text{Inoltre} \quad \frac{d\sigma(\text{exp.})}{d\sigma(\text{th.})} \sim K \sim 2 \div 3 !$$

$K$  factor = misura delle  
correzioni pQCD  
oltre il QPM

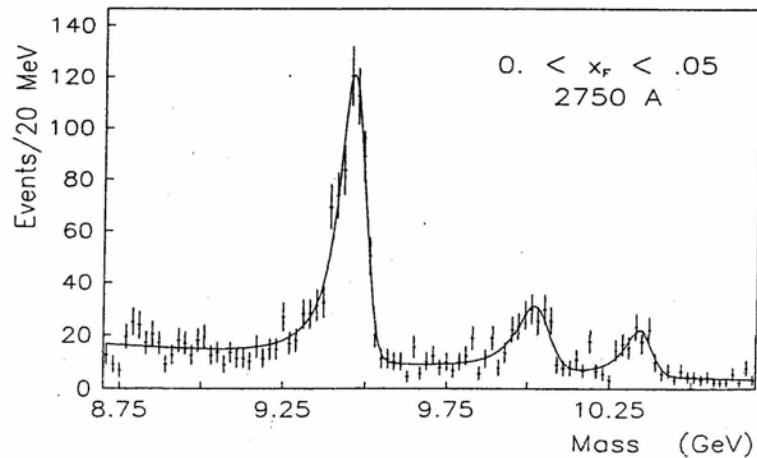
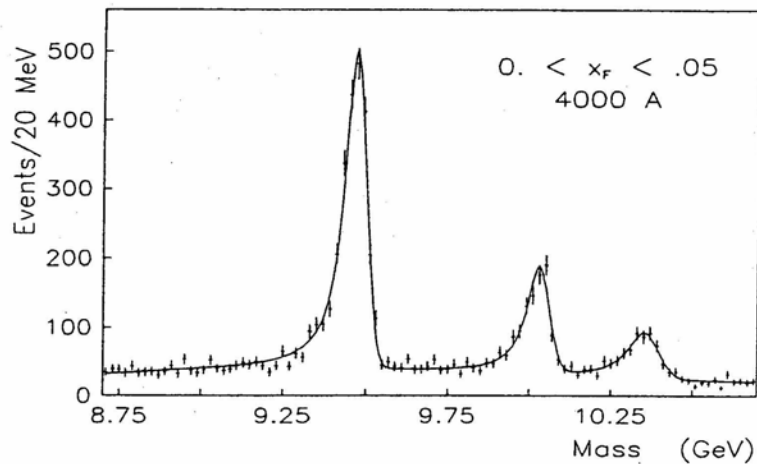


FIG. 16. The fit function used to extract  $\Upsilon$  cross sections superimposed on the raw mass spectrum of the two data sets.

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31-Ott-07

spettro della famiglia  $\Upsilon$  ( $b\bar{b}$ )

differente distribuzione in  $q_T$  della coppia di leptoni  $\rightarrow$  nuovo meccanismo ?

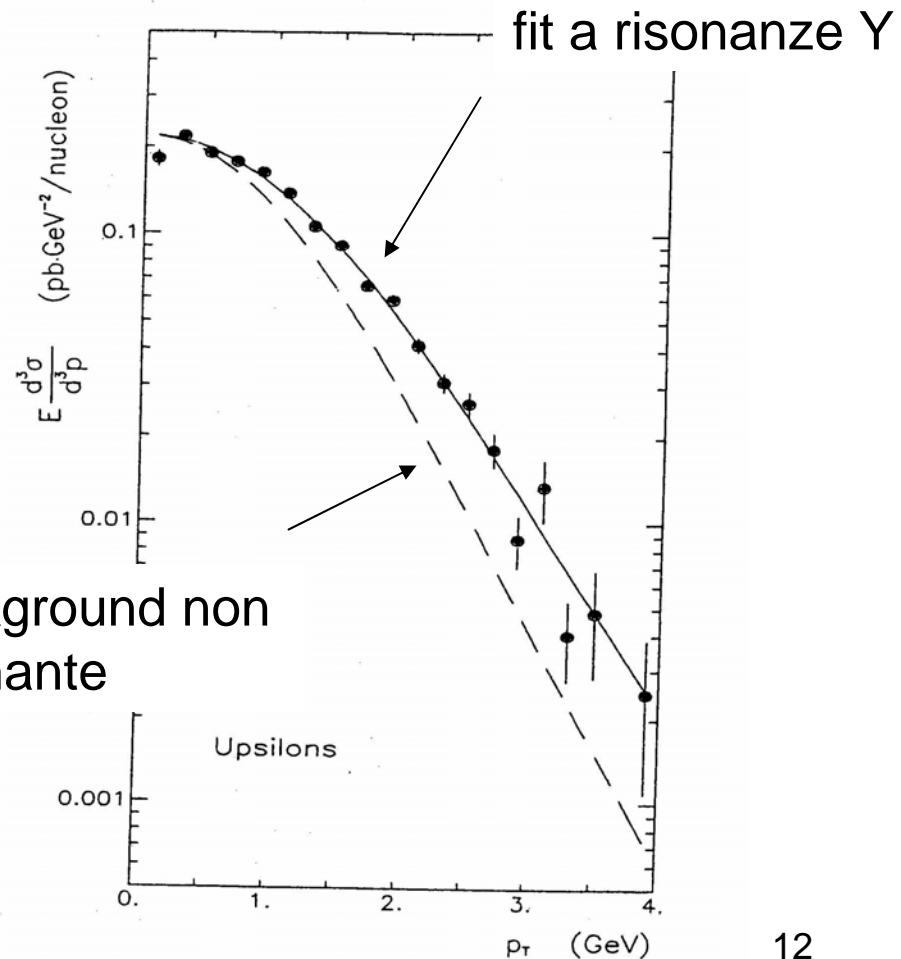
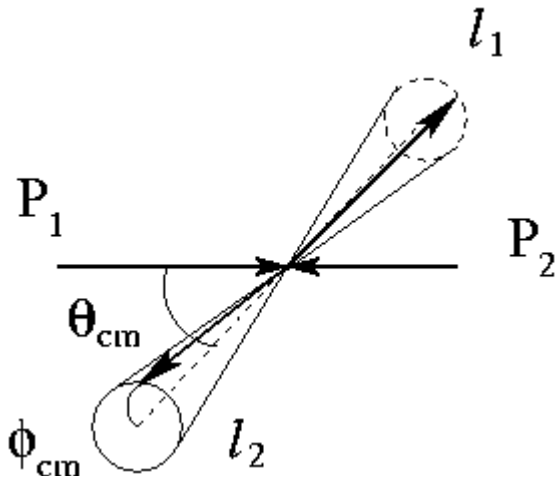


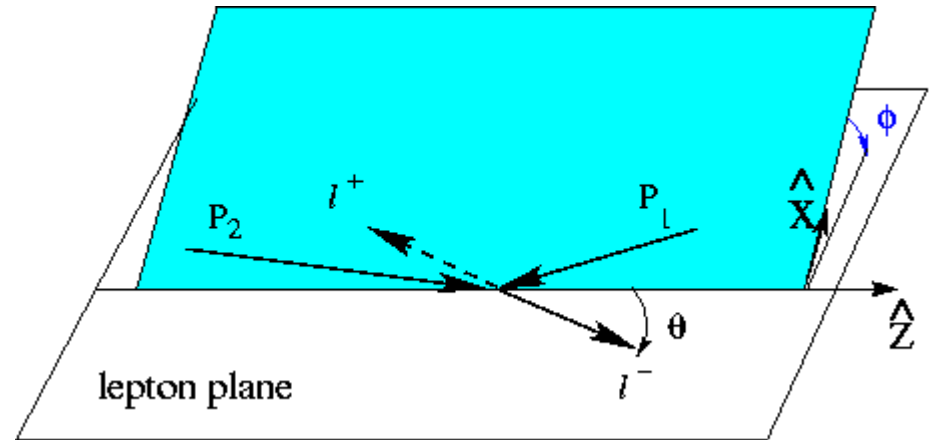
FIG. 19.  $p_T$  distribution for the sum of the three  $\Upsilon$ 's. The solid curve is a fit to the data, the dashed curve gives the

# Distribuzione angolare della coppia leptonica

c.m. degli adroni

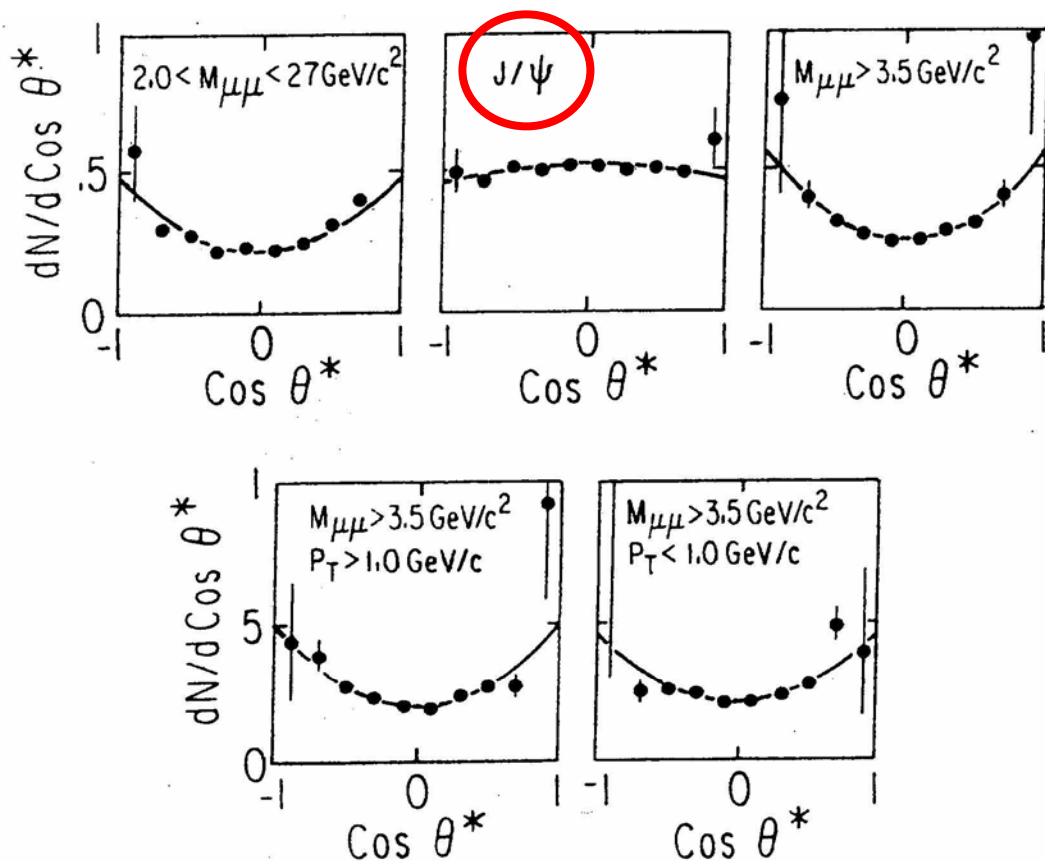


c.m. dei leptoni (Collins-Soper frame)



se  $q_T (l_1 l_2) \neq 0 \rightarrow$  direzione di annichilazione non nota  
Collins-Soper frame: asse z = direzione "media"

$$\begin{aligned} \underline{\frac{d\sigma}{d\Omega}} &= \frac{1}{N_c} \bar{\sum}_f \int_0^1 dx_1 dx_2 \phi_f(x_1) \frac{d\sigma^{el}(e^+e^- \rightarrow \mu^+\mu^-)}{d\Omega} e_f^2 \phi_{\bar{f}}(x_2) \delta(x_1 x_2 s - Q^2) \\ &= \frac{\alpha^2}{12Q^4} \underline{(1 + \cos^2 \theta)} \bar{\sum}_f e_f^2 \int_0^1 dx_1 dx_2 \phi_f(x_1) \phi_{\bar{f}}(x_2) \delta\left(x_1 x_2 \frac{s}{Q^2} - 1\right) \end{aligned}$$



distribuzione angolare  
 $\sim (1 + \cos^2 \theta)$   
 data dal processo elementare  
 $e^+e^- \rightarrow \mu^+\mu^-$

però sulla **risonanza J/ψ**  
 distribuzione piatta  
 $\rightarrow$  meccanismo diverso

da  $\gamma^* \rightarrow \mu^+\mu^-$   
 $J/\psi \rightarrow \mu^+\mu^-$  ?

FIG. 3. Helicity angular distributions in three different mass intervals. The  $M > 3.5 \text{ GeV}/c^2$  interval is also shown divided in two  $p_T$  intervals. The Collins-Soper angle ( $\theta^*$ ) is defined in the text.

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# Distribuzione angolare e discrepanze da QPM

$$\begin{aligned} \frac{d\sigma}{d^4q d\Omega} &= \frac{\alpha^2}{2sQ^4} L_{\mu\nu} W^{\mu\nu} \rightarrow \frac{d\sigma}{dq^0 d\mathbf{q} d\Omega} = \dots \rightarrow \frac{d\sigma}{dx_1 dx_2 d\mathbf{q}_T d\Omega} = \dots \\ &\rightarrow \frac{d\sigma}{dx_F d\tau d\mathbf{q}_T d\Omega} = \dots \rightarrow \frac{d\sigma}{dx_F dM d\mathbf{q}_T d\Omega} = \dots \end{aligned}$$

- $W^{\mu\nu}$  :
- 3 vettori indipendenti  $P_1, P_2, q$
  - conservazione parità, no polarizzazione  $\rightarrow$  struttura simmetrica
  - base tensoriale:  $b_1 = g^{\mu\nu}$ ,  $b_2 = q^\mu q^\nu$ ,  $b_3 = P_1^\mu P_1^\nu$ ,  
 $b_4 = (P_1^\mu q^\nu + P_1^\nu q^\mu)$ ,  $b_5 = (P_2^\mu q^\nu + P_2^\nu q^\mu)$ ,  
 $b_6 = (P_1^\mu P_2^\nu + P_1^\nu P_2^\mu)$ ,  $b_7 = P_2^\mu P_2^\nu$
  - tensore adronico  $W^{\mu\nu} = \sum_i c_i (q^2, P_1 \cdot q, P_2 \cdot q, P_1 \cdot P_2) b_i$
  - conservazione della corrente  $q_\mu W^{\mu\nu} = W^{\mu\nu} q_\nu = 0$   
 $\rightarrow$  **3 relazioni**
  - gauge-invarianza QED  $\rightarrow$  termini lineari in  $q^\mu = 0$

$$W^{\mu\nu} = -W_1 g^{\mu\nu} + W_2 \frac{P_1^\mu P_1^\nu}{M^2} + \frac{W_3}{2M^2} (P_1^\mu P_2^\nu + P_1^\nu P_2^\mu) + W_4 \frac{P_2^\mu P_2^\nu}{M^2}$$

# Distribuzione angolare (continua)

$$\frac{d\sigma}{dx_F d\mathbf{q}_T dM d\Omega} \propto \left[ \underset{\substack{\uparrow \\ \text{trasversa}}}{W_T} (1 + \cos^2 \theta_{cm}) + \underset{\substack{\uparrow \\ \text{longitudinale}}}{W_L} \sin^2 \theta_{cm} \right]$$

polarizz. del  $\gamma^*$

trasversa

longitudinale

$$+ \underset{\substack{\uparrow \\ \text{1 spin flip}}}{W_{\uparrow}} \sin 2\theta_{cm} \cos \phi_{cm} + \underset{\substack{\uparrow\uparrow \\ \text{2 spin flip}}}{W_{\uparrow\uparrow}} \sin^2 \theta_{cm} \cos 2\phi_{cm}]$$

elementi non diagonali  
della matrice densita`  
di elicit` della coppia  $H_1 H_2$

1 spin flip

2 spin flip



$$\left[ \frac{d\sigma}{dx_F dM d\mathbf{q}_T} \right]^{-1} \frac{d\sigma}{dx_F dM d\mathbf{q}_T d \cos \theta_{cm}} \propto 1 + \alpha \cos^2 \theta_{cm} , \quad \alpha = \frac{W_T - W_L}{W_T + W_L}$$

on-shell (anti)quark  
con spin 1/2

$\alpha = 1 \Rightarrow W_L = 0$

$\alpha \neq 1 \Rightarrow \left\{ \begin{array}{l} \text{frame differente con } q_T \neq 0 \\ \text{(Collis-Soper frame } \rightarrow \alpha = 0.85) \\ \text{meccanismo differente } \rightarrow q_T \neq 0 \end{array} \right.$



$$p p \rightarrow \mu^+ \mu^- X$$

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$M > 4 \text{ GeV}$  (no  $J/\psi$ )

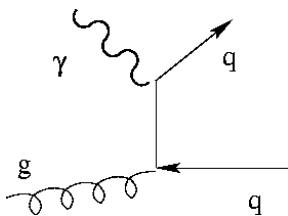
$$d\sigma \sim 1 + \alpha \cos^2 \theta_{cm}$$

$$\alpha = \frac{W_T - W_L}{W_T + W_L}$$

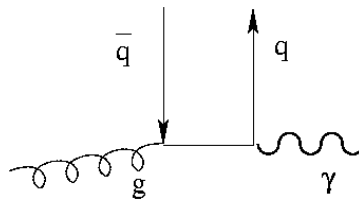
$\alpha \sim 1 \rightarrow W_T$  dominante

Ma sulla **risonanza  $J/\psi$**  meccanismo puo` essere diverso.

DY appartiene a classe piu` generale di processi  $A+B \rightarrow C+X$  dove meccanismo elementare puo` essere piu` complicato:



**QCD Compton**



**$\gamma - g$  fusion**

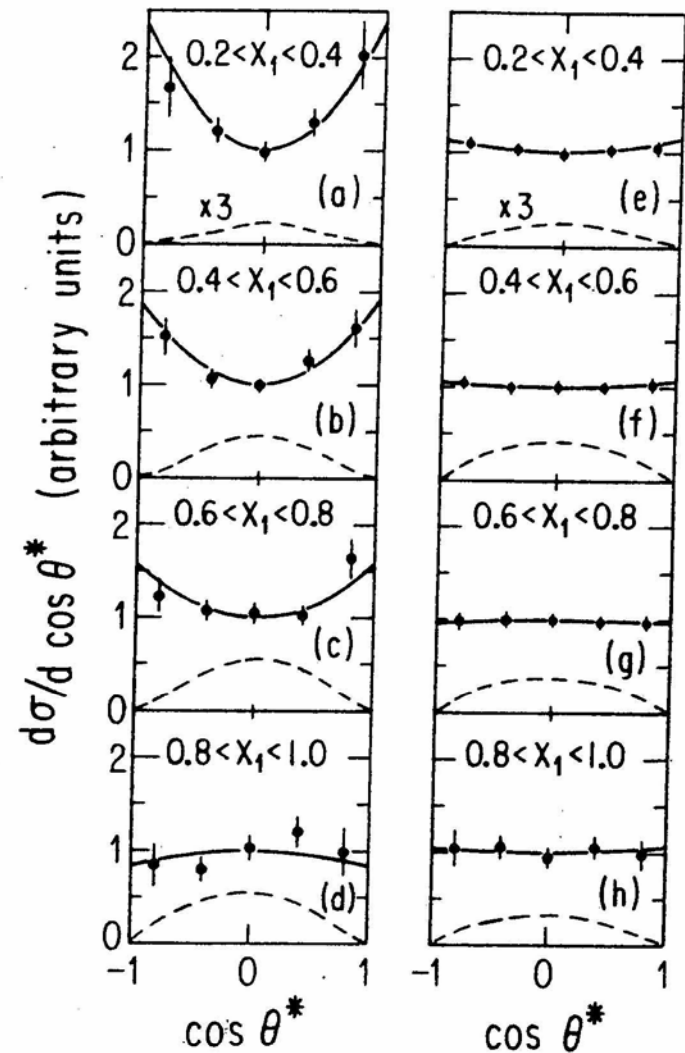


FIG. 1.  $d\sigma/d \cos \theta^*$  in the  $t$ -channel helicity frame for various  $x_1$  intervals. (a)–(d) Results for the mass continuum with  $M > 4 \text{ GeV}$ ; (e)–(h) results for the  $J/\psi$  resonance in the same  $x_1$  intervals. Data are integrated over  $P_T$ . The dashed curve shows the variation of detection efficiency with  $\cos \theta^*$ . The same arbitrary efficiency

correzioni radiative pQCD  $\rightarrow$  dipendenza  $q_T$   
 $q$  e' off-shell  $\rightarrow W_L \neq 0 \rightarrow \alpha(q_T) < 1$

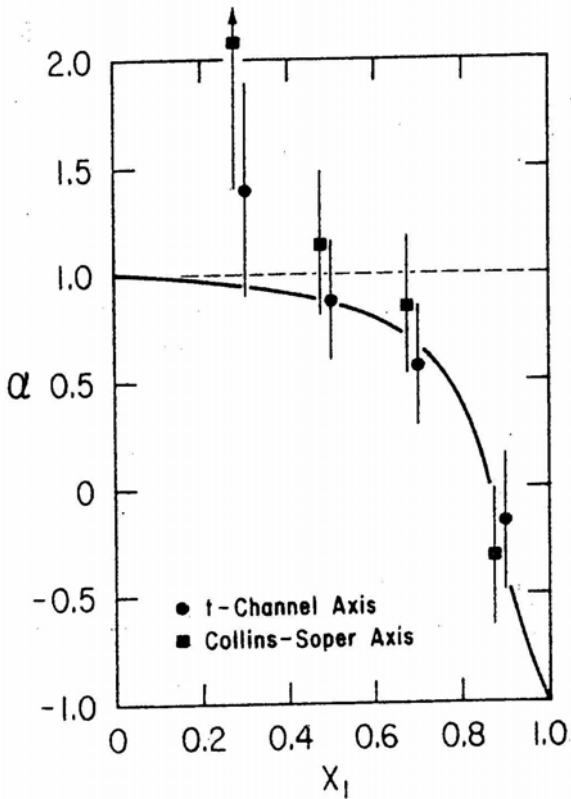


FIG. 2. The dependence of  $\alpha$  on  $x_1$  for data with  $M > 4$  GeV. The dashed line is the expected result for the naive Drell-Yan model. The solid curve is the QCD prediction of Berger and Brodsky (Ref. 8).

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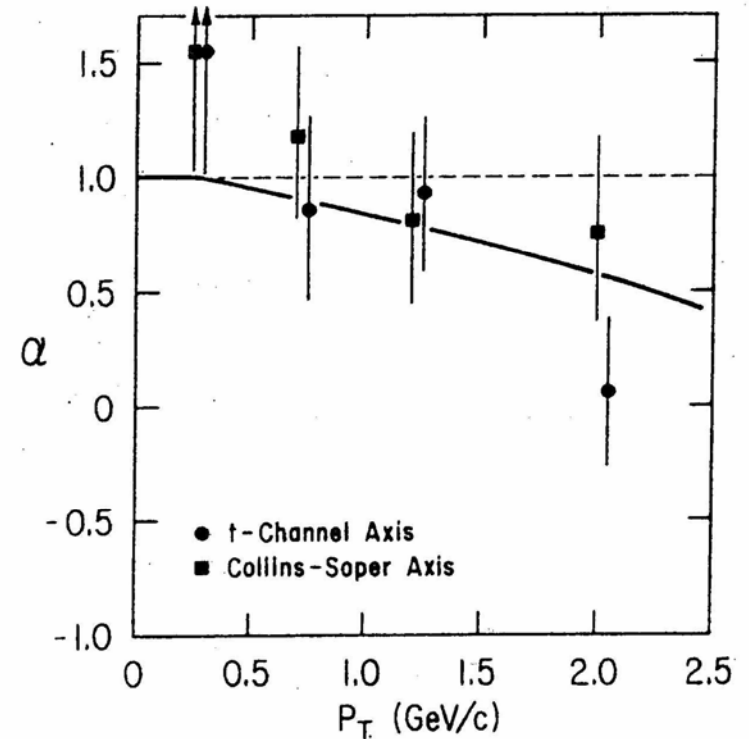


FIG. 3. The dependence of  $\alpha$  on  $P_T$  for data with  $M > 4$  GeV. The smooth curve is the QCD prediction from Kajantie *et al.* (Ref. 7). These authors consider only  $x_F = 0$  while the data are integrated over  $x_F$ .