# Schema

- riassunto precedente lezione

- estensione del QPM a reazioni di DIS con (anti)neutrino: teoria elettrodebole

- correnti cariche e neutre

- funzioni di struttura e distribuzioni di (anti)quark di valenza e del "mare"; partoni senza carica: i gluoni

- regole di somma

- definizione del Quark Parton Model (QPM); interpretazione e giustificazione intuitiva delle approssimazioni di base del modello

- scattering su costituenti elementari puntiformi del bersaglio: calcolo delle funzioni di struttura e predizione dello scaling

- relazione di Callan-Gross e dominanza della risposta trasversa per $Q^2 \to \infty$ : i partoni sono fermioni ;
  identificazione tra partoni e i quark

- estrazione dai dati di DIS su N delle distribuzioni di quark :
  predominanza dei quark di valenza $u_p$ e $d_n$ per $x \to 1$
  predominanza dei quark del "mare" per $x \to 0$

- concetto del quark costituente: $m_{\rm cq} \sim m_N/3$ e $F_2^{\rm cq} \sim \delta(x - 1/3)$
  oltre a correzioni dovute a moto di Fermi e pQCD

- approssimazione di usare solo 2 flavors, $u(x)$ e $d(x)$, insufficiente : nello spettro mesonico e barionico evidenza del terzo flavor $s(x)$
  1974: scoperta della risonanza $J/\psi$ , interpretabile come stato $c\bar{c}$

- osservazione di processi deboli con cambio di stranezza : $K^{\pm} \to \mu^{\pm} \nu$

- osservazione di correnti “neutre” in processi $\nu/e^{-} + p \to \nu/e^{-} + p$

Unificazione delle teorie dell’interazione elettromagnetica e debole

prime idee ~ ’60 (Feynmann, Gell-Mann, Glashow, Weinberg..)

- Meccanismo di Goldstone / Higgs
- Teorie non-abeliane rinormalizzabili

$\boxed{\text{Settore elettrodebole del Modello Standard}}$



Nobel Glashow, Weinberg, Salam

# Genesi del Modello Standard elettrodebole

• prime ipotesi (Feynmann Gell-Mann, '58 ; Glashow, '61) :
interazioni deboli cariche ($W^{\pm}$) legate a interazione e.m. isovettoriale ($\gamma$) da rotazione di isospin ; i leptoni e i quark sinistrorsi (left-handed) sono quindi organizzati in doppietti di isospin debole $T$ secondo la simmetria $SU(2)_T$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \quad \begin{pmatrix} ? \\ s_\theta \end{pmatrix}_L$$

dove $d_\theta = d \cos \theta_C + s \sin \theta_C$ ; $s_\theta = - d \sin \theta_C + s \cos \theta_C$ $\theta_C$ **angolo di Cabibbo**
$d,s$ autostati di interazione forte
$d_\theta,,s_\theta$ autostati di interazione debole

Commenti: • necessita` di un quarto flavor, il quark charm (scoperto nel '74)

• transizioni left-handed tra $\nu$ e $e^-/\mu^-$ , tra quarks, via $W^{\pm}$
$d_\theta$ , $s_\theta$ spiegano reazioni del tipo $K^{\pm} \to \mu^{\pm} \nu$

## Genesi….. (continua)

- ipotesi della carica debole $Y$ (Glashow, '61) : ulteriore struttura $U(1)_Y$

 i quark hanno carica e.m. $e_f = Y + ½ T_3$

 carica debole $Y = B + S$

 riepilogo dei numeri quantici

| | $B$ | $S$ | $Y$ | $T_3$ | $e_f$ |
|---|---|---|---|---|---|
| $u$ | $\frac{1}{3}$ | $0$ | $\frac{1}{3}$ | $\frac{1}{2}$ | $\frac{2}{3}$ |
| $d$ | $\frac{1}{3}$ | $0$ | $\frac{1}{3}$ | $-\frac{1}{2}$ | $-\frac{1}{3}$ |
| $s$ | $\frac{1}{3}$ | $-1$ | $-\frac{2}{3}$ | $0$ | $-\frac{1}{3}$ |

- teoria elettrodebole: i fermioni interagiscono attraverso i bosoni di gauge ***W***,*B*

$$\mathcal{L}_{\text{weak}} = g\bar{\psi}\,\frac{\mathbf{T}}{2}\,\psi\,\cdot\mathbf{W} + g'\bar{\psi}\,Y\,\psi\,B^0$$

***g*,*g*' couplings incognite**

invarianza per $SU(2)_T \otimes U(1)_Y$ e fermioni / bosoni di gauge massless
$\Rightarrow$ teoria rinormalizzabile non-abeliana, perche` $[W_i,W_j]=i\varepsilon_{ijk}\,W_k$

Ma $m_W \neq 0$ ! Altrimenti si vedrebbe in $\beta$ / $K$ decays

# Genesi…. (continua)

- ('t Hooft, '71) : teorie non-abeliane rimangono rinormalizzabili se masse sono generate dinamicamente da rottura spontanea della simmetria di gauge (meccanismi di Goldstone, '64; Higgs, '64…)

- rottura spontanea della simmetria implica $\boldsymbol{W}, B \rightarrow W^{\pm}, Z^{0}, A$
  in particolare

$$
\begin{aligned}
A &= \cos\theta_W B + \sin\theta_W W_3 \\
Z^0 &= -\sin\theta_W B + \cos\theta_W W_3
\end{aligned}
$$

**$\theta_W$ angolo di Weinberg**

$$
\mathcal{L}_{\text{weak}} = g\bar{\psi}\,\frac{\mathbf{T}}{2}\,\psi\,\cdot\mathbf{W} + g'\bar{\psi}\,Y\,\psi\,B^0
$$

$$
\begin{aligned}
e_f &= Y + \frac{T_3}{2} \\
g' &= g\tan\theta_W
\end{aligned}
$$

$$
\mathcal{L}_{\text{weak}} = \frac{g}{\sqrt{2}}\bar{\psi}\,\frac{T^{\mp}}{2}\,\psi\,\cdot W^{\pm}
$$

$$
+g\sin\theta_W\bar{\psi}\,e_f\,\psi\,A + \frac{g}{\cos\theta_W}\,\bar{\psi}\left(\frac{T_3}{2} - e_f\sin^2\theta_W\right)\psi\,Z^0
$$

$g\sin\theta_W \equiv e$

$$
+e\bar{\psi}\,e_f\,\psi\,A + \frac{2e}{2\sin\theta_W\cos\theta_W}\,\bar{\psi}\left(\frac{T_3}{2} - e_f\sin^2\theta_W\right)\psi\,Z^0
$$

**corrente e.m.** $\rightarrow$ ***A*** $\equiv \gamma$

**correnti debole neutre**

# Rottura spontanea della simmetria : meccanismo di Goldstone

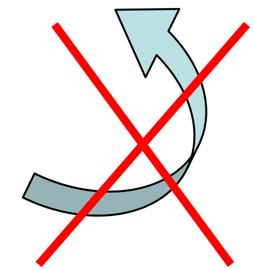
Esempio: teoria di campo per particella scalare $\phi$

$$\mathcal{L}(\phi) = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi)$$

$$V(\phi) = \mu^2 \phi^2 + \lambda \phi^4 \quad , \qquad \lambda > 0$$

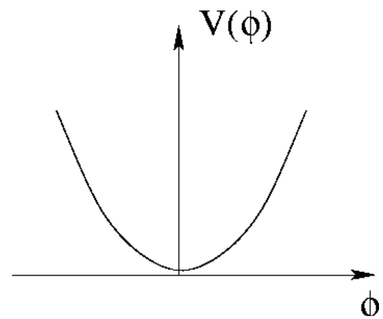
simmetria

$$\mathcal{L}(-\phi) = \mathcal{L}(\phi)$$

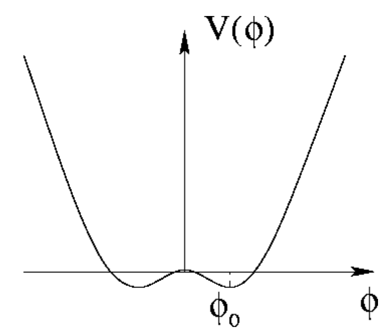


vuoto $\equiv \dfrac{\partial V}{\partial \phi} = 0$

$\mu^2 > 0 \longrightarrow \phi_0 = 0$




$\mu^2 < 0 \longrightarrow \phi_0 = \pm\sqrt{\dfrac{-\mu^2}{2\lambda}}$




nuovo campo $\phi' = \phi - \phi_0$ ; $\qquad \phi'_0 = 0$

$$V(\phi') = \mu^2 (\phi' + \phi_0)^2 + \lambda (\phi' + \phi_0)^4 = -2\mu^2 \phi'^2 + o(\phi'^3)$$

$$m_{\phi'} = \sqrt{-4\mu^2}$$

$$\mathcal{L}(\phi') = \frac{1}{2}\left(\partial_\mu \phi'\right)^2 + 2\mu^2 \phi'^2 + .... = \underbrace{\frac{1}{2}\left(\partial_\mu \phi'\right)^2 - \frac{1}{2} m^2 \phi'^2}_{\mathcal{L}_{free}(\phi')} + ....$$

Riepilogo settore elettrodebole del Modello Standard

=

teoria non-abeliana rinormalizzabile
delle interazioni e.m. e debole unificate
in simmetria di gauge $SU(2)_T \otimes U(1)_Y$

Predizioni :

- necessita` di un quarto flavor, il quark charm
- 4 bosoni di gauge: $\gamma$ , $W^{\pm}$ , $Z^0$
- $\gamma$ accoppiato a corrente conservata $\rightarrow$ massless (ok con QED)
- rapporto $\frac{\textbf{weak strength}}{\textbf{e.m. strength}}$ sperimentale si spiega se il coupling

  costante di Fermi $G_F = \frac{e^2}{4\sqrt{2} M_W^2 \sin\theta_W^2}$ con $M_W \sim 75$ GeV

  risulta inoltre $M_W^2 = M_Z^2 \cos^2\theta_W \rightarrow M_Z \geq M_W$

- correnti deboli cariche: $W^{\pm}$ producono transizioni

  $\nu \leftrightarrow e^-$ , $u \leftrightarrow d$ , $u \leftrightarrow s$ (cambio di stranezza) , ….

- correnti deboli neutre: $\nu + p \rightarrow \nu + p$ , …

Conferme sperimentali:

- quark charm osservato nella risonanza $J / \psi$ (BNL, 1974)

- bosoni di gauge $W^{\pm}$ , $Z^0$ osservati nell'exp. UA1 (CERN, 1983)

Nobel Rubbia, van der Meer

- dal Particle Data Group: $M_W = 80.22 \quad \pm 0.0026$ GeV

$$M_Z = 91.187 \pm 0.007 \text{ GeV}$$
$$\sin^2 \theta_W (M_Z) = 0.2319 \pm 0.0005$$

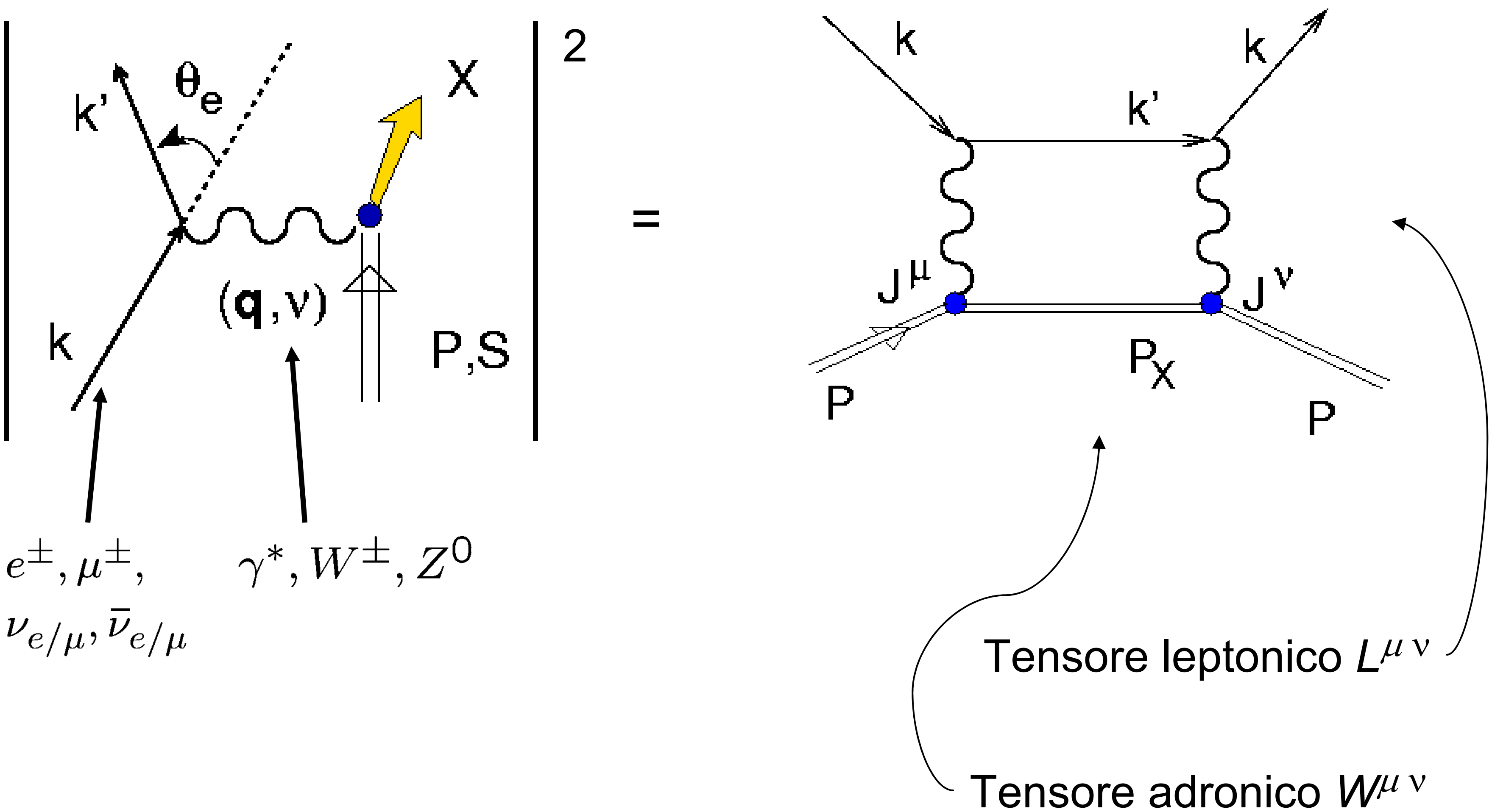
- si spiegano correnti deboli cariche con cambio di stranezza $K^{\pm} \to \mu^{\pm} \nu$

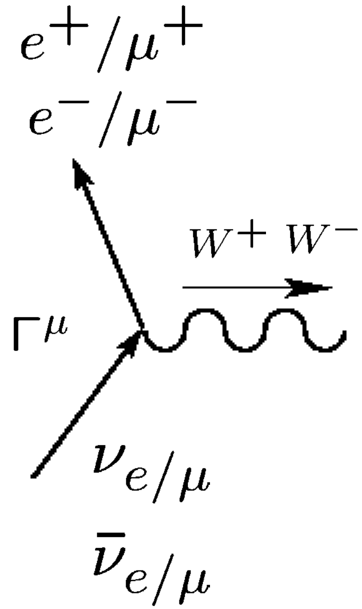
- correnti deboli neutre osservate al CERN nel 1973

Benvenuti *et al*., PRL **32** 800 (74)
Hasert et al., PL **B46** 138 (73)

- correnti deboli neutre non cambiano la stranezza (no $K^0 \to \mu^+ \mu^-$ )
  cancellazioni seguono da $m_q \ll M_W$ e da esistenza di quark $c$ con
  mixing $c \leftrightarrow - d \sin \theta_W + s \cos \theta_W$

# Deep Inelastic Scattering






# Tensore leptonico

interazione e.m. ($e^-$ / $\mu^-$ left-handed) $\rightarrow$ scambio di $\gamma$

$$\Gamma^\mu = e\gamma^\mu$$

fascio di neutrini (left-handed) $\rightarrow$ scambio di $W^+$

**(ma anche in reazioni inverse del tipo** $e^+/\mu^+ \rightarrow \bar{\nu}_{e/\mu}$ **)**

$$\Gamma^\mu = \frac{e}{2\sqrt{2}\sin\theta_W}\frac{T_3(=+1)}{2}\gamma^\mu(1-\gamma_5) \qquad V - A$$

fascio di antineutrini (right-handed) $\rightarrow$ scambio di $W^-$

**(ma anche in reazioni inverse del tipo** $e^-/\mu^- \rightarrow \nu_{e/\mu}$ **)**

$$\Gamma^\mu = \frac{e}{2\sqrt{2}\sin\theta_W}\frac{T_3(=-1)}{2}\gamma^\mu(1+\gamma_5) \qquad V + A$$

$$\begin{aligned} L^{\mu\nu} &= \mathrm{Tr}\left[\Gamma^\mu \, \not{k}' \, \Gamma^\nu \, \not{k}\right] \\ &= \frac{e^2}{8\sin^2\theta_W}\frac{1}{4}\Big\{\underbrace{\mathrm{Tr}\left[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta\right]}_{V\text{-}V} k'_\alpha k_\beta + \underbrace{\mathrm{Tr}\left[\gamma^\mu\gamma_5\gamma^\alpha\gamma^\nu\gamma_5\gamma^\beta\right]}_{A\text{-}A} k'_\alpha k_\beta \\ &\mp \mathrm{Tr}\left[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma_5\gamma^\beta\right] k'_\alpha k_\beta \mp \mathrm{Tr}\left[\gamma^\mu\gamma_5\gamma^\alpha\gamma^\nu\gamma^\beta\right] k'_\alpha k_\beta\Big\} \longleftarrow V\text{-}A \end{aligned}$$

## Tensore leptonico (continua)

$$\begin{aligned}\mathrm{Tr}\left[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta\right] &= \mathrm{Tr}\left[\gamma^\mu\gamma_5\gamma^\alpha\gamma^\nu\gamma_5\gamma^\beta\right] = 4(g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha} - g^{\mu\nu}g^{\alpha\beta}) \\ \mathrm{Tr}\left[\gamma^\mu\gamma_5\gamma^\alpha\gamma^\nu\gamma^\beta\right] &= \mathrm{Tr}\left[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma_5\gamma^\beta\right] = 4i\epsilon^{\mu\nu\alpha\beta}\end{aligned}$$

$$L^{\mu\nu} = \frac{e^2}{8\sin^2\theta_W}\,2\left(k'^\mu k^\nu + k'^\nu k^\mu - k\cdot k' g^{\mu\nu} \mp i\epsilon^{\mu\nu\alpha\beta}k'_\alpha k_\beta\right)$$

$$\begin{aligned}k' &= k + q \\ k^2 &= k'^2 = m_e^2 \sim 0\end{aligned}$$

**e.m.**

$$= \frac{e^2}{8\sin^2\theta_W}\left[\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)Q^2 + 4\tilde{p}^\mu\tilde{p}^\nu \pm 2i\epsilon^{\mu\nu\alpha\beta}k_\alpha q_\beta\right]$$

$$\equiv \frac{e^2}{8\sin^2\theta_W}\left[\quad L^{\mu\nu(S)} \quad \pm \quad L^{\mu\nu(A)} \quad\right]$$

$$\tilde{p} = p - \frac{p\cdot q}{q^2}q \overset{elastic}{=} p + \frac{q}{2}$$

parte antisimmetrica del tensore e` memoria dell'interferenza tra corrente debole vettoriale ed assiale

# Sezione d'urto

flusso

$$d\sigma = \frac{1}{\mathcal{F}} |\mathcal{M}|^2 \, dR \qquad \mathcal{F} = 4\sqrt{(P\cdot k)^2 - P^2k^2} \overset{\mathrm{TRF}}{=} 4ME$$

spazio fasi

$$dR = (2\pi)^4 \delta(P + q - P_X) \, \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} \, \frac{d\mathbf{k}'}{(2\pi)^3 2E'}$$

$$\frac{E' dE' d\Omega}{16\pi^3}$$

ampiezza di scattering

$$|\mathcal{M}|^2 = \frac{1}{2}\sum_{S,X} \left| \bar{u}(k')\Gamma_\mu u(k) \, \frac{1}{Q^2 + M_W^2} \, \frac{e}{2\sqrt{2}\sin\theta_W} \, \frac{T_3}{2} \, \langle P_X | J^\mu(0) | P, S\rangle \right|^2$$

$$= \frac{e^4}{64\sin^4\theta_W} \, \frac{1}{(Q^2+M_W^2)^2} \, \frac{1}{4} \, L_{\mu\nu}\, H^{\mu\nu}$$

$$\frac{d\sigma}{dE' d\Omega} = \frac{e^4}{64\sin^4\theta_W} \, \frac{E'}{E} \, \frac{1}{4M8\pi^2} \, \frac{1}{(Q^2+M_W^2)^2} \, L_{\mu\nu}$$

$$\times \frac{1}{2\pi} \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} \, (2\pi)^4 \, \delta(P + q - P_X) \, H^{\mu\nu}$$

$$= \frac{\alpha^2}{64\sin^4\theta_W} \, \frac{E'}{E} \, \frac{1}{(Q^2+M_W^2)^2} \, \frac{1}{2M} \, L_{\mu\nu}\, 2MW^{\mu\nu}$$

N.B. Il propagatore del bosone vettore di gauge si approssima

$$\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) \frac{1}{q^2 - M_W^2} \sim -\frac{g^{\mu\nu}}{q^2 - M_W^2}$$

perche` $q^\mu q^\nu \sim \left(\frac{m_e}{M_W}\right)^2 \sim 0$

# Tensore adronico

- 2 vettori indipendenti *P, q*
- base tensoriale: $b_1 = g^{\mu\nu}$, $b_2 = q^\mu q^\nu$, $b_3 = P^\mu P^\nu$,
  $b_4 = (P^\mu q^\nu + P^\nu q^\mu)$ , $b_5 = (P^\mu q^\nu - P^\nu q^\mu)$,
  $b_6 = \varepsilon_{\mu\nu\rho\sigma}\, q^\rho P^\sigma$
- tensore adronico $W^{\mu\nu} = \Sigma_i\, c_i\,(q^2, P\cdot q)\ b_i$
  Hermiticity $\rightarrow$ $c_i$ sono reali
- invarianza per time-reversal,
  conservazione della corrente $q_\mu W^{\mu\nu} = W^{\mu\nu} q_\nu = 0$
- sistema lineare con $c_5=0$ , $c_1$ e $c_3$ dipendenti da $c_2$ e $c_4$
- Risultato finale :

$$W^{\mu\nu} = \underbrace{\underbrace{\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) c_2(q^2, P\cdot q)}_{W_1} + \underbrace{\frac{\tilde{P}^\mu \tilde{P}^\nu}{M^2}\, c_4(q^2, P\cdot q)}_{W_2}}_{W^{(S)\mu\nu}} \underbrace{- i\epsilon^{\mu\nu\rho\sigma}\frac{P_\rho q_\sigma}{M^2}\, c_6(q^2, P\cdot q)}_{\boxed{W_3}\ \to\ \boxed{W^{(A)\mu\nu}}}$$

violazione di parita`

$$L_{\mu\nu} = L_{\mu\nu}^{(S)} \pm L_{\mu\nu}^{(A)}$$
$$W^{\mu\nu} = W^{(S)\mu\nu} + W^{(A)\mu\nu}$$

$$\Rightarrow \quad L_{\mu\nu} W^{\mu\nu} = L_{\mu\nu}^{(S)} W^{(S)\mu\nu} \pm L_{\mu\nu}^{(A)} W^{(A)\mu\nu}$$

N.B. le costanti di accoppiamento sono esplicitamente fattorizzate

$$L^{\mu\nu} = \frac{e^2}{8\sin^2\theta_W}\left[\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) Q^2 + 4\tilde{p}^\mu \tilde{p}^\nu \pm 2i\epsilon^{\mu\nu\alpha\beta} k_\alpha q_\beta\right]$$

$$W^{\mu\nu} = \frac{e^2}{8\sin^2\theta_W}\left[\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1 + \frac{\tilde{P}^\mu \tilde{P}^\nu}{M^2} W_2 - i\epsilon^{\mu\nu\rho\sigma}\frac{P_\rho q_\sigma}{2M^2} W_3\right]$$

e.m.

$$L_{\mu\nu} W^{\mu\nu} \stackrel{TRF}{=} \frac{e^4}{64\sin^4\theta_W} 4EE' \cos^2\frac{\theta_e}{2} \times \left[2W_1 \tan^2\frac{\theta_e}{2} + W_2 \pm \frac{E+E'}{M}\tan^2\frac{\theta_e}{2} W_3\right]$$

Interferenza *VA* $\rightarrow$ antisimmetria tra leptoni / antileptoni

# Sezione d'urto

$$
\begin{aligned}
\frac{d\sigma^{\nu/\bar\nu}}{dE'd\Omega} &= \frac{\alpha^2}{64\sin^4\theta_W}\,\frac{E'}{E}\,\frac{1}{(Q^2+M_W^2)^2}\,L_{\mu\nu}\,W^{\mu\nu} \\
&= \frac{\alpha^2}{64\sin^4\theta_W}\,\frac{E'}{E}\,\frac{1}{(Q^2+M_W^2)^2}\,4EE'\cos^2\frac{\theta_e}{2} \\
&\quad \times\left[2W_1\tan^2\frac{\theta_e}{2} + W_2 \pm \frac{E+E'}{M}\,\tan^2\frac{\theta_e}{2}\,W_3\right]
\end{aligned}
$$

**Limite DIS :**

$$
W_1 \to \frac{F_1}{M} \qquad W_2 \to \frac{F_2}{\nu} \qquad W_3 \to \frac{F_3}{\nu}
$$

$$
\begin{aligned}
&= \frac{G_F^2}{8\pi^2}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 E'^2\cos^2\frac{\theta_e}{2} \\
&\quad \times\left[2\frac{F_1}{M}\tan^2\frac{\theta_e}{2} + \frac{F_2}{\nu} \pm \frac{E+E'}{M\nu}\,\tan^2\frac{\theta_e}{2}\,F_3\right]
\end{aligned}
$$

$$
G_F = \frac{e^2}{4\sqrt{2}M_W^2\sin^2\theta_W}
$$

## Sezione d'urto (continua)

$$
\begin{aligned}
Q^2 &= 4EE' \sin^2 \frac{\theta_e}{2} \\
dQ^2 &= 4EE' 2 \sin \frac{\theta_e}{2} \cos \frac{\theta_e}{2} \frac{d\theta_e}{2} = \frac{EE'}{\pi} \sin \theta_e d\theta_e d\phi = \frac{EE'}{\pi} d\Omega \\
dx_B &= \frac{dQ^2}{2M\nu}
\end{aligned}
$$

$$
\begin{aligned}
dE' &= d(E+\nu) = d\nu \\
dy &= d\left(\frac{\nu}{E}\right) = \frac{d\nu}{E}
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\sigma^{\nu/\bar{\nu}}}{dx_B dy} &= \frac{dE' d\Omega}{dx_B dy} \frac{d\sigma}{dE' d\Omega} = \frac{\pi 2 M\nu}{E'} \frac{d\sigma}{dE' d\Omega} \\
&= \frac{G_F^2}{4\pi} \left(\frac{M_W^2}{Q^2 + M_W^2}\right)^2 MyEE' \\
&\quad \times \left[\frac{F_1}{M} \frac{Q^2}{2EE'} + \frac{F_2}{\nu} \left(1 - \frac{Q^2}{4EE'}\right) \pm \frac{E+E'}{M\nu} \frac{Q^2}{4EE'} F_3\right] \\
&= \frac{G_F^2}{4\pi} \left(\frac{M_W^2}{Q^2 + M_W^2}\right)^2 ME \left[F_1 x_B y^2 + F_2 \left(\frac{E'}{E} - \frac{Mx_B y}{2E}\right) \pm x_B y \frac{E+E'}{2E} F_3\right] \\
&= \frac{G_F^2}{4\pi} \left(\frac{M_W^2}{Q^2 + M_W^2}\right)^2 ME \left[F_1 x_B y^2 + F_2 \left(1 - y - \frac{Mx_B y}{2E}\right) \pm x_B \left(y - \frac{y^2}{2}\right) F_3\right]
\end{aligned}
$$

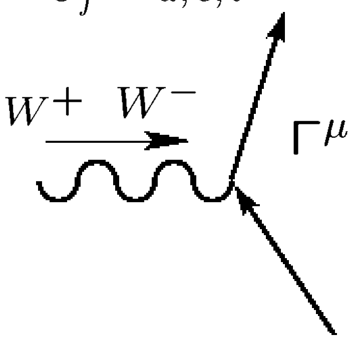
# Vertice elettrodebole elementare

$$W^{\text{el}\,\mu\nu} = \frac{1}{2m}\frac{1}{2\pi}\int \frac{d\mathbf{p}'}{(2\pi)^3 2p'^0}(2\pi)^4\delta(p'-xp-q)\,H^{\text{el}\,\mu\nu}$$

$$= \frac{1}{2m}\delta(2xp\cdot q - Q^2)\,H^{\text{el}\,\mu\nu} = \frac{1}{2mQ^2}\delta\left(\frac{x}{x_B}-1\right)\,H^{\text{el}\,\mu\nu}$$

$D_f = d, s, b$
$U_f = u, c, t$

$\bar{U}_{\bar{f}} = \bar{u}, \bar{c}, \bar{t}$
$\bar{D}_{\bar{f}} = \bar{d}, \bar{s}, \bar{b}$




$U_f = u, c, t$ $\quad \bar{D}_{\bar{f}} = \bar{d}, \bar{s}, \bar{b}$

$D_f = d, s, b$ $\quad \bar{U}_{\bar{f}} = \bar{u}, \bar{c}, \bar{t}$

interazione e.m.$\rightarrow$ scambio di $\gamma$

$$\Gamma^\mu = e\gamma^\mu$$

quark (left-handed)

$$\Gamma^\mu = \frac{e}{2\sqrt{2}\sin\theta_W}\,\frac{T_3(=+1)}{2}\,\gamma^\mu(1-\gamma_5)\,\sum_{f'} V_{ff'}$$

antiquark (right-handed)

$$\Gamma^\mu = \frac{e}{2\sqrt{2}\sin\theta_W}\,\frac{T_3(=-1)}{2}\,\gamma^\mu(1+\gamma_5)\,\sum_{\bar{f}'} V_{\bar{f}\bar{f}'}$$

$$\begin{pmatrix} U_f \\ D_f \end{pmatrix} \equiv \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

$$V_{U_f D_f} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \epsilon\; SU_f(3)$$

$$\sim \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0 \\ -\sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\sum_{f'}\left|V_{U_f D_{f'}}\right|^2 = 1$$

# Tensore adronico elementare

$$H^{\mathrm{el}\,\mu\nu} = \frac{e_f^2}{4} \mathrm{Tr}\left[\gamma^\mu (1 \mp \gamma_5) (x\,\not{p} + \not{q}) \gamma^\nu (1 \mp \gamma_5)\, x\,\not{p}\right] \sum_{f'} \left| V_{U_f D_{f'}} \right|^2$$

$$= H^{\mathrm{el}(S)\mu\nu} \pm H^{\mathrm{el}(A)\mu\nu}$$

Poi $$L_{\mu\nu}\, H^{\mathrm{el}\,\mu\nu} = L^{(S)}_{\mu\nu}\, H^{\mathrm{el}(S)\mu\nu} \pm L^{(A)}_{\mu\nu}\, H^{\mathrm{el}(A)\mu\nu}$$

Metodo alternativo

# DIS elastico di elettrone su fermione puntiforme

$$\frac{d\sigma}{dE'd\Omega} = \frac{4\alpha^2 E'^2}{Q^4}\left(\cos^2\frac{\theta_e}{2} + \frac{Q^2}{2M^2}\sin^2\frac{\theta_e}{2}\right)\delta\left(\nu - \frac{Q^2}{2M}\right)$$

helicity-flip magnetico, cioe`
scattering $f_L - f_R$ e $f_R - f_L$

$1 - \sin^2\frac{\theta_e}{2} \sim 1$

non helicity-flip, cioe`
scattering $f_L - f_L$ e $f_R - f_R$

**$f_h$ = fermione$_{\text{helicity}}$**

Variabili di Mandelstam

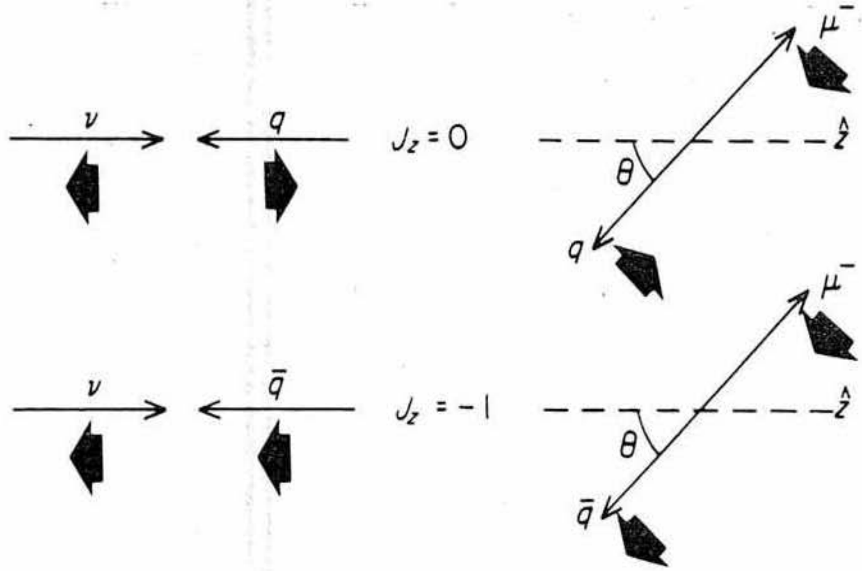
$$\begin{cases} s = (P+k)^2 \sim 2ME \\ t = (k-k')^2 = -Q^2 \\ u = (P-k')^2 \sim -2ME' \end{cases}$$

$$\begin{aligned} d\nu &= dE' = -\frac{1}{2M}du \\ dQ^2 &= \frac{EE'}{\pi}d\Omega = -dt \end{aligned}$$

$$\begin{aligned} \frac{d\sigma}{dt\,du} &= \frac{\pi}{2MEE'}\frac{d\sigma}{dE'd\Omega} \\ &\sim \frac{4\pi\alpha^2}{t^2}\frac{1}{2}\frac{2E'}{E}\left(1 + \frac{Q^2}{2M^2}\frac{Q^2}{4EE'}\right)\delta(2M\nu - Q^2) \\ &= \frac{4\pi\alpha^2}{t^2}\frac{1}{2}\left(\frac{t^2}{s^2} - \frac{2u}{s}\right)\delta(s+t+u) \\ &= \frac{4\pi\alpha^2}{t^2}\frac{1}{2}\left(1 + \frac{u^2}{s^2}\right)\delta(s+t+u) \end{aligned}$$

non helicity flip

helicity flip




isotropico

½ (1+cosθ) = 1-y

## DIS elastico di elettrone su quark

$$s \to xs \qquad u \to xu$$

$$\frac{d\sigma}{dtdu} = \frac{4\pi\alpha^2}{t^2}\frac{1}{2}\left(1 + \frac{u^2}{s^2}\right)\delta(s+t+u)$$

$$\frac{d\sigma^{\mathrm{el}}}{dtdu} = \frac{4\pi\alpha^2 e_f^2}{t^2}\frac{1}{2}\left(1 + \frac{u^2}{s^2}\right) x\,\delta\left(t + x(s+u)\right)$$

## DIS elastico di (anti)neutrino su quark

Accoppiamento debole richiede leptoni left-handed e antileptoni right-handed

Combinazioni possibili :

$$\left.\begin{array}{llll} \nu_{e/\mu} \to (e^-/\mu^- + W^+) & + & (d + W^+) \to u & f_L - f_L \\ \bar{\nu}_{e/\mu} \to (e^+/\mu^+ + W^-) & + & (\bar{d} + W^-) \to \bar{u} & f_R - f_R \end{array}\right\} \text{no helicity flip}$$

$$\left.\begin{array}{llll} \bar{\nu}_{e/\mu} \to (e^+/\mu^+ + W^-) & + & (u + W^-) \to d & f_R - f_L \\ \nu_{e/\mu} \to (e^-/\mu^- + W^+) & + & (\bar{u} + W^+) \to \bar{d} & f_L - f_R \end{array}\right\} \text{helicity flip}$$

## DIS elastico di (anti)neutrino su quark (continua)

no helicity flip

$V \mp A \quad V \mp A$

$$\frac{d\sigma^{\text{el}}}{dtdu} = \frac{G_F^2 e_f^2}{8\pi} \left( \frac{M_W^2}{Q^2 + M_W^2} \right)^2 \; 1 \, x\delta \left( t + x(s+u) \right)$$

helicity flip

$V \mp A \quad V \pm A$

$$\frac{d\sigma^{\text{el}}}{dtdu} = \frac{G_F^2 e_f^2}{8\pi} \left( \frac{M_W^2}{Q^2 + M_W^2} \right)^2 \frac{u^2}{s^2} x\delta \left( t + x(s+u) \right)$$

**Non c'e` fattore ½ per media su elicita` del ν**

Quindi

$$\frac{d\sigma^{\text{el}\nu}}{dx_B dy} = \frac{\pi 2M\nu}{E'} \frac{2MEE'}{\pi} \frac{d\sigma^{\text{el}}}{dtdu}$$

$$= \frac{G_F^2 e_f^2}{8\pi} \left( \frac{M_W^2}{Q^2 + M_W^2} \right)^2 s \left[ 1|_{q(x)} + (1-y)^2|_{\bar{q}(x)} \right] x\,\delta(x - x_B)$$

$$\frac{d\sigma^{\text{el}\bar{\nu}}}{dx_B dy} = \frac{\pi 2M\nu}{E'} \frac{2MEE'}{\pi} \frac{d\sigma^{\text{el}}}{dtdu}$$

$$1 + \frac{u}{s} = y$$

$$= \frac{G_F^2 e_f^2}{8\pi} \left( \frac{M_W^2}{Q^2 + M_W^2} \right)^2 s \left[ 1|_{\bar{q}(x)} + (1-y)^2|_{q(x)} \right] x\,\delta(x - x_B)$$

$$
\begin{aligned}
\frac{d\sigma^{\nu/\bar{\nu}}}{dx_B dy} &= \frac{G_F^2}{4\pi}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 ME\left[F_1 x_B y^2 + F_2\left(1-y-\frac{Mx_B y}{2E}\right) \pm x_B\left(y-\frac{y^2}{2}\right) F_3\right] \\
&\sim \frac{G_F^2}{8\pi}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 s\,\frac{F_2}{2}\left[1 \pm \frac{x_B F_3}{F_2} + (1-y)^2\left(1 \mp \frac{x_B F_3}{F_2}\right)\right] \\
&= \sum_f \int_0^1 dx\, \frac{d\sigma^{\mathrm{el}\,\nu/\bar{\nu}}}{dx dy}\,\phi_f(x) \\
&= \frac{G_F^2}{8\pi}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 s \sum_f e_f^2 \int_0^1 dx\, x\, \delta(x-x_B)\,\left[\phi_f(x)/\bar{\phi}_f(x) + (1-y)^2\,\bar{\phi}_f(x)/\phi_f(x)\right] \\
&= \frac{G_F^2}{8\pi}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 s \sum_f e_f^2\, x_B\,\left[\phi_f(x_B)/\bar{\phi}_f(x_B) + (1-y)^2\,\bar{\phi}_f(x_B)/\phi_f(x_B)\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{F_2}{2}\left(1+\frac{x_B F_3}{F_2}\right) &= x_B \sum_f e_f^2\,\phi_f(x_B) \\
\frac{F_2}{2}\left(1-\frac{x_B F_3}{F_2}\right) &= x_B \sum_f e_f^2\,\bar{\phi}_f(x_B)
\end{aligned}
\quad\Longrightarrow\quad
\begin{aligned}
F_2(x_B) &= x_B \sum_f e_f^2\left[\phi_f(x_B) + \bar{\phi}_f(x_B)\right] \\
F_3(x_B) &= \sum_f e_f^2\left[\phi_f(x_B) - \bar{\phi}_f(x_B)\right]
\end{aligned}
$$

$\text{SU}_f(3) \rightarrow$ 12 incognite :

$$u_p, d_p, s_p, \bar{u}_p, \bar{d}_p, \bar{s}_p$$
$$u_n, d_n, s_n, \bar{u}_n, \bar{d}_n, \bar{s}_n$$

8 misure possibili :

$$F_2^{W^+p}, F_2^{W^-p}, F_3^{W^+p}, F_3^{W^-p}$$
$$F_2^{W^+n}, F_2^{W^-n}, F_3^{W^+n}, F_3^{W^-n}$$

invarianza di isospin : $u_p \equiv d_n \qquad d_p \equiv u_n$

simmetria di isospin del “mare” : $\bar{u} = \bar{d} = \bar{s}$

Sistema determinato: da DIS (anti)neutrino – Nucleone si possono estrarre le distribuzioni degli (anti)quark per i tre flavor

Esempio : $\nu_{e/\mu} + p \to e^-/\mu^- + X$

$$\begin{aligned} J^\mu_{W^+} &\propto \bar{u}\,\gamma^\mu(1-\gamma_5)\,[d\cos\theta_C + s\sin\theta_C] \\ &+ \bar{d}\,\gamma^\mu(1+\gamma_5)\,\bar{u} \end{aligned}$$

$$\begin{aligned} \frac{F_2(x_B)}{x_B} &= 2F_1(x_B) = 2\,[d(x_B) + s(x_B) + \bar{u}(x_B)] \\ F_3(x_B) &= 2\,[d(x_B) + s(x_B) - \bar{u}(x_B)] \end{aligned}$$

$$\bar{\nu}_{e/\mu} + p \to e^+/\mu^+ + X$$

$$\begin{aligned} J^\mu_{W^-} &\propto [\bar{d}\cos\theta_C + \bar{s}\sin\theta_C]\,\gamma^\mu(1-\gamma_5)\,u \\ &+ \overline{(\bar{u})}\,\gamma^\mu(1+\gamma_5)\,\bar{d} \end{aligned}$$

$$\begin{aligned} \frac{F_2(x_B)}{x_B} &= 2F_1(x_B) = 2\left[\bar{d}(x_B) + \bar{s}(x_B) + u(x_B)\right] \\ F_3(x_B) &= 2\left[-\bar{d}(x_B) - \bar{s}(x_B) + u(x_B)\right] \end{aligned}$$

## Verifiche sperimentali

(anti)neutrino DIS su nuclei isoscalari ($Z$=$N \rightarrow$ n° $u$ = n° $d$ quarks)

$$\frac{\sigma(\nu A)}{\sigma(\bar{\nu} A)} \sim 3$$

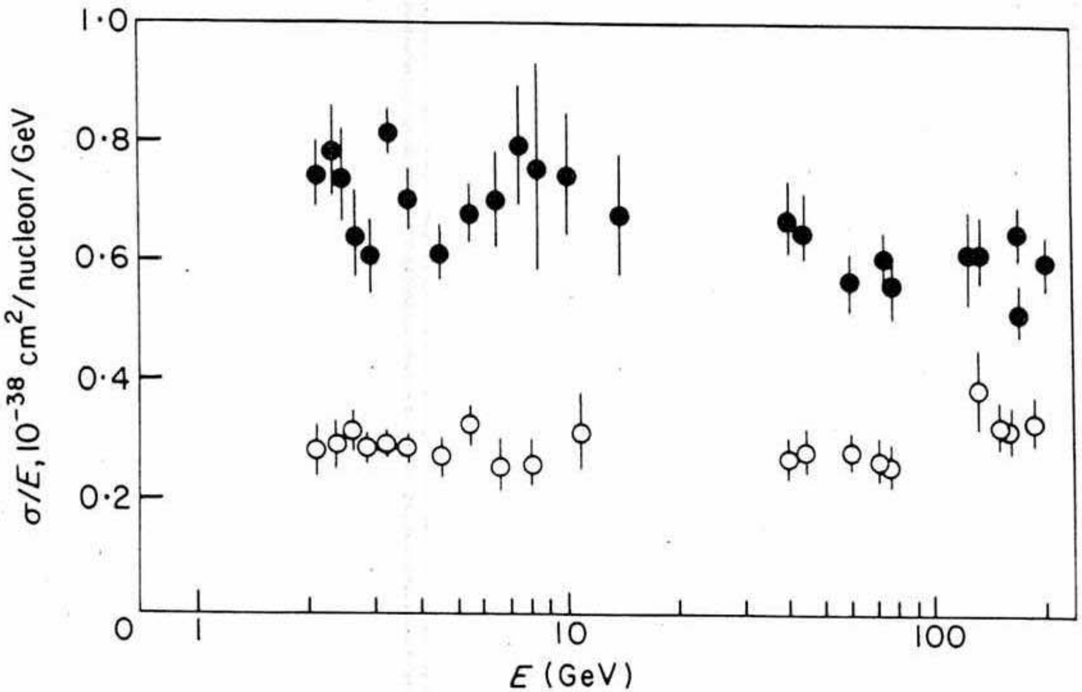



FIG. 11.12. $\sigma^{\bar{\nu}}/E$ and $\sigma^{\nu}/E$ for $E \leqslant 200$ GeV.

Dati dell'esp. Gargamelle
Perkins, Contemp. Phys. **16** 173 (75)

Interpretazione in QPM :

approssimazioni : $\bar{u} = \bar{d} = \bar{c} = \bar{s} = \bar{t} = \bar{b} = 0$

$$s \sim c \sim K \qquad t \sim b \sim 0$$

$$\frac{d\sigma^{\nu A}}{dx_B dy} = N^{\nu A} \left[ \left( 1 - y + \frac{y^2}{2} - \frac{M_A x_B y}{2E} \right) F_2 + \left( y - \frac{y^2}{2} \right) x_B F_3 \right]$$

$$\sim N^{\nu A} 2x_B (d + K) \qquad\qquad F_2 = x_B F_3 = 2x_B (d + s)$$

$$\frac{d\sigma^{\bar{\nu} A}}{dx_B dy} = N^{\bar{\nu} A} \left[ \left( 1 - y + \frac{y^2}{2} - \frac{M_A x_B y}{2E} \right) F_2 - \left( y - \frac{y^2}{2} \right) x_B F_3 \right]$$

$$\sim N^{\bar{\nu} A} 2x_B (u + K) (1 - y)^2 \qquad\qquad F_2 = x_B F_3 = 2x_B (u + c)$$

$$\frac{\sigma^{\nu A}}{\sigma^{\bar{\nu} A}} = \frac{\int_0^1 dx dy \, d\sigma^{\nu A}}{\int_0^1 dx dy \, d\sigma^{\bar{\nu} A}} = \frac{N^{\nu A} \int_0^1 dx 2x (d + K)}{N^{\bar{\nu} A} \int_0^1 dx 2x (u + K) \frac{1}{3}} \sim 3$$

*N* = { partoni a spin ½ con stessa interazione elettrodebole dei leptoni ; antipartoni soppressi }

Inoltre per $x_B \gtrsim 0.2$ no antiquark

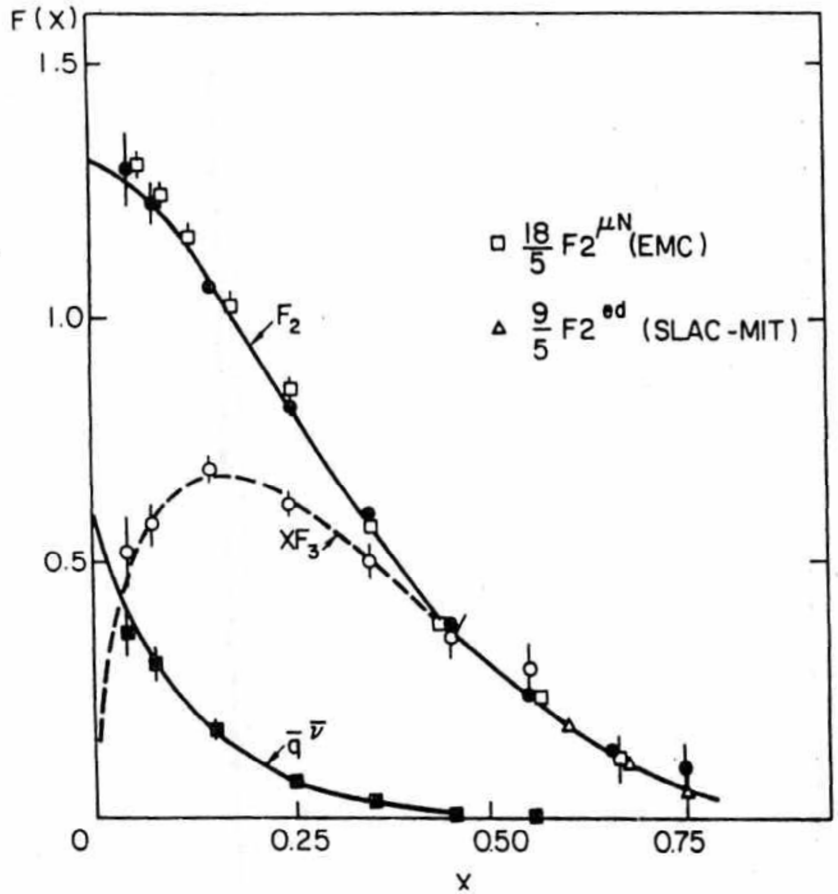
$$\frac{\frac{d\sigma^{\nu A}}{dx_B dy}\Big|_{y=0}}{\frac{d\sigma^{\bar{\nu} A}}{dx_B dy}\Big|_{y=0}} \sim \frac{N^{\nu A}\, 2x_B\,(d+s)}{N^{\bar{\nu} A}\, 2x_B\,(u+c)} \stackrel{A=p}{=} \frac{d(x_B)}{u(x_B)} \xrightarrow{x_B \to 1} 0$$

consistente con la dominanza di quark *u* in *p* (*d* in *n*) dei dati di DIS di elettrone

$$1 \xleftarrow{x_B \to 0} \frac{F_2^{e^- n}}{F_2^{e^- p}} \xrightarrow{x_B \to 1} \frac{1}{4}$$

rapporto di carica : DIS di elettrone e (anti)neutrino su nuclei isoscalari

$$\frac{F_2^{e^- A}}{F_2^{\nu A}+F_2^{\bar{\nu} A}} \sim \frac{5}{18}$$




Collab. BCDMS, P.L. **B195** 91 (87)
" " , " **B237** 592 (90)
" " , " " 599 (90)
" CCFR , Z. Phys. **C26** 1 (84)

Interpretazione in QPM :

$$0 \xleftarrow{x_B \to 0} \frac{x_B F_3}{F_2} \sim \frac{q-\bar{q}}{q+\bar{q}} \xrightarrow{x_B \to 1} 1$$

$$\frac{\bar{q}}{q} \xrightarrow{x_B \to 0} 1 \qquad\qquad \frac{\bar{q}}{q} \xrightarrow{x_B \to 1} 0$$

per $x_B \gtrsim 0.2$ e $f = u,d$

$$\frac{F_2^{e^- A}}{F_2^{\nu A} + F_2^{\bar{\nu} A}} \sim \frac{A x_B \left(\frac{4}{9}u + \frac{1}{9}d\right)}{A 2 x_B (d+u)} \sim \frac{1}{2}\left(\frac{4}{9} + \frac{1}{9}\right) = \frac{5}{18}$$

(carica)$^2$ media

deviazioni per $x < 0.2$ dovute a $s(x)$, $c(x)$ e correzioni pQCD

## Regole di somma

$$\int_0^1 dx\, u(x) - \bar{u}(x) = n_u \begin{cases} 2 & \textbf{p} \\ 1 & \textbf{n} \end{cases}$$

$$\int_0^1 dx\, d(x) - \bar{d}(x) = n_d \begin{cases} 1 & \textbf{p} \\ 2 & \textbf{n} \end{cases}$$

$$\int_0^1 dx\, s(x) - \bar{s}(x) = 0$$

Adler s.r. $$\int_0^1 \frac{dx}{2x} \left( F_2^{\bar{\nu}p} - F_2^{\nu p} \right) = n_u - n_d + n_c - n_s = 1$$

dato exp. $1.01 \pm 0.20$

Allasia *et al.*, P.L. **B135** 231 (84)
Z. Phys. **C28** 321 (85)

unpolarized Bjorken s.r. $$\int_0^1 dx \left( F_1^{\bar{\nu}p} - F_1^{\nu p} \right) = n_u - n_d + n_c - n_s = 1$$

Gross-Lewellin Smith s.r. $$\int_0^1 \frac{dx}{2} \left( F_3^{\bar{\nu}p} + F_3^{\nu p} \right) = n_u + n_d + n_c + n_s = 3$$

dato exp. $2.50 \pm 0.08$

correzioni pQCD evidenti!

## Gottfried s.r.

$$\int_0^1 \frac{dx}{x}\left(F_2^{e^-p} - F_2^{e^-n}\right) = \frac{1}{3}\int_0^1 dx\,(u + \bar{u} - d - \bar{d})$$

$$= \frac{1}{3}\int_0^1 dx\,(u^v - d^v) + \frac{1}{3}\int_0^1 dx\,(u^{sea} - d^{sea})$$

$$\sim \frac{1}{3}(n_u - n_d) = \frac{1}{3}$$

**simmetria isospin**

***u* sea = *d* sea**

dato exp. $0.240 \pm 0.016$

NMC coll., P.R.L. **66** 2712 (91)
Arneodo, P.Rep. **240** 301 (94)

correzioni QCD $\to u^{sea} \neq d^{sea}$

$$d\bar{d} > u\bar{u}$$

## Momentum s.r.

$$\int_0^1 dx\, x(u + \bar{u} + d + \bar{d} + s + \bar{s}) = \frac{9(1+\delta)}{5+2\delta}\int_0^1 dx(F_2^{e^-p} + F_2^{e^-n}) = 1 - \varepsilon$$

$$\delta = \frac{\int_0^1 dx x(s+\bar{s})}{\int_0^1 dx x(u+\bar{u}+d+\bar{d})}$$

dati $\to \delta < 0.06 \to \varepsilon \sim 0.5 \pm 0.04$

$$= \int_0^1 dx\left[\frac{9}{2}(F_2^{e^-p} + F_2^{e^-n}) - \frac{3}{4}(F_2^{\nu p} + F_2^{\nu n})\right] = 1 - \varepsilon$$

gluoni portano circa meta\`
del momento del N !