# Teorema di Robertson

H.P. Robertson, Phys. Rev. 34 (1929) 163–164

$$\forall\, A = A^\dagger,\;\; B = B^\dagger \;\; \Rightarrow \;\; \boxed{\Delta A\,\Delta B \geq \tfrac{1}{2}\,|\langle [A,B]\rangle|}$$

infatti:

$$a = A - \langle A\rangle, \quad b = B - \langle B\rangle, \quad \lambda = \text{reale}, \quad |\Phi\rangle = (a + i\,\lambda\, b)|\Psi\rangle$$

$$0 \leq \langle \Phi|\Phi\rangle = (\Delta A)^2 + \lambda^2 (\Delta B)^2 + i\,\lambda\,\langle [A,B]\rangle \equiv F(\lambda)$$

$$\text{ma} \quad \langle [A,B]\rangle^* = -\langle [A,B]\rangle \;\; \Rightarrow \;\; \langle [A,B]\rangle = i\,\langle C\rangle, \quad \langle C\rangle^* = \langle C\rangle$$

$$\Rightarrow \;\; F(\lambda)\ \text{reale} \geq 0 \quad \Rightarrow \quad \langle C\rangle^2 - 4(\Delta A)^2(\Delta B)^2 \leq 0 \quad \text{q.e.d.}$$

p.es. $A = x,\;\; B = p,\;\; [x,p] = i\,\hbar \;\; \Rightarrow \;\; \boxed{\Delta x\,\Delta p \geq \tfrac{1}{2}\,\hbar}$

p.es. $\boxed{\Delta L_i\,\Delta L_j = \tfrac{1}{2}\,\hbar |\epsilon_{ijk}\langle L_k\rangle|}$

p.es. $\boxed{\Delta L^2\,\Delta L_i = 0}$