$$\bullet \quad \frac{d}{dt}\langle x\rangle = \int dx\, x\, \frac{\partial}{\partial t}|\Psi|^2 = -\int dx\, x\, \frac{\partial j}{\partial x} = \int dx\, j$$

$$= -\frac{i\hbar}{2m}\int dx\, \left(\Psi^*\frac{\partial \Psi}{\partial x} - \Psi\frac{\partial \Psi^*}{\partial x}\right)$$

$$= 2\,\mathrm{Re}\left[-\frac{i\hbar}{2m}\int dx\, \Psi^*\frac{\partial \Psi}{\partial x}\right]$$

ma $\quad 2\,\mathrm{Im}\left[-\frac{i\hbar}{m}\int dx\, \Psi^*\frac{\partial \Psi}{\partial x}\right] = -i\hbar\int dx\, \Psi^*\frac{\partial \Psi}{\partial x} - i\hbar\int dx\, \Psi\frac{\partial \Psi^*}{\partial x}$

$$= -i\hbar\int dx\, \frac{\partial}{\partial x}|\Psi|^2 = -i\hbar\left[|\Psi|^2\right]_{-\infty}^{+\infty} = 0$$

$\Rightarrow \frac{d}{dt}\langle x\rangle = -\frac{i\hbar}{m}\int dx\, \Psi^*\frac{\partial \Psi}{\partial x} \equiv \frac{1}{m}\langle p_x\rangle$ $\quad$ reale

$$\bullet \quad \frac{d}{dt}\langle p_x\rangle = -i\hbar\frac{d}{dt}\int dx\, \Psi^*\frac{\partial \Psi}{\partial x}$$

$$= -i\hbar\int dx\, \frac{\partial \Psi^*}{\partial t}\frac{\partial \Psi}{\partial x} - i\hbar\int dx\, \Psi^*\frac{\partial}{\partial x}\frac{\partial \Psi}{\partial t}$$

$$= \int dx\, \left(-\frac{\hbar^2}{2m}\nabla^2\Psi^* + V\Psi^*\right)\frac{\partial \Psi}{\partial x} - \int dx\, \Psi^*\frac{\partial}{\partial x}\left(-\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi\right)$$

$$= \int dx\, \Psi^*\left[V\frac{\partial \Psi}{\partial x} - \frac{\partial}{\partial x}(V\Psi)\right] = -\int dx\, \Psi^*\frac{\partial V}{\partial x}\Psi$$

$$\Rightarrow \frac{d}{dt}\langle p_x\rangle = \left\langle -\frac{\partial V(x)}{\partial x}\right\rangle$$