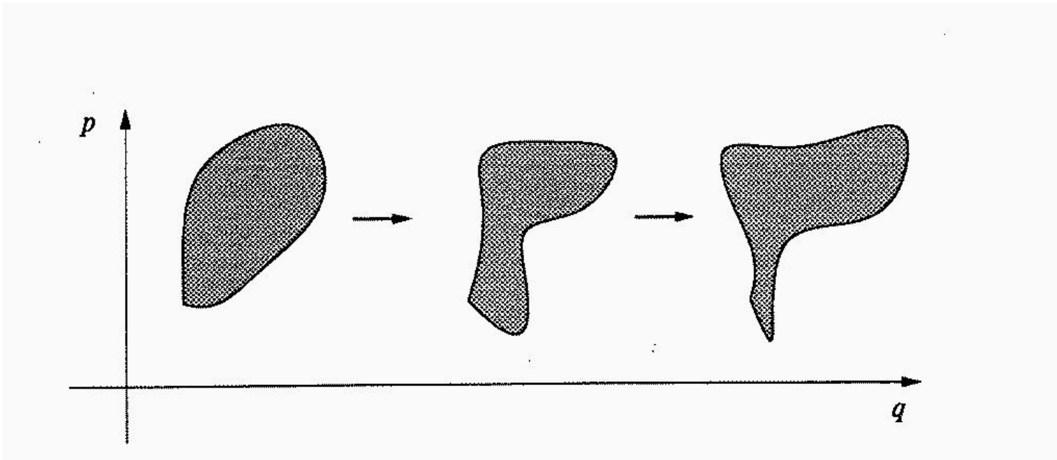
Spazio delle fasi e probabilità

Joseph Liouville (1809–1882)



probabilità: $\rho(q,p,t)\, dq\, dp$ $\left(\int dq \int dp\, \rho(q,p,t) = 1\right)$

eq. di continuità: $\boxed{\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho\, \vec{u}) = 0}$

i.e. $\frac{\partial \rho}{\partial t} + \sum \left(\frac{\partial \rho}{\partial q} \dot{q} + \frac{\partial \rho}{\partial p} \dot{p} \right) + \rho \vec{\nabla} \cdot \vec{u} = 0$

ma: $\vec{\nabla} \cdot \vec{u} = \sum \left(\frac{\partial \dot{q}}{\partial q} + \frac{\partial \dot{p}}{\partial p} \right) = \frac{\partial^2 H}{\partial q \partial p} - \frac{\partial^2 H}{\partial p \partial q} = 0$

eq. di Liouville : $\boxed{\frac{\partial \rho}{\partial t} + \{\rho, H\} = 0}$

I teorema di Liouville: $\frac{d\rho}{dt} = 0$ (ρ costante)

II teorema di Liouville: $\vec{\nabla} \cdot \vec{u} = 0$

(fluido incomprimibile)