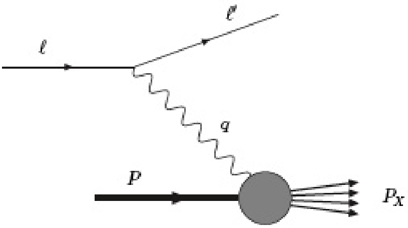
The nucleon response to an external probe

Sigfrido Boffi

- inclusive deep inelastic scattering
- parton distributions
- semi-inclusive deep inelastic scattering
- transverse momentum dependent distributions
- deeply virtual Compton scattering
- generalized parton distributions

(inclusive) deep inelastic scattering (DIS)



$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{2MQ^4}\,\frac{E'}{E}\, L_{\mu\nu}\, W^{\mu\nu}$$

cross section: $d\sigma = \frac{1}{\mathcal{F}}|\mathcal{M}|^2 d\mathcal{R}$

incident flux: $\mathcal{F} = 4\sqrt{(P\cdot l)^2 - M^2 m^2}$

phase space element: $d\mathcal{R} = (2\pi)^4\delta^4(l + P - l' - P_X)\frac{d^3P_X}{(2\pi)^3 2P_X^0}\,\frac{d^3 l'}{(2\pi)^3 2E'}$

invariant amplitude: $\mathcal{M} = \bar{u}(l', s')\gamma^\mu u(l, s)\frac{e^2}{Q^2}\langle P_X|J_\mu(0)|P, S\rangle$

lepton tensor: $L_{\mu\nu}(l, s; l', s') = [\bar{u}(l', s')\ \gamma_\mu\ u(l, s)]^*\ [\bar{u}(l', s')\ \gamma_\nu\ u(l, s)]$

hadron tensor: $W_{\mu\nu}(q; P, S) = \frac{1}{2\pi}\int \frac{d^3P_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta^4(q + P - P_X)\langle P, S|J_\mu(0)|P_X\rangle\langle P_X|J_\nu(0)|P, S\rangle$

the lepton tensor

$$L_{\mu\nu}(l,s;l',s') = L^{(S)}_{\mu\nu}(l;l') + iL^{(A)}_{\mu\nu}(l,s;l') + L'^{(S)}_{\mu\nu}(l,s;l',s') + iL'^{(A)}_{\mu\nu}(l;l',s')$$

$$\begin{aligned}
L^{(S)}_{\mu\nu}(l;l') &= l_\mu l'_\nu + l'_\mu l_\nu - g_{\mu\nu}\,(l\cdot l' - m^2) \\
L^{(A)}_{\mu\nu}(l,s;l') &= m\,\varepsilon_{\mu\nu\alpha\beta}\,s^\alpha\,(l-l')^\beta \\
&= \lambda\,\varepsilon_{\mu\nu\alpha\beta}\,l^\alpha\,q^\beta \\
L'^{(S)}_{\mu\nu}(l,s;l',s') &= (l\cdot s')\,(l'_\mu s_\nu + s_\mu l'_\nu - g_{\mu\nu}\,l'\cdot s) \\
&\quad -(l\cdot l' - m^2)\,(s_\mu s'_\nu + s'_\mu s_\nu - g_{\mu\nu}\,s\cdot s') \\
&\quad +(l'\cdot s)(s'_\mu l_\nu + l_\mu s'_\nu) - (s\cdot s')(l_\mu l'_\nu + l'_\mu l_\nu) \\
L'^{(A)}_{\mu\nu}(l;l',s') &= m\,\varepsilon_{\mu\nu\alpha\beta}\,s'^\alpha(l-l')^\beta \\
&= \lambda\,\varepsilon_{\mu\nu\alpha\beta}\,l'^\alpha\,q^\beta
\end{aligned}$$

the hadron tensor

$$W_{\mu\nu}(q;P,S) = W^{(S)}_{\mu\nu}(q;P) + i\ W^{(A)}_{\mu\nu}(q;P,S)$$

the DIS cross section

$$\frac{d^2\sigma}{d\Omega\ dE'}(l,s,P,S;l',s') = \frac{\alpha^2}{2MQ^4}\ \frac{E'}{E}\left[L^{(S)}_{\mu\nu}\ W^{\mu\nu(S)} + L'^{(S)}_{\mu\nu}\ W^{\mu\nu(S)} - L^{(A)}_{\mu\nu}\ W^{\mu\nu(A)} - L'^{(A)}_{\mu\nu} W^{\mu\nu(A)}\right]$$

- unpolarized cross section proportional to $W^{\mu\nu(S)}$:

$$\frac{d^2\sigma^{\mathrm{unp}}}{d\Omega\, dE'}\left(l,P;l'\right) = \frac{1}{4}\sum_{s,s',S}\frac{d^2\sigma}{d\Omega\, dE'}(l,s,P,S;l',s') = \frac{\alpha^2}{2MQ^4}\ \frac{E'}{E}\ 2L^{(S)}_{\mu\nu}\ W^{\mu\nu(S)}$$

- differences of cross sections single out $W^{\mu\nu(A)}$ term:

$$\sum_{s'}\left[\frac{d^2\sigma}{d\Omega\, dE'}(l,s,P,-S;l',s') - \frac{d^2\sigma}{d\Omega\, dE'}(l,s,P,S;l',s')\right] = \frac{\alpha^2}{2MQ^4}\ \frac{E'}{E}\ 4L^{(A)}_{\mu\nu}\ W^{\mu\nu(A)}$$

- if target spin unobserved (only two independent vectors q^μ, P^μ)

$$\begin{aligned}
\frac{1}{2M} W_{\mu\nu}^{(S)} &= \frac{1}{2M} W_{\nu\mu}^{(S)} \\
&= -W_1 g_{\mu\nu} + W_2 \frac{1}{M^2} P_\mu P_\nu + W_3 \frac{1}{M^2} q_\mu q_\nu + W_4 \frac{1}{M^2} (P_\mu q_\nu + q_\mu P_\nu) \\
\frac{1}{2M} W_{\mu\nu}^{(A)} &= -\frac{1}{2M} W_{\nu\mu}^{(A)} \\
&= W_5 \frac{1}{M^2} (P_\mu q_\nu - q_\mu P_\nu) \qquad \text{N.B. } W_i \equiv W_i(P\cdot q, Q^2)
\end{aligned}$$

- gauge invariance (current conservation, i.e. $q^\mu J_\mu = 0$) $\Longrightarrow$ $q^\mu W_{\mu\nu}^{(S)} = q^\mu W_{\mu\nu}^{(A)} = 0$

$$-W_1 q^\nu + W_2 \frac{1}{M^2} q\cdot P\, P^\nu + W_3 \frac{1}{M^2} q_\mu^2 q^\nu + W_4 \frac{1}{M^2} (q\cdot P\, q^\nu + q_\mu^2 P^\nu) = 0$$

$$W_5 \frac{1}{M^2} (q\cdot P\, q^\nu - q_\mu^2 P^\nu) = 0$$

i.e.

$$\begin{aligned}
&-W_1 + W_3 \frac{1}{M^2} q_\mu^2 + W_4 \frac{1}{M^2} q\cdot P = 0 \\
&W_2 \frac{1}{M^2} q\cdot P + W_4 \frac{1}{M^2} q_\mu^2 = 0 \\
&W_5 = 0
\end{aligned}$$

$$W_5 = 0 \quad \Longrightarrow \quad W^{(A)}_{\mu\nu} = 0$$

$$W_4 = -W_2 \frac{q \cdot P}{q^2_\mu}, \qquad W_3 = W_2 \left(\frac{q \cdot P}{q^2_\mu} \right)^2 + W_1 M^2 \frac{1}{q^2_\mu}$$

$$\Longrightarrow \quad \frac{1}{2M} W_{\mu\nu} = \frac{1}{2M} W^{(S)}_{\mu\nu} = -W_1\, \tilde{g}_{\mu\nu} + W_2 \frac{1}{M^2} \tilde{P}_\mu \tilde{P}_\nu$$

where

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2_\lambda}, \qquad \tilde{P}^\mu = P^\mu - \frac{P \cdot q}{q^2_\lambda}\, q^\mu$$

$$\Longrightarrow \quad \frac{d^2\sigma^{\mathrm{unp}}}{d\Omega\ dE'} = \frac{4\alpha^2 E'^2}{Q^4} \left[2W_1 \sin^2 \tfrac{1}{2}\theta + W_2 \cos^2 \tfrac{1}{2}\theta \right]$$

Similarly, including spin d.o.f.,

$$\frac{1}{2M} W^{(A)}_{\mu\nu}(q; P, S)$$
$$= \varepsilon_{\mu\nu\alpha\beta}\, q^{\alpha} \left\{ MS^{\beta} G_1(P\cdot q, Q^2) + [(P\cdot q)S^{\beta} - (S\cdot q)P^{\beta}]\, \frac{G_2(P\cdot q, Q^2)}{M} \right\}$$

$\Longrightarrow$

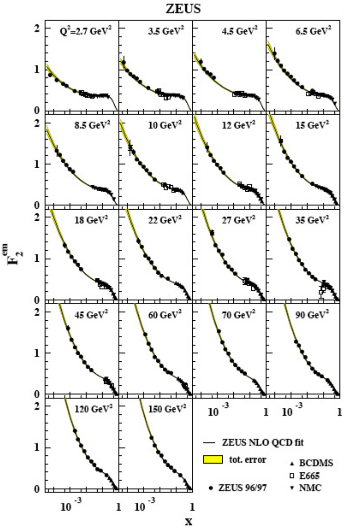
$$\sum_{s'} \left[\frac{d^2\sigma}{d\Omega\, dE'}(l, s, P, S; l', s') - \frac{d^2\sigma}{d\Omega\, dE'}(l, s, P, -S; l', s') \right] \equiv \frac{d^2\sigma^{s,S}}{d\Omega\, dE'} - \frac{d^2\sigma^{s,-S}}{d\Omega\, dE'}$$
$$= \frac{8m\alpha^2}{Q^4} \frac{E'}{E} \left\{ \left[(q\cdot S)(q\cdot s) + Q^2 (s\cdot S) \right] M G_1 \right.$$
$$\left. + Q^2 \left[(s\cdot S)(P\cdot q) - (q\cdot S)(P\cdot s) \right] \frac{G_2}{M} \right\},$$

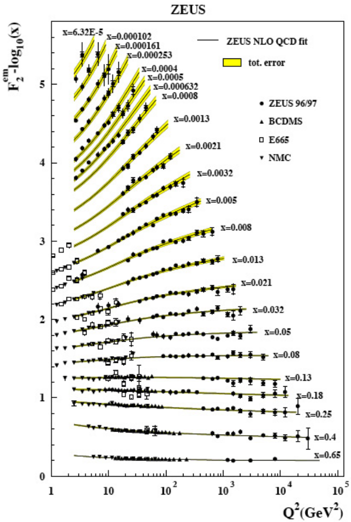
the Bjorken limit

$$-q^2 = Q^2 \to \infty \qquad \nu = E - E' \to \infty \qquad x = \frac{Q^2}{2P \cdot q} = \frac{Q^2}{2M\nu}\ \text{(fixed)}$$

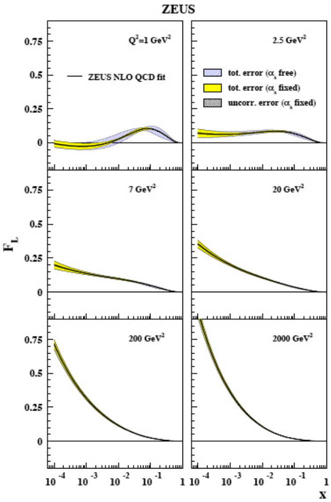
$$\begin{aligned}
\lim_{Bj}\ M\, W_1(P \cdot q, Q^2) &= F_1(x) \\
\lim_{Bj}\ \nu\, W_2(P \cdot q, Q^2) &= F_2(x) \\
\lim_{Bj}\ M^2\nu\, G_1(P \cdot q, Q^2) &= g_1(x) \\
\lim_{Bj}\ M\nu^2\, G_2(P \cdot q, Q^2) &= g_2(x)
\end{aligned}$$

$$F_{\mathrm{L}} = F_2\left(1 + \frac{4M^2x^2}{Q^2}\right) - 2xF_1 \to F_2 - 2xF_1$$



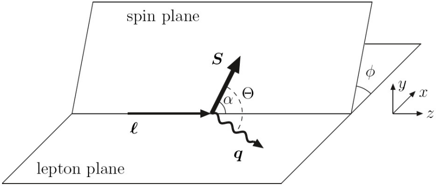


The ZEUS NLO QCD fit compared to ZEUS 96/97 and proton fixed-target F_2 data. Chekanov *et al.*, PR D 67 (2003) 012007



The longitudinal structure function F_L from ZEUS NLO QCD fit. Chekanov *et al.*, PR D 67 (2003) 012007

extracting spin structure functions from data



$$A_{\parallel} \equiv \frac{d\sigma^{\rightarrow\Leftarrow} - d\sigma^{\rightarrow\Rightarrow}}{d\sigma^{\rightarrow\Rightarrow} + d\sigma^{\rightarrow\Leftarrow}} = \frac{Q^2\ [(E + E'\cos\theta) M G_1 - Q^2 G_2]}{2EE'\ [2W_1 \sin^2\frac{1}{2}\theta + W_2\cos^2\frac{1}{2}\theta)]}$$

$$A_{\perp} \equiv \frac{d\sigma^{\rightarrow\Downarrow} - d\sigma^{\rightarrow\Uparrow}}{d\sigma^{\rightarrow\Uparrow} + d\sigma^{\rightarrow\Downarrow}} = \frac{Q^2 \sin\theta (M G_1 + 2E G_2)}{2E\,[2W_1 \sin^2\frac{1}{2}\theta + W_2\cos^2\frac{1}{2}\theta]} \cos\phi$$

with $S^\mu = S_\parallel^\mu + S_\perp^\mu$

$$\frac{1}{2M} W_{\mu\nu}^{(A)}(q;P,S) = \varepsilon_{\mu\nu\alpha\beta}\, q^\alpha \left\{ \frac{M S^\beta G_1 + [(P\cdot q)S^\beta - (S\cdot q)P^\beta]\, G_2}{M} \right\}$$

$$\to_{Bj} \quad \frac{1}{P\cdot q} \varepsilon_{\mu\nu\alpha\beta}\, q^\alpha \left[S_\parallel^\beta\, g_1 + S_\perp^\beta\, (g_1 + g_2) \right]$$

i.e.

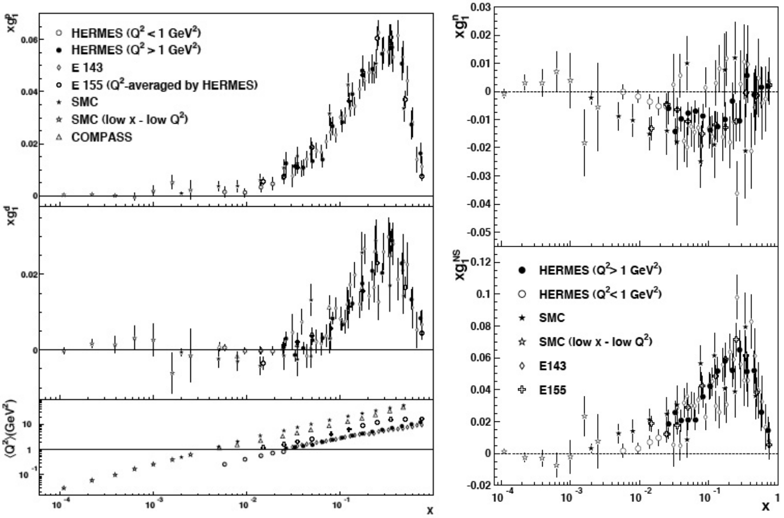
g_1 describes the longitudinal polarization, $g_1 + g_2$ describes the transverse polarization

From

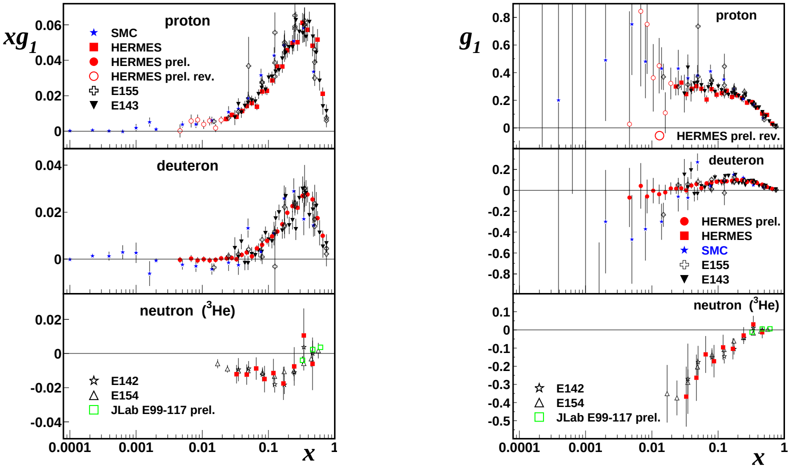
$$g_2(x) = \int_x^1 \frac{dy}{y}\, g_1(y) - g_1(x)$$

$$\Longrightarrow \quad \int_0^1 dx\, x^{J-1} \left\{ \frac{J-1}{J}\, g_1(x) + g_2(x) \right\} = 0$$ Wandzura-Wilczek sum rule

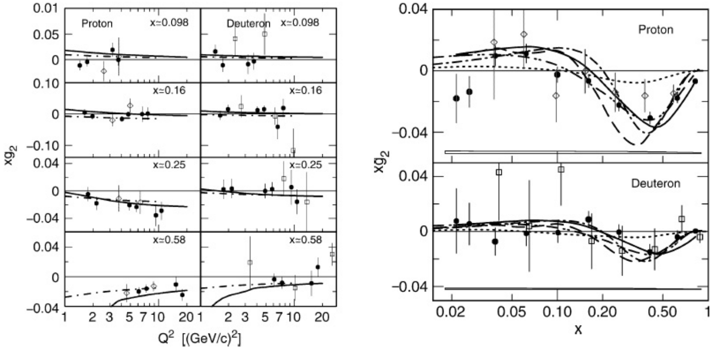
$$\int_0^1 dx\, g_2(x) = 0$$ Burkhardt-Cottingham sum rule



Left panel: HERMES results on xg_1^p and xg_1^d vs. x. Right panel: xg_1^n from data for g_1^p and g_1^d (top), the x-weighted non-singlet spin structure function xg_1^{NS} obtained by HERMES (bottom). Airapetian *et al.*, hep-ex/0609039.



Left (right) panel: the world data on xg_1 (g_1). taken from Bass, RMP 77 (2005) 1257



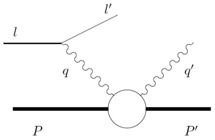
Left panel: the SLAC data for xg_2 for the proton and deuteron as a function of Q^2 for selected values of x. Data are from E155-03 (solid), E143 (open diamond) and E155-99 (open square). The curves show the twist-two xg_2^{WW} (solid) and the bag model calculation by Stratmann (dash-dot). Right panel: the Q^2-averaged structure function xg_2 from E155-03 (solid circle), E143 (open diamond) and E155-99. Also shown is the twist-two g_2^{WW} at the average Q^2 of E155-03 at each value of x (solid line), the bag model calculations by Stratmann (dash-dot-dot) and Song (dot) and the chiral soliton model of Weigel and Gamberg (dash-dot) and Wakamatsu (dash). Anthony *et al.*, Pl B 553 (2003) 18

light-cone dominance in DIS

$$W_{\mu\nu}(q;P,S) = \frac{1}{2\pi}\int \frac{d^3P_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta^4(q+P-P_X)\langle P,S|J_\mu(0)|P_X\rangle\langle P_X|J_\nu(0)|P,S\rangle$$

$$J_\mu(\xi) = e^{i\hat{P}\cdot\xi}J_\mu(0)\,e^{-i\hat{P}\cdot\xi} \quad\Longrightarrow\quad \langle P,S|J_\mu(0)|P_X\rangle = e^{i\xi\cdot(P-P_X)}\langle P,S|J_\mu(\xi)|P_X\rangle$$

$$\begin{aligned} W_{\mu\nu}(q,P) = \tfrac{1}{2}\sum_S W_{\mu\nu}(q;P,S) &= \frac{1}{2\pi}\int d^4\xi\, e^{i\xi\cdot q}\langle P|J_\mu(\xi)J_\nu(0)|P\rangle \\ &= \frac{1}{2\pi}\int d^4\xi\, e^{i\xi\cdot q}\langle P|[J_\mu(\xi),J_\nu(0)]|P\rangle \\ &= 2\pi\,\mathrm{Im}\,T_{\mu\nu} \end{aligned}$$



virtual Compton scattering

$$T_{\mu\nu} = i\int d^4\xi\, e^{i\xi\cdot q}\langle P|T[J_\mu(\xi)J_\nu(0)]|P\rangle$$

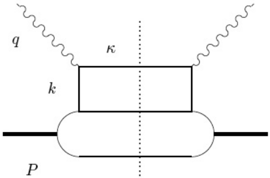
$$q^\mu = (\nu,0,0,-\sqrt{\nu^2+Q^2}) \;\to_{Bj}\; (\nu,0,0,-\nu-Mx)$$

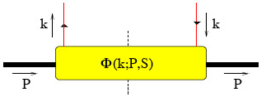
$$q^+ = -\frac{Mx}{\sqrt{2}}\ \text{fixed},\quad q^- = \frac{2\nu+Mx}{\sqrt{2}} \to \sqrt{2}\,\nu \to \infty$$

$$\exp(iq\cdot\xi) \to \exp(iq^+\xi^-) \quad\Longrightarrow\quad \xi^+ \to 0,\ \boldsymbol{\xi}_T \to 0$$

Jaffe, 1986

the quark-quark correlation function





$$W_{\mu\nu}(q;P,S) = \frac{1}{2\pi}\int \frac{d^3P_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta^4(q+P-P_X)\langle P,S|J_\mu(0)|P_X\rangle\langle P_X|J_\nu(0)|P,S\rangle$$

$$= \sum_a e_a^2 \int \frac{d^3P_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta^4(q+P-P_X)\int \frac{d^4\kappa}{(2\pi)^4}\,\delta(\kappa^2)\int d^4k\,\delta^4(k+q-\kappa)$$

$$\times\,[\overline{u}(\kappa)\gamma_\mu\phi(k;P,S)]^*\,[\overline{u}(\kappa)\gamma_\nu\phi(k;P,S)]$$

$$= \sum_a e_a^2\int d^4k\,\delta\left((k+q)^2\right)\,\mathrm{Tr}\,[\Phi(k;P,S)\gamma_\mu(\not k+\not q)\gamma_\nu]$$

with $\phi_i(k;P,S) = \langle P_X|\psi_i(0)|P,S\rangle$ and

$$\Phi_{ij}(k;P,S) = \frac{1}{(2\pi)^4}\int\frac{d^3P_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta^4(P-k-P_X)\langle P,S|\overline{\psi}_j(0)|P_X\rangle\langle P_X|\psi_i(0)|P,S\rangle$$

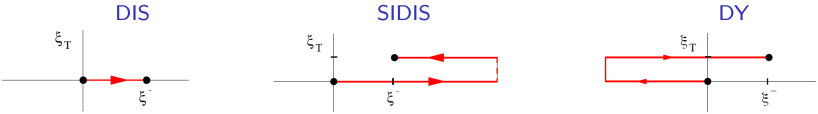
$$= \frac{1}{(2\pi)^4}\int d^4\xi\,e^{ik\cdot\xi}\langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle,$$

colour gauge invariance

$$\Phi_{ij}(P,k,S|n) = \frac{1}{(2\pi)^4}\int d^4\xi\, e^{ik\cdot\xi}\langle PS|\overline{\psi}_j(0)\,\mathcal{U}(0,\xi|n)\psi_i(\xi)|PS\rangle$$

Wilson line:

$$\begin{aligned}\mathcal{U}(0,\xi|n) &= \mathcal{P}\exp\left[-ig\int_0^{\xi} d\tau\, n^\mu A_\mu(\tau n)\right]\\ &= [0,0,\mathbf{0}_T;\infty^-,0,\mathbf{0}_T]\times[\infty^-,0,\mathbf{0}_T;\infty^-,\xi^+,\boldsymbol{\xi}_T]\times[\infty^-,\xi^+,\boldsymbol{\xi}_T;\xi^-,\xi^+,\boldsymbol{\xi}_T]\end{aligned}$$



the choice of the contour depends on the process under consideration

- hermiticity and parity:

$$\begin{aligned}\Phi^\dagger(P,p,S|n) &= \gamma_0\Phi(P,p,S|n)\gamma_0\\ \Phi(P,p,S|n) &= \gamma_0\,\Phi(\bar{P},\bar{p},-\bar{S}|\bar{n})\gamma_0, \qquad \bar{P}^\mu = (P^0,-\vec{P})\end{aligned}$$

- time-reversal does not give an additional constraint
- due to the Wilson line Lorentz invariance is violated

Goeke *et al.*, P.L. B 618 (2005) 90

$$
\begin{aligned}
\Phi(P,k,S|n) \;=\;& M A_1 + \not{P} A_2 + \not{k} A_3 + \frac{i}{2M}[\not{P},\not{k}] A_4 + i(k\cdot S)\gamma_5 A_5 + M\not{S}\gamma_5 A_6 \\
&+ \frac{k\cdot S}{M}\not{P}\gamma_5 A_7 + \frac{k\cdot S}{M}\not{k}\gamma_5 A_8 + \frac{[\not{P},\not{S}]}{2}\gamma_5 A_9 + \frac{[\not{k},\not{S}]}{2}\gamma_5 A_{10} \\
&+ \frac{(k\cdot S)}{2M^2}[\not{P},\not{k}]\gamma_5 A_{11} + \frac{1}{M}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu k_\rho S_\sigma A_{12} \\
&+ \frac{M^2}{P\cdot n_-}\not{n} B_1 + \frac{iM}{2P\cdot n_-}[\not{P},\not{n}] B_2 + \frac{iM}{2P\cdot n_-}[\not{k},\not{n}] B_3 \\
&+ \frac{1}{P\cdot n_-}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_5 P_\nu k_\rho n_{-\sigma} B_4 \\
&+ \frac{1}{P\cdot n_-}\varepsilon^{\mu\nu\rho\sigma}P_\mu k_\nu n_{-\rho} S_\sigma B_5 + \frac{iM^2}{P\cdot n_-}(n_-\cdot S)\gamma_5 B_6 \\
&+ \frac{M}{P\cdot n_-}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu n_{-\rho} S_\sigma B_7 + \frac{M}{P\cdot n_-}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu k_\nu n_{-\rho} S_\sigma B_8 \\
&+ \frac{(k\cdot S)}{M(P\cdot n_-)}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu k_\rho n_{-\sigma} B_9 + \frac{M(n_-\cdot S)}{(P\cdot n_-)^2}\varepsilon^{\mu\nu\rho\sigma}\gamma_\mu P_\nu k_\rho n_{-\sigma} B_{10} \\
&+ \frac{M}{P\cdot n_-}(n_-\cdot S)\not{P}\gamma_5 B_{11} + \frac{M}{P\cdot n_-}(n_-\cdot S)\not{k}\gamma_5 B_{12} \\
&+ \frac{M}{P\cdot n_-}(k\cdot S)\not{n}\gamma_5 B_{13} + \frac{M^3}{(P\cdot n_-)^2}(n_-\cdot S)\not{n}\gamma_5 B_{14} \\
&+ \frac{M^2}{2P\cdot n_-}[\not{n}_-,\not{S}]\gamma_5 B_{15} + \frac{(k\cdot S)}{2P\cdot n_-}[\not{P},\not{n}_-]\gamma_5 B_{16} + \frac{(k\cdot S)}{2P\cdot n_-}[\not{k},\not{n}_-]\gamma_5 B_{17} \\
&+ \frac{(n_-\cdot S)}{2P\cdot n_-}[\not{P},\not{k}]\gamma_5 B_{18} + \frac{M^2(n_-\cdot S)}{2(P\cdot n_-)^2}[\not{P},\not{n}_-]\gamma_5 B_{19} + \frac{M^2(n_-\cdot S)}{2(P\cdot n_-)^2}[\not{k},\not{n}_-]\gamma_5 B_{20}
\end{aligned}
$$

$$\Phi_{ij}(P,k,S|n) = \frac{1}{(2\pi)^4}\int d^4\xi\, e^{ik\cdot\xi}\langle PS|\overline{\psi}_j(0)\mathcal{U}(0,\xi|n)\psi_i(\xi)|PS\rangle$$

- k_T-dependent correlation function:

$$\begin{aligned}\Phi_{ij}(x,k_T) &= \int dk^-\Phi_{ij}(P,k,S|n)\\ &= \frac{1}{(2\pi)^3}\int d\xi^-\, d^2\boldsymbol{\xi}_T\, e^{i(k^+\xi^- - \mathbf{k}_T\cdot\boldsymbol{\xi}_T)}\langle P,S\,|\,\overline{\psi}_j(0)\,\mathcal{U}(0,\xi)\,\psi_i(\xi)\,|\,P,S\rangle\Big|_{\xi^+=0}\end{aligned}$$

$$\mathcal{U}(0,\xi) = \mathcal{U}(0,\xi|n)|_{\xi^+=0}$$

- integrating over k_T:

$$\begin{aligned}\Phi_{ij}(x) &= \int d^2\mathbf{k}_T\,\Phi_{ij}(x,k_T)\\ &= \frac{1}{2\pi}\int d\xi^-\, e^{ik^+\xi^-}\langle P,S\,|\,\overline{\psi}_j(0)\,\mathcal{U}(0,\xi)\,\psi_i(\xi)\,|\,P,S\rangle\Big|_{\xi^+=\boldsymbol{\xi}_T=0}\end{aligned}$$

- fully integrated:

$$\Phi_{ij} = \int d^4k\,\Phi_{ij}(P,k,S|n) = \langle P,S|\overline{\psi}_j(0)\psi_i(0)|P,S\rangle$$

- orthonormal basis set of Γ matrices: $\{\mathbf{1}, i\gamma_5, \gamma^\mu, \gamma^\mu\gamma_5, i\sigma^{\mu\nu}\gamma_5\}$
- inner product $(\Gamma_1, \Gamma_2) = \mathrm{Tr}\,[\Gamma_1^{-1}\Gamma_2]/4$

  $\gamma_5^{-1} = \gamma_5$, $(\gamma^\mu)^{-1} = \gamma_\mu$, and $\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$ with $(\sigma^{\mu\nu})^{-1} = \sigma_{\mu\nu}$
- thus

$$\Psi = \frac{1}{4}\left\{\mathbf{1}\mathrm{Tr}\,[\Psi] - i\gamma_5\mathrm{Tr}\,[i\gamma_5\Psi] + \gamma_\mu\mathrm{Tr}\,[\gamma^\mu\Psi] + \gamma_5\gamma_\mu\mathrm{Tr}\,[\gamma^\mu\gamma_5\Psi] + i\gamma_5\sigma_{\nu\mu}\mathrm{Tr}\,[i\sigma^{\mu\nu}\gamma_5\Psi]\right\}$$
$$= \frac{1}{2}\left\{\mathbf{1}\Psi^{[1]} - i\gamma_5\Psi^{[i\gamma_5]} + \gamma_\mu\Psi^{[\gamma^\mu]} + \gamma_5\gamma_\mu\Psi^{[\gamma^\mu\gamma_5]} - i\gamma_5\sigma_{\mu\nu}\Psi^{[i\sigma^{\mu\nu}\gamma_5]}\right\}$$

$$\Psi^{[\Gamma]} \equiv \frac{1}{2}\mathrm{Tr}\,[\Psi\Gamma]$$

- for example: $\Phi = \langle P, S|\overline{\psi}(0)\psi(0)|P, S\rangle$

$$= \frac{1}{2}\left\{M g_S + g_V \not{P} + M g_A \gamma_5 \not{S} + g_T \frac{1}{2}[\not{S}, \not{P}]\gamma_5\right\}$$

scalar	$\Phi^{[1]} = g_S M$
pseudoscalar	$\Phi^{[i\gamma_5]} = 0$
vector	$\Phi^{[\gamma^\mu]} = g_V P^\mu$
axial	$\Phi^{[\gamma^\mu\gamma_5]} = g_A M\, S^\mu$
tensor	$\Phi^{[i\sigma^{\mu\nu}\gamma_5]} = g_T\,(S^\mu P^\nu - S^\nu P^\mu)$

$$\Phi(x) = \tfrac{1}{2}\left\{ f_1(x)\, \not{n}_+ + \lambda\, g_1(x)\, \gamma_5\, \not{n}_+ + h_1(x)\, \gamma_5\, \tfrac{1}{2}[\, \not{S}_\perp,\, \not{n}_+] \right\}$$
$$+ \frac{M}{2P^+}\left\{ e(x) + g_T(x)\, \gamma_5\, \not{S}_\perp + \lambda\, h_L(x)\, \gamma_5\, \tfrac{1}{2}[\, \not{n}_+,\, \not{n}_-] \right\}$$
$$+ \frac{M}{2P^+}\left\{ -\lambda\, e_L(x)\, i\gamma_5 - f_T(x)\, \varepsilon_T^{\rho\sigma}\, \gamma_\rho\, S_{\perp\sigma} + h(x)\, i\,\tfrac{1}{2}[\, \not{n}_+,\, \not{n}_-] \right\}$$
$$+ \frac{M^2}{2(P^+)^2}\left\{ f_3(x)\, \not{n}_- + \lambda\, g_3(x)\, \gamma_5\, \not{n}_- + h_3(x)\, \gamma_5\, \tfrac{1}{2}[\, \not{S}_\perp,\, \not{n}_-] \right\}$$

twist-2

$$\Phi^{[\gamma^+]}(x) = f_1(x)$$
$$\Phi^{[\gamma^+\gamma_5]}(x) = \lambda\, g_1(x)$$
$$\Phi^{[i\sigma^{i+}\gamma_5]}(x) = S_\perp^i\, h_1(x)$$

$$n_+^\mu = [0, 1, \mathbf{0}_T]$$
$$n_-^\mu = [1, 0, \mathbf{0}_T]$$

twist-3

$$\Phi^{[1]}(x) = \frac{M}{P^+}\, e(x)$$
$$\Phi^{[i\gamma_5]}(x) = \frac{M}{P^+}\, e_L(x)$$
$$\Phi^{[\gamma^i]}(x) = -\frac{M\varepsilon_T^{i\rho}\, S_{\perp\rho}}{P^+}\, f_T(x)$$
$$\Phi^{[\gamma^i\gamma_5]}(x) = \frac{MS_\perp^i}{P^+}\, g_T(x)$$
$$\Phi^{[i\sigma^{+-}\gamma_5]}(x) = \frac{M}{P^+}\, \lambda\, h_L(x)$$
$$\Phi^{[i\sigma^{ij}\gamma_5]}(x) = \frac{M}{P^+}\, \varepsilon_T^{ij}\, \lambda\, h(x)$$

twist-4

$$\Phi^{[\gamma^-]}(x) = f_3(x)$$
$$\Phi^{[\gamma^-\gamma_5]}(x) = \lambda\, g_3(x)$$
$$\Phi^{[i\sigma^{i-}\gamma_5]}(x) = S_\perp^i\, h_3(x)$$

$$\varepsilon_T^{\alpha\beta}:\ \varepsilon_T^{11} = \varepsilon_T^{22} = -1$$
$$\varepsilon_T^{12} = -\varepsilon_T^{21} = 1$$

parton distributions

with $\lambda = 1, S_\perp^i = (1,0)$

$$\begin{bmatrix} f_1(x) \\ g_1(x) \\ h_1(x) \end{bmatrix} = \begin{bmatrix} \Phi^{[\gamma^+]}(x) \\ \Phi^{[\gamma^+\gamma_5]}(x) \\ \Phi^{[i\sigma^{i+}\gamma_5]}(x) \end{bmatrix} = \frac{1}{2}\int \frac{d\xi^-}{2\pi}\, e^{ixP^+\xi^-}\, \langle PS|\overline{\psi}(0) \begin{bmatrix} \gamma^+ \\ \gamma^+\gamma_5 \\ \gamma^+\gamma^1\gamma_5 \end{bmatrix} \psi(0,\xi^-,\mathbf{0}_T)|PS\rangle$$

- decompose into "good" and "bad" components: $\psi = \psi_{(+)} + \psi_{(-)}$, $\psi_{(\pm)} = \frac{1}{2}\gamma^\mp\gamma^\pm\psi$

$$\begin{bmatrix} f_1(x) \\ g_1(x) \\ h_1(x) \end{bmatrix} = \frac{1}{\sqrt{2}}\int \frac{d\xi^-}{2\pi}\, e^{ixP^+\xi^-}\, \langle PS|\psi^\dagger_{(+)}(0) \begin{bmatrix} 1 \\ \gamma_5 \\ \gamma^1\gamma_5 \end{bmatrix} \psi_{(+)}(0,\xi^-,\mathbf{0}_T)|PS\rangle$$

- define projectors $\mathcal{P}_\pm = \frac{1}{2}(1\pm\gamma^5)$ (for helicity) and $\mathcal{P}_{\uparrow\downarrow} = \frac{1}{2}(1\pm\gamma^1\gamma^5)$ (for transversity)

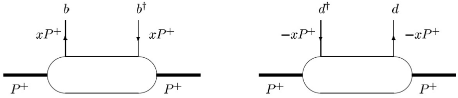
$$\begin{bmatrix} f_1(x) \\ g_1(x) \\ h_1(x) \end{bmatrix} = \frac{1}{\sqrt{2}}\sum_n \delta\left((1-x)P^+ - P_n^+\right) \begin{bmatrix} \left|\langle PS|\psi_{(+)}(0)|n\rangle\right|^2 \\ \left|\langle PS|\mathcal{P}_+\psi_{(+)}(0)|n\rangle\right|^2 - \left|\langle PS|\mathcal{P}_-\psi_{(+)}(0)|n\rangle\right|^2 \\ \left|\langle PS|\mathcal{P}_\uparrow\psi_{(+)}(0)|n\rangle\right|^2 - \left|\langle PS|\mathcal{P}_\downarrow\psi_{(+)}(0)|n\rangle\right|^2 \end{bmatrix}$$

parton distributions and Fock-state decomposition

$$\psi^{q}_{(+)}(z^-, \boldsymbol{z}_\perp) = \int \frac{dk^+ d\boldsymbol{k}_\perp}{2k^+(2\pi)^3}\,\Theta(k^+) \sum_{\mu} \left\{ b_q(w) u_+(k,\mu)\, e^{-ik^+z^- + i\boldsymbol{k}_\perp\cdot\boldsymbol{z}_\perp} + d_q^\dagger(w) v_+(k,\mu)\, e^{+ik^+z^- - i\boldsymbol{k}_\perp\cdot\boldsymbol{z}_\perp} \right\}$$

$$\begin{aligned}
\int \frac{dy^-}{2\pi}\, e^{ixP^+y^-}\, \overline{\psi}(-\tfrac{1}{2}y)\,\gamma^+\,\psi(\tfrac{1}{2}y) \quad = \quad & 2\sqrt{2}\int \frac{dk'^+ d\boldsymbol{k}'_\perp}{2k'^+(2\pi)^3}\Theta(k'^+) \int \frac{dk^+ d\boldsymbol{k}_\perp}{2k^+(2\pi)^3}\Theta(k^+) \\
& \times \sum_{\mu,\mu'} \Big\{ \delta(2xP^+ - k'^+ - k^+)\, b_q^\dagger(w') b_q(w)\, u_+^\dagger(k',\mu') u_+(k,\mu) \\
& \quad + \delta(2xP^+ + k'^+ + k^+)\, d_q(w') d_q^\dagger(w)\, v_+^\dagger(k',\mu') v_+(k,\mu) \\
& \quad + \delta(2xP^+ + k'^+ - k^+)\, d_q(w') b_q(w)\, v_+^\dagger(k',\mu') u_+(k,\mu) \\
& \quad + \delta(2xP^+ - k'^+ + k^+)\, b_q^\dagger(w') d_q^\dagger(w)\, u_+^\dagger(k',\mu') v_+(k,\mu) \Big\}
\end{aligned}$$

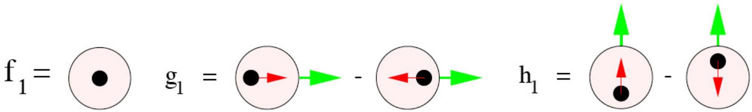
$$f_1^q(x) = \frac{1}{2(2\pi)^3} \int \frac{dk^+ d\boldsymbol{k}_\perp}{2k^+(2\pi)^3}\Theta(k^+) \sum_{\mu} \left\{ \delta\left(x - \frac{k^+}{P^+}\right) \langle P | b_q^\dagger(w) b_q(w) | P\rangle + \delta\left(x + \frac{k^+}{P^+}\right) \langle P | d_q(w) d_q^\dagger(w) | P \rangle \right\}$$



(a) (b)

parton distributions as probabilities

$$\begin{bmatrix} f_1(x) \\ g_1(x) \\ h_1(x) \end{bmatrix} = \frac{1}{\sqrt{2}} \sum_n \delta\big((1-x)P^+ - P_n^+\big) \begin{bmatrix} |\langle PS|\psi_{(+)}(0)|n\rangle|^2 \\ \big|\langle PS|\mathcal{P}_+\psi_{(+)}(0)|n\rangle\big|^2 - \big|\langle PS|\mathcal{P}_-\psi_{(+)}(0)|n\rangle\big|^2 \\ \big|\langle PS|\mathcal{P}_\uparrow\psi_{(+)}(0)|n\rangle\big|^2 - \big|\langle PS|\mathcal{P}_\downarrow\psi_{(+)}(0)|n\rangle\big|^2 \end{bmatrix}$$



$$|g_1^a(x)| \leq f_1^a(x), \qquad |h_1^a(x)| \leq f_1^a(x)$$

- Soffer inequality: $|h_1^a(x)| \leq \frac{1}{2}[f_1^a(x) + g_1^a(x)]$

including antiquarks

$$\bar{\Phi}_{ij}(P,k,S|n) = \frac{1}{(2\pi)^4}\int d^4\xi\, e^{ik\cdot\xi}\langle PS|\psi_j(0)\overline{\psi}_i(\xi)|PS\rangle$$

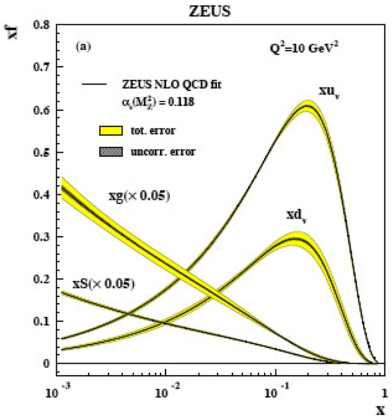
$$\langle PS|\overline{\psi}_j(0)\psi_i(\xi)|PS\rangle = -\langle PS|\psi_j(0)\overline{\psi}_i(\xi)|PS\rangle$$

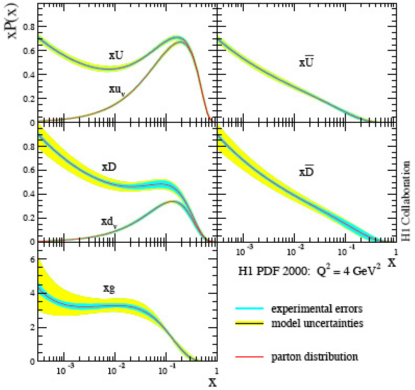
$$\Longrightarrow \quad \begin{aligned} \bar{f}_1(x) &= -f_1(-x) \\ \bar{g}_1(x) &= g_1(-x) \\ \bar{h}_1(x) &= h_1(-x) \end{aligned}$$

N.B. $W^{(S)}_{\mu\nu} = \sum_a e_a^2 (n_\mu\, P^\nu + n_\nu\, P^\mu - g_{\mu\nu})\ \left[f_1^a(x) + \bar{f}_1^a(x)\right]$

$$\Longrightarrow \quad F_2 = 2x\, F_1 = \sum_a e_a^2\, x\ \left[f_1^a(x) + \bar{f}_1^a(x)\right] \qquad \text{Callan-Gross}$$

similarly: $W^{(A)}_{\mu\nu} = \lambda\, \varepsilon_{\mu\nu\rho\sigma} n^\rho p^\sigma \frac{1}{2} \sum_a e_a^2 \left[g_1^a(x) + \bar{g}_1^a(x)\right]$

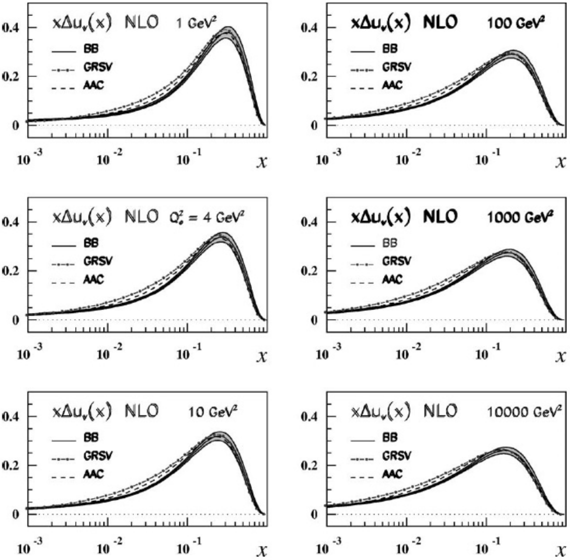




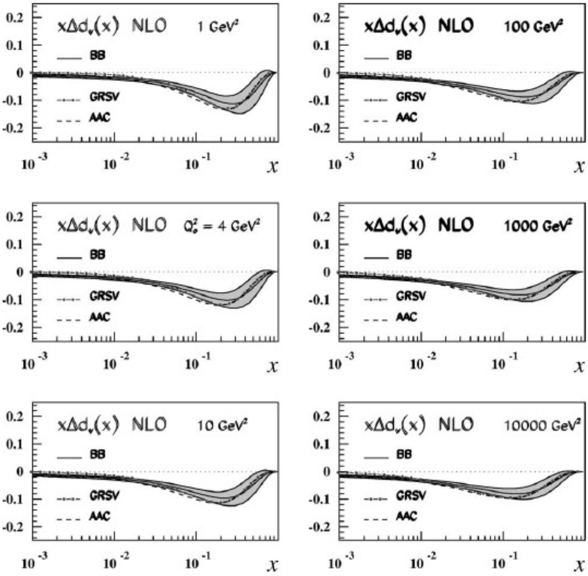
the gluon, sea, u and d valence distributions extracted from

the ZEUS NLO QCD fit at $Q^2 = 10$ GeV^2
Chekanov *et al.*, PR D 67 (2003) 012007

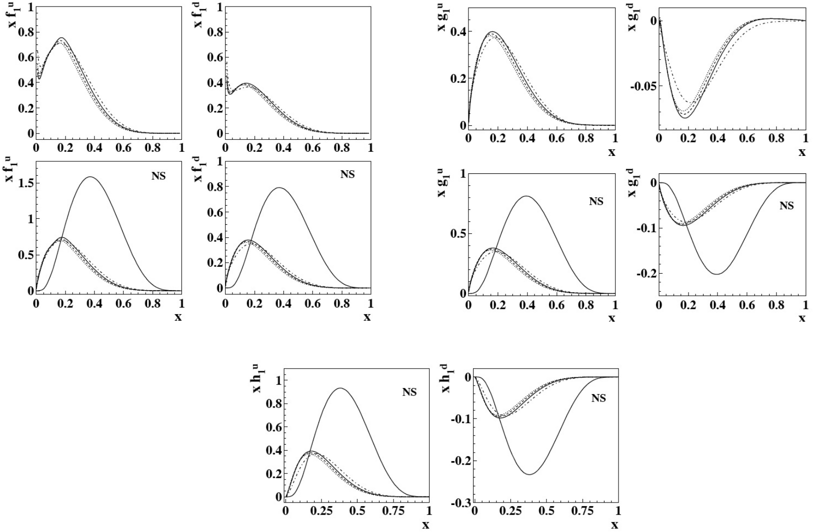
the H1 starting scale at $Q^2 = 4$ GeV^2
Adloff *et al.*, EPJ C 30 (2003) 1



the polarized helicity distributions $x\Delta u_v$ evolved up to $Q^2 = 10\ 000\ \mathrm{GeV}^2$ Blümlein, Böttcher, NP B 636 (2002) 225



the polarized helicity dstributions $x\Delta d_v$ evolved up to $Q^2 = 10\ 000\ \mathrm{GeV}^2$ Blümlein, Böttcher, NP B 636 (2002) 225



B. Pasquini, M. Pincetti, S. B., hep-ph/0612094

first moments of parton distributions

$$\int_{-1}^{+1} dx\, f_1(x) \quad = \int_0^1 dx\left[f_1(x) - \overline{f}_1(x)\right] = g_V \qquad \text{vector charge} = \text{valence number}$$

$$\text{non} - \text{singlet}$$

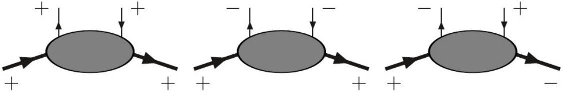
$$\int_{-1}^{+1} dx\, g_1(x) \quad = \int_0^1 dx\left[g_1(x) + \overline{g}_1(x)\right] = g_A \qquad \text{axial charge} = \text{net number}$$

$$\text{singlet} \qquad \text{of L quarks in L proton}$$

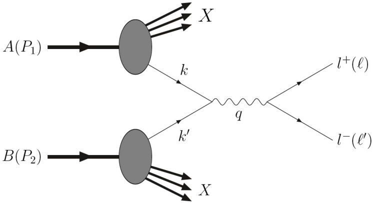
$$\int_{-1}^{+1} dx\, h_1(x) \quad = \int_0^1 dx\left[h_1(x) - \overline{h}_1(x)\right] = g_T \qquad \text{tensor charge} = \text{net number}$$

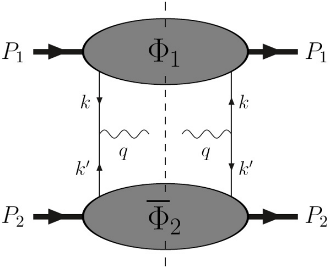
$$\text{non} - \text{singlet} \qquad \text{of T quarks in T proton}$$

no access to transversity in inclusive DIS

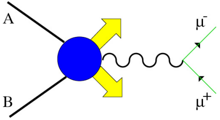


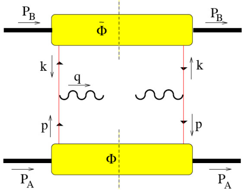
possible access to transversity in Drell-Yan processes





the Drell-Yan process with Born diagram





for unpolarized hadrons

$$\frac{d\sigma_{UU}(AB \;\rightarrow\; \mu^+\mu^- X)}{dx_A\, dx_B\, dy} = \frac{4\pi\alpha^2}{3Q^2}\left(\tfrac{1}{2} - y + y^2\right) f_1(x_A)\overline{f}_1(x_B)$$

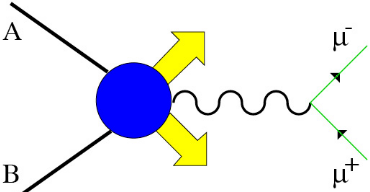
for longitudinally polarized hadrons

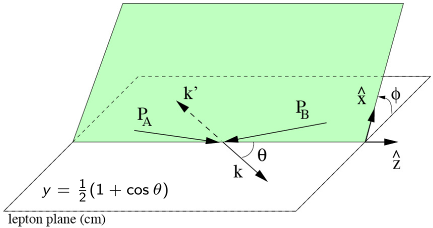
$$\frac{d\sigma_{LL}(\vec{A}\vec{B} \;\rightarrow\; \mu^+\mu^- X)}{dx_A\, dx_B\, dy} = \frac{4\pi\alpha^2}{3Q^2}\left(\tfrac{1}{2} - y + y^2\right) \lambda_A\, \lambda_B\, g_1(x_A)\overline{g}_1(x_B)$$

for transversely polarized hadrons

$$\frac{d\sigma_{TT}(\vec{A}\vec{B} \;\rightarrow\; \mu^+\mu^- X)}{dx_A\, dx_B\, dy} = \frac{4\pi\alpha^2}{3Q^2}\, y\,(1-y)\, |\boldsymbol{S}_{A\perp}|\, |\boldsymbol{S}_{B\perp}|\, h_1(x_A)\overline{h}_1(x_B)$$

transversity and the Drell-Yan dilepton production



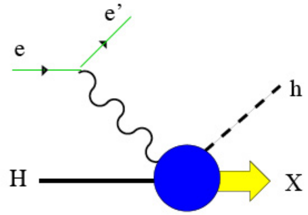


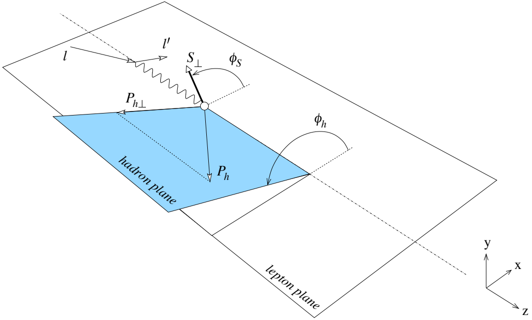
double transverse-spin asymmetry

$$A_{TT}^{pp} = \frac{d\sigma^{\Uparrow\Uparrow} - d\sigma^{\Uparrow\Downarrow}}{d\sigma^{\Uparrow\Uparrow} + d\sigma^{\Uparrow\Downarrow}}$$

$$= \frac{\sin^2\theta}{1+\cos^2\theta}\cos(2\phi)\,\frac{\sum_a e_a^2\left[h_1^a(x_1,Q^2)\bar{h}_1^a(x_2,Q^2) + (1\leftrightarrow 2)\right]}{\sum_a e_a^2\left[f_1^a(x_1,Q^2)\bar{f}_1^a(x_2,Q^2) + (1\leftrightarrow 2)\right]}$$

semi-inclusive DIS (SIDIS)



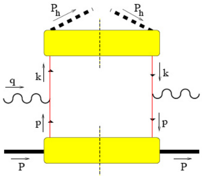


$$\frac{d\sigma}{dx\, dy\, d\psi\, dz\, d\phi_h\, dP_{h\perp}^2} = \frac{\alpha^2 y}{8z\, Q^4}\, \frac{1}{2M}\, L_{\mu\nu}\, \mathcal{W}^{\mu\nu}$$

$$x = \frac{Q^2}{2\, P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot l}, \quad z = \frac{P\cdot P_h}{P\cdot q}, \quad z_h = -\frac{Q^2}{2P_h\cdot q}, \quad \text{in DIS} : d\psi \approx d\phi_S$$

Trento convention, Bacchetta *et al.*, hep-ph/0410050

the SIDIS hadron tensor



$$\mathcal{W}_{\mu\nu}(q; PS; P_h S_h)$$

$$= \frac{1}{(2\pi)^4} \int \frac{d^3 P_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta^4(q + P - P_X - P_h) \langle PS|J_\mu(0)|P_X; P_h S_h\rangle \langle P_X; P_h S_h|J_\mu(0)|PS\rangle$$

$$= \sum_a e_a^2 \int \frac{d^3 P_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta^4(P - p - P_X) \int \frac{d^4 p}{(2\pi)^4} \delta(p + q - k) \int \frac{d^4 k}{(2\pi)^4} \delta^4(k - P_h - P_{X'})$$

$$\times [\overline{\chi}(k; P_h, S_h)\gamma_\mu \phi(p; P, S)]^* [\overline{\chi}(k; P_h, S_h)\gamma_\nu \phi(p; P, S)]$$

$$= \int d^4 p \int d^4 k\, \delta^4(p + q - k) \mathrm{Tr}\left[\Phi(p; P, S)\gamma_\mu \Delta(k; P_h, S_h)\gamma_\nu\right] + \begin{Bmatrix} q \leftrightarrow -q \\ \mu \leftrightarrow \nu \end{Bmatrix}$$

with $\chi_i(k; P_h, S_h) = \langle 0|\psi_i(0)|P_X; P_h S_h\rangle$, $\phi_i(p; P, S) = \langle P_X|\psi_i(0)|PS\rangle$ and

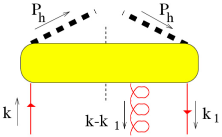
$$\Phi_{ij}(k; P, S) = \frac{1}{(2\pi)^4} \int \frac{d^3 P_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta^4(P - p - P_X) \langle P, S|\overline{\psi}_j(0)|P_X\rangle \langle P_X|\psi_i(0)|P, S\rangle$$

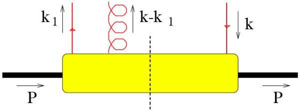
$$= \frac{1}{(2\pi)^4} \int d^4\xi\, e^{ik\cdot\xi} \langle P, S|\overline{\psi}_j(0)\psi_i(\xi)|P, S\rangle,$$

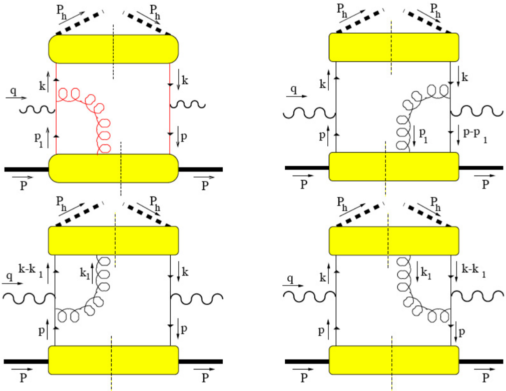
$$\Delta_{ij}(k; P_h, S_h) = \frac{1}{(2\pi)^4} \int \frac{d^3 P_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \delta^4(P_h + P_X - k) \langle 0|\psi_i(0)|P_X; P_h S_h\rangle \langle P_X; P_h S_h|\overline{\psi}_j(0)|0\rangle$$

$$= \sum_X \frac{1}{(2\pi)^4} \int d^4\xi\, e^{ik\cdot\xi} \langle 0|\psi_i(\xi)|X; P_h S_h\rangle \langle X; P_h S_h|\overline{\psi}_j(0)|0\rangle$$

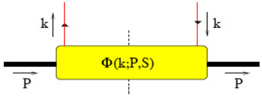
including gluons







k_T-dependent correlation functions



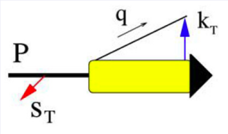
$$
\begin{aligned}
\Phi(x, k_T) &= \int dk^- \, \Phi(k; P, S)\Big|_{k^+ = xP^+, k_T} \\
&= \frac{1}{2}\Big\{ f_1 \, \not{n}_+ - f_{1T}^{\perp} \frac{\epsilon_T^{\rho\sigma} k_{T\rho} S_{T\sigma}}{M} \, \not{n}_+ + g_{1s} \gamma_5 \, \not{n}_+ \\
&\qquad + h_{1T} \frac{[\not{S}_T, \not{n}_+]\gamma_5}{2} + h_{1s}^{\perp} \frac{[\not{k}_T, \not{n}_+]\gamma_5}{2M} + h_1^{\perp} \, i \, \frac{[\not{k}_T, \not{n}_+]}{2M} \Big\} \\
&+ \frac{M}{2P^+}\Big\{ e - e_s \, i \, \gamma_5 - e_T^{\perp} \frac{\epsilon_T^{\rho\sigma} k_{T\rho} S_{T\sigma}}{M} \\
&\qquad + f^{\perp} \frac{\not{k}_T}{M} - f_T' \, \epsilon_T^{\rho\sigma} \gamma_\rho S_{T\sigma} - f_s^{\perp} \frac{\epsilon_T^{\rho\sigma} \gamma_\rho k_{T\sigma}}{M} \\
&\qquad + g_T' \, \gamma_5 \, \not{S}_T + g_s^{\perp} \gamma_5 \frac{\not{k}_T}{M} - g^{\perp} \gamma_5 \frac{\epsilon_T^{\rho\sigma} \gamma_\rho k_{T\sigma}}{M} \\
&\qquad + h_s \frac{[\not{n}_+, \not{n}_-]\gamma_5}{2} + h_T^{\perp} \frac{[\not{S}_T, \not{k}_T]\gamma_5}{2M} + h \, i \, \frac{[\not{n}_+, \not{n}_-]}{2} \Big\}
\end{aligned}
$$

N.B. subscript s, e.g.: $g_{1s}(x, k_T^2) = S_L \, g_{1L}(x, k_T^2) - \dfrac{k_T \cdot S_T}{M} \, g_{1T}(x, k_T^2)$

N.B. e.g.: $f_1(x) = \int d^2\boldsymbol{k}_T f_1(x, k_T^2)$

correlation between target transverse polarization and quark transverse momentum

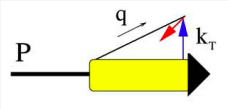
Sivers function: $f_{1T}^{\perp}$



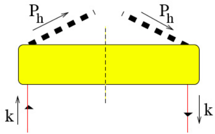
N.B. $f_{1T}^{\perp}(x, k_T^2)|_{SIDIS} = -f_{1T}^{\perp}(x, k_T^2)|_{DY}$

correlation between quark transverse spin and momentum

Boer-Mulders function: $h_1^{\perp}$



quark fragmentation function



$$\Delta_{kl}(k;P_h,S_h) = \sum_X \frac{1}{(2\pi)^4}\int d^4\xi\, e^{ik\cdot\xi}\langle 0|\psi_k(\xi)|X;P_hS_h\rangle\langle X;P_hS_h|\overline{\psi}_l(0)|0\rangle$$

$$= \tfrac{1}{2}\left\{\mathcal{S}\,1 + \mathcal{P}\,i\gamma_5 + \mathcal{V}_\mu\,\gamma^\mu + \mathcal{A}_\mu\,\gamma^\mu\gamma_5 + \mathcal{T}_{\mu\nu}\,i\tfrac{1}{2}\sigma^{\mu\nu}\gamma_5\right\}$$

$$\Delta_{ij}(z,k_T) = \frac{1}{2z}\int dk^+\,\Delta_{ij}(k;P_h,S_h)\Big|_{k^-=P_h^-/z,\,k_T}$$

$$= \frac{1}{2z}\sum_X\int\frac{d\xi^+d^2\boldsymbol{\xi}_T}{(2\pi)^3}\,e^{ik\cdot\xi}\,\langle 0|\,\psi_i(\xi)|h,X\rangle\langle h,X|\bar{\psi}_j(0)\,|0\rangle\Big|_{\xi^-=0}$$

$$\Delta(z) = z^2\int d^2\mathbf{k}_T\,\Delta(z,k_T)$$

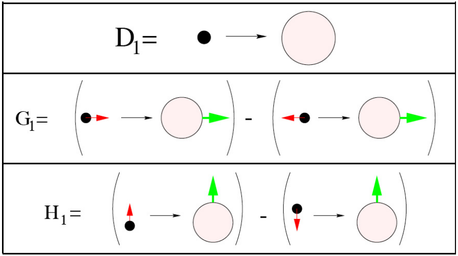
z^2 because probability density w.r.t. $k_T' = -zk_T$

$$\Delta(z) = z^2 \int d^2\mathbf{k}_T\, \Delta(z, k_T) = \frac{z}{2} \int dk^+ d\mathbf{k}_T \Delta(k; P_h, S_h)$$

$$= \tfrac{1}{4} \left\{ D_1(z)\, \not{n}_- - \lambda_h\, G_1(z)\, \not{n}_- \gamma_5 + H_1(z)\, \tfrac{1}{2}\, [\not{S}_{hT},\, \not{n}_-]\gamma_5 \right\}$$

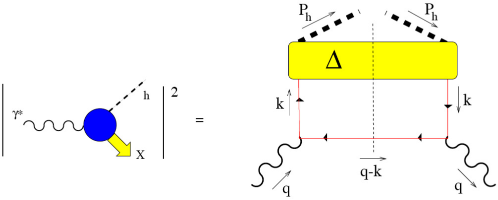
$$+ \frac{M_h}{4P_h^-} \left\{ D_T(z)\, \varepsilon_T^{\rho\sigma}\, \gamma_\rho\, S_{hT\sigma} + E(z) - \lambda_h\, E_L(z)\, i\gamma_5 \right.$$

$$\left. - G_T(z)\, \not{S}_{hT}\, \gamma_5 + \lambda_h\, H_L(z)\, \tfrac{1}{2} [\not{n}_-,\, \not{n}_+]\gamma_5 + i H(z) \tfrac{1}{2}\, [\not{n}_-,\, \not{n}_+] \right\}$$



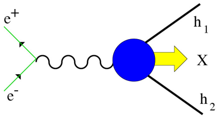
quark fragmentation function

- one-hadron inclusive e^+-e^- annihilation



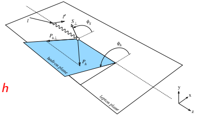
$$\frac{d\sigma(e^+e^- \to h\,X)}{d\Omega\, dz_h} \sim \sum_a e_a^2\, D_1^a(z_h)$$

- two-hadron inclusive e^+-e^- annihilation



$$\frac{d\sigma(e^+e^- \to h_1 h_2 X)}{d\Omega\, dz_1\, dz_2} \sim \sum_a e_a^2\, D_1^a(z_1) D_1^{\bar{a}}(z_2)$$

SIDIS



$$
\begin{aligned}
\frac{d\sigma}{dx\, dy\, d\psi\, dz\, d\phi_h\, dP_{h\perp}^2} = {} & \\
\frac{\alpha^2}{xy\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right) \Bigg\{ & F_{UU,T} + \varepsilon\, F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \cos\phi_h\, F_{UU}^{\cos\phi_h} \\
& + \varepsilon \cos(2\phi_h)\, F_{UU}^{\cos 2\phi_h} + \lambda_e\, \sqrt{2\,\varepsilon(1-\varepsilon)}\, \sin\phi_h\, F_{LU}^{\sin\phi_h} \\
& + S_{\parallel} \left[\sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_h\, F_{UL}^{\sin\phi_h} + \varepsilon \sin(2\phi_h)\, F_{UL}^{\sin 2\phi_h} \right] \\
& + S_{\parallel}\, \lambda_e \left[\sqrt{1-\varepsilon^2}\, F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_h\, F_{LL}^{\cos\phi_h} \right] \\
& + |\mathbf{S}_\perp| \Bigg[\sin(\phi_h-\phi_S) \left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{\sin(\phi_h-\phi_S)} \right) \\
& \qquad + \varepsilon\, \sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\, \sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)} \\
& \qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin\phi_S\, F_{UT}^{\sin\phi_S} + \sqrt{2\,\varepsilon(1+\varepsilon)}\, \sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)} \Bigg] \\
& + |\mathbf{S}_\perp|\lambda_e \Bigg[\sqrt{1-\varepsilon^2}\, \cos(\phi_h-\phi_S)\, F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos\phi_S\, F_{LT}^{\cos\phi_S} \\
& \qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\, \cos(2\phi_h-\phi_S)\, F_{LT}^{\cos(2\phi_h-\phi_S)} \Bigg] \Bigg\}
\end{aligned}
$$

N.B. $F \equiv F(x, Q^2, z, P_{h\perp}^2)$ Bacchetta *et al.*, hep-ph/0611265 introduce the unit vector $\hat{\mathbf{h}} = \mathbf{P}_{h\perp}/|\mathbf{P}_{h\perp}|$ and the notation

$$\mathcal{C}\left[w\, f\, D\right] = x \sum_a e_a^2 \int d^2\mathbf{p}_T\, d^2\mathbf{k}_T\, \delta^{(2)}\left(\mathbf{p}_T - \mathbf{k}_T - \mathbf{P}_{h\perp}/z\right) w(\mathbf{p}_T, \mathbf{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2),$$

$$F_{UU,T} = \mathcal{C}\left[f_1 D_1\right],$$

$$F_{UU,L} = 0,$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\, \mathcal{C}\left[-\frac{\hat{\mathbf{h}}\cdot\mathbf{k}_T}{M_h}\left(xh\, H_1^{\perp} + \frac{M_h}{M}\, f_1\, \frac{\tilde{D}^{\perp}}{z}\right) - \frac{\hat{\mathbf{h}}\cdot\mathbf{p}_T}{M}\left(xf^{\perp} D_1 + \frac{M_h}{M}\, h_1^{\perp}\, \frac{\tilde{H}}{z}\right)\right]$$

$$\approx \frac{2M}{Q}\, \mathcal{C}\left[-\frac{\hat{\mathbf{h}}\cdot\mathbf{p}_T}{M}\, f_1 D_1\right], \qquad \text{Cahn effect}$$

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\mathbf{h}}\cdot\mathbf{k}_T\right)\left(\hat{\mathbf{h}}\cdot\mathbf{p}_T\right) - \mathbf{k}_T\cdot\mathbf{p}_T}{MM_h}\, h_1^{\perp} H_1^{\perp}\right], \qquad \text{Boer-Mulders and Collins functions}$$

$$[\lambda_e] \quad F_{LU}^{\sin\phi_h} = \frac{2M}{Q}\, \mathcal{C}\left[-\frac{\hat{\mathbf{h}}\cdot\mathbf{k}_T}{M_h}\left(xe\, H_1^{\perp} + \frac{M_h}{M}\, f_1\, \frac{\tilde{G}^{\perp}}{z}\right) + \frac{\hat{\mathbf{h}}\cdot\mathbf{p}_T}{M}\left(xg^{\perp} D_1 + \frac{M_h}{M}\, h_1^{\perp}\, \frac{\tilde{E}}{z}\right)\right],$$

$$[S_{\parallel}] \quad F_{UL}^{\sin\phi_h} = \frac{2M}{Q}\, \mathcal{C}\left[-\frac{\hat{\mathbf{h}}\cdot\mathbf{k}_T}{M_h}\left(xh_L H_1^{\perp} + \frac{M_h}{M}\, g_{1L}\, \frac{\tilde{G}^{\perp}}{z}\right) + \frac{\hat{\mathbf{h}}\cdot\mathbf{p}_T}{M}\left(xf_L^{\perp} D_1 - \frac{M_h}{M}\, h_{1L}^{\perp}\, \frac{\tilde{H}}{z}\right)\right],$$

$$[S_{\parallel}] \quad F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\mathbf{h}}\cdot\mathbf{k}_T\right)\left(\hat{\mathbf{h}}\cdot\mathbf{p}_T\right) - \mathbf{k}_T\cdot\mathbf{p}_T}{MM_h}\, h_{1L}^{\perp} H_1^{\perp}\right],$$

$$[S_{\parallel}\lambda_e] \quad F_{LL} = \mathcal{C}\left[g_{1L} D_1\right],$$

$$[S_{\parallel}\lambda_e] \quad F_{LL}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[\frac{\hat{\mathbf{h}}\cdot\mathbf{k}_T}{M_h}\left(x e_L H_1^{\perp} - \frac{M_h}{M}\, g_{1L}\,\frac{\tilde{D}^{\perp}}{z}\right) - \frac{\hat{\mathbf{h}}\cdot\mathbf{p}_T}{M}\left(x g_L^{\perp} D_1 + \frac{M_h}{M}\, h_{1L}^{\perp}\,\frac{\tilde{E}}{z}\right)\right],$$

$$[|\boldsymbol{S}_{\perp}|] \quad F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\mathbf{h}}\cdot\mathbf{p}_T}{M}\, f_{1T}^{\perp} D_1\right],$$ Sivers function

$$[|\boldsymbol{S}_{\perp}|] \quad F_{UT,L}^{\sin(\phi_h-\phi_S)} = 0,$$

$$[|\boldsymbol{S}_{\perp}|] \quad F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\mathbf{h}}\cdot\mathbf{k}_T}{M_h}\, h_1 H_1^{\perp}\right],$$ transversity and Collins function

$$[|\boldsymbol{S}_{\perp}|] \quad F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[\frac{2\,(\hat{\mathbf{h}}\cdot\mathbf{p}_T)\,(\mathbf{p}_T\cdot\mathbf{k}_T) + \mathbf{p}_T^2\,(\hat{\mathbf{h}}\cdot\mathbf{k}_T) - 4\,(\hat{\mathbf{h}}\cdot\mathbf{p}_T)^2\,(\hat{\mathbf{h}}\cdot\mathbf{k}_T)}{2M^2 M_h}\, h_{1T}^{\perp} H_1^{\perp}\right],$$

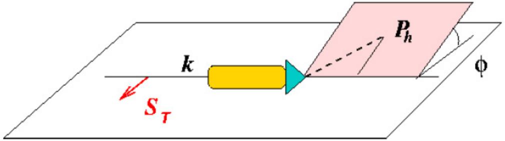
$$[|\boldsymbol{S}_{\perp}|] \quad F_{UT}^{\sin\phi_S} = \frac{2M}{Q}\,\mathcal{C}\left\{\left(x f_T D_1 - \frac{M_h}{M}\, h_1\,\frac{\tilde{H}}{z}\right) - \frac{\mathbf{k}_T\cdot\mathbf{p}_T}{2MM_h}\left[\left(x h_T H_1^{\perp} + \frac{M_h}{M}\, g_{1T}\,\frac{\tilde{G}^{\perp}}{z}\right) - \left(x h_T^{\perp} H_1^{\perp} - \frac{M_h}{M}\, f_{1T}^{\perp}\,\frac{\tilde{D}^{\perp}}{z}\right)\right]\right\},$$

$$[|\boldsymbol{S}_{\perp}|] \quad F_{UT}^{\sin(2\phi_h-\phi_S)} = \frac{2M}{Q}\,\mathcal{C}\left\{\frac{2\,(\hat{\mathbf{h}}\cdot\mathbf{p}_T)^2 - \mathbf{p}_T^2}{2M^2}\left(x f_T^{\perp} D_1 - \frac{M_h}{M}\, h_{1T}^{\perp}\,\frac{\tilde{H}}{z}\right) - \frac{2\,(\hat{\mathbf{h}}\cdot\mathbf{k}_T)\,(\hat{\mathbf{h}}\cdot\mathbf{p}_T) - \mathbf{k}_T\cdot\mathbf{p}_T}{2MM_h}\left[\left(x h_T H_1^{\perp} + \frac{M_h}{M}\, g_{1T}\,\frac{\tilde{G}^{\perp}}{z}\right) + \left(x h_T^{\perp} H_1^{\perp} - \frac{M_h}{M}\, f_{1T}^{\perp}\,\frac{\tilde{D}^{\perp}}{z}\right)\right]\right\},$$

$$[|S_\perp|\lambda_e] \quad F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\left[\frac{\hat{\mathbf{h}}\cdot\mathbf{p}_T}{M} g_{1T} D_1\right],$$

$$[|S_\perp|\lambda_e] \quad F_{LT}^{\cos\phi_S} = \frac{2M}{Q}\,\mathcal{C}\Big\{-\Big(x g_T D_1 + \frac{M_h}{M}\, h_1 \frac{\tilde{E}}{z}\Big) + \frac{\mathbf{k}_T\cdot\mathbf{p}_T}{2MM_h}\Big[\Big(x e_T H_1^\perp - \frac{M_h}{M} g_{1T}\frac{\tilde{D}^\perp}{z}\Big) + \Big(x e_T^\perp H_1^\perp + \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{G}^\perp}{z}\Big)\Big]\Big\},$$

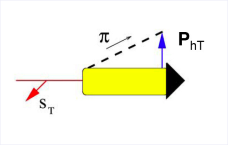
$$[|S_\perp|\lambda_e] \quad F_{LT}^{\cos(2\phi_h-\phi_S)} = \frac{2M}{Q}\,\mathcal{C}\Big\{-\frac{2\,(\hat{\mathbf{h}}\cdot\mathbf{p}_T)^2 - \mathbf{p}_T^2}{2M^2}\Big(x g_T^\perp D_1 + \frac{M_h}{M}\, h_{1T}^\perp \frac{\tilde{E}}{z}\Big) + \frac{2\,(\hat{\mathbf{h}}\cdot\mathbf{k}_T)\,(\hat{\mathbf{h}}\cdot\mathbf{p}_T) - \mathbf{k}_T\cdot\mathbf{p}_T}{2MM_h}\Big[\Big(x e_T H_1^\perp - \frac{M_h}{M} g_{1T}\frac{\tilde{D}^\perp}{z}\Big) - \Big(x e_T^\perp H_1^\perp + \frac{M_h}{M} f_{1T}^\perp \frac{\tilde{G}^\perp}{z}\Big)\Big]\Big\}$$



plane of final hadron

correlation between hadron transverse polarization and quark transverse momentum

Collins function: $H_1^\perp$

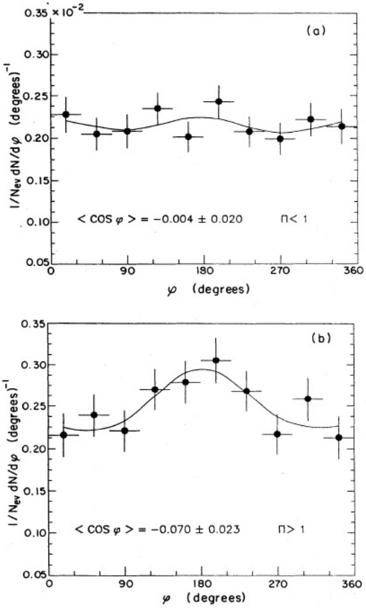


Cahn effect:
$\cos\phi$ modulation of SIDIS
unpolarized cross section

normalized ϕ distributions of hadrons about the virtual photon direction

$$\frac{1}{N_{\rm ev}}\frac{dN}{d\phi} = A + B\cos\phi + C\cos 2\phi + D\sin\phi$$

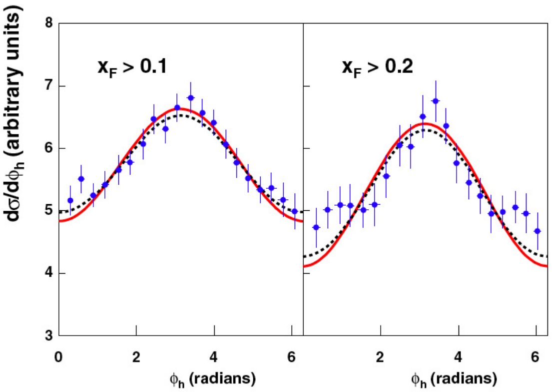
(a) $\Pi < 1.0$
(b) $\Pi > 1.0$



Adams *et al.* (E665), P.R. D 48 (1993) 5057

Cahn effect: $\cos\phi$ modulation of the SIDIS unpolarized cross section in charged hadron production

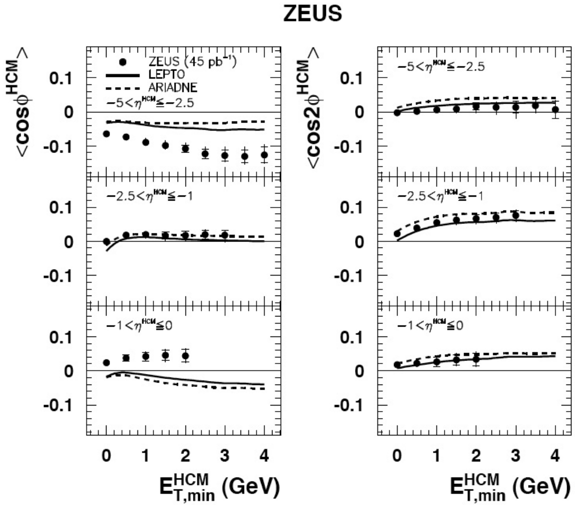
$$x_F = \frac{2P_{hL}}{\sqrt{(P+q)^2}}$$



dashed line with exact kinematics, solid line includes terms up to $\mathcal{O}(k_\perp/Q)$
evidence for $\cos 2\phi_h$ at small values of ϕ_h

Anselmino *et al.*, P.R. D 71 (2005) 074006, data from Arneodo *et al.* (EMC), Z. Phys. C 34 (1987) 277

azimuthal asymmetries in SIDIS $ep \to e' h X \Rightarrow F_{UU}^{\cos\phi_h}, F_{UU}^{\cos 2\phi_h}$



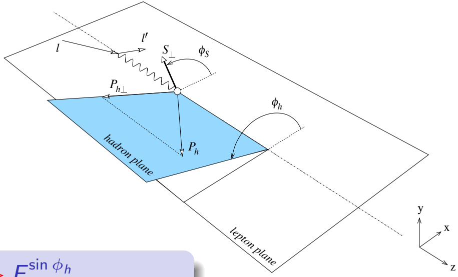
pseudorapidity: $\eta = -\ln\left(\tan\frac{1}{2}\theta\right)$

$100 < Q^2 < 8000\ \mathrm{GeV}^2, \qquad 0.01 < x < 0.1$

Chekanov *et al.*, hep-ex/0608053

single-spin asymmetries (SSA) in SIDIS

$l + p \rightarrow l' + h + X$



longitudinal SSA: beam-spin asymmetry $\Longrightarrow F_{LU}^{\sin\phi_h}$

$$A(\phi_h) = \frac{d\sigma^{\rightarrow}(\phi_h) - d\sigma^{\leftarrow}(\phi_h)}{d\sigma^{\rightarrow}(\phi_h) + d\sigma^{\leftarrow}(\phi_h)}$$

longitudinal SSA: target-spin asymmetry $\Longrightarrow F_{UL}^{\sin\phi_h}$

$$A(\phi_h) = \frac{d\sigma^{\Rightarrow}(\phi_h) - d\sigma^{\Leftarrow}(\phi_h)}{d\sigma^{\Rightarrow}(\phi_h) + d\sigma^{\Leftarrow}(\phi_h)}$$

SSA for transverse target polarization $\Longrightarrow F_{UT}^{\sin(\phi_h \pm \phi_S)}$

$$A(\phi_h, \phi_S) = \frac{d\sigma(\phi_h, \phi_S) - d\sigma(\phi_h, \phi_S + \pi)}{d\sigma(\phi_h, \phi_S) + d\sigma(\phi_h, \phi_S + \pi)}.$$

CLAS beam-spin asymmetry in $ep \to e'\pi^+ X$ at 4.3 GeV

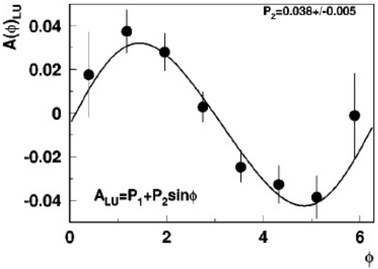


FIG. 3. The beam-spin azimuthal asymmetry as a function of azimuthal angle ϕ, measured in the range $z=0.5$–0.8.

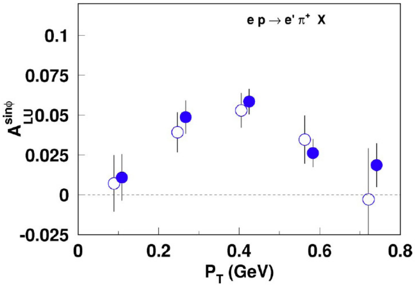
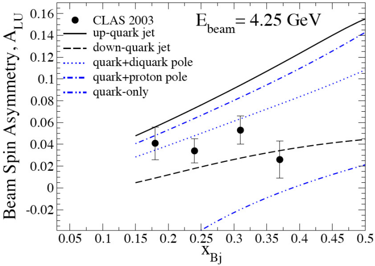
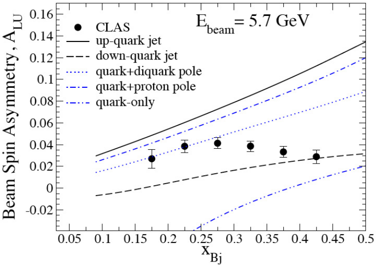


FIG. 7. Beam SSA as a function of $P_\perp$ for $M_X > 1.1$ GeV (filled circles) and $M_X > 1.4$ GeV (open circles).

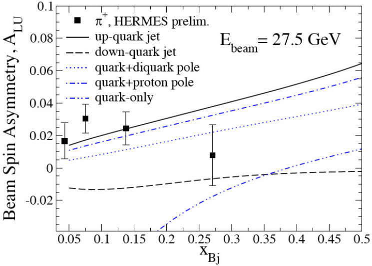
Avakian *et al.*, P.R. D 69 (2004) 112004 + P.R.L. 84 (2000) 4047

beam-spin asymmetry in $ep \to e'\pi^+ X$





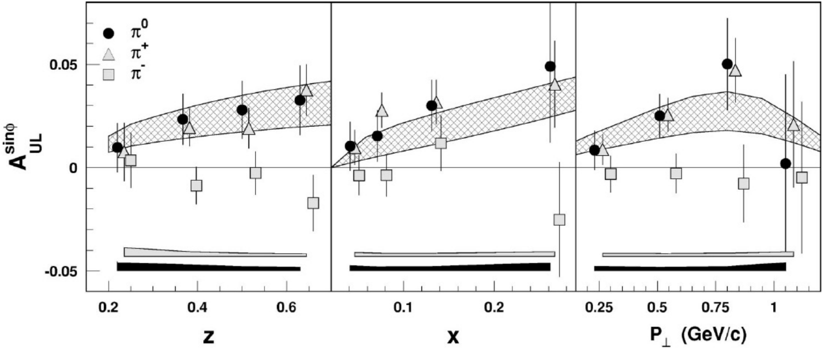
$$F_{LU}^{\sin\phi} \propto x\, g^{\perp} D_1, \quad A_{LU}^{\sin\phi} \propto$$



Afanasev, Carlson, P.R. D 74 (2006) 112027

HERMES single-spin azimuthal asymmetry in $e\vec{p} \to e'\pi^{\pm,0} X$

$$\text{assuming} \quad F_{UL}^{\sin 2\phi_h} \approx 0, \quad F_{UL}^{\sin \phi_h} \propto h_L H_1^{\perp}, \quad h_L \approx h_1$$



range of predictions between $h_1 = g_1$ (non-relativistic limit) and $h_1 = \frac{1}{2}(f_1 + g_1)$ (Soffer limit)

Airapetian *et al.*, P.R. D 64 (2001) 097101

HERMES SSA
on transversely polarized proton

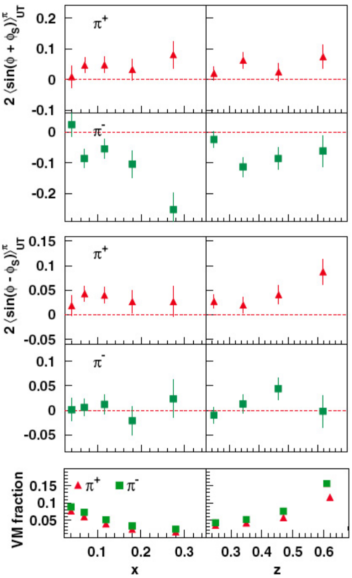
Collins azimuthal moment

$$F_{UT}^{\sin(\phi_h+\phi_S)} \propto h_1 H_1^{\perp}$$

Sivers azimuthal moment

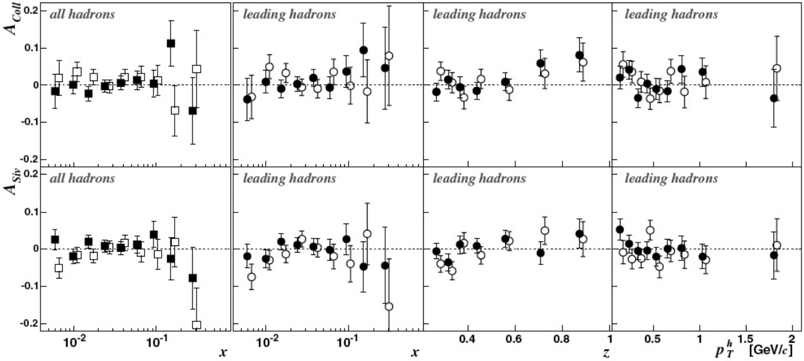
$$F_{UT,T}^{\sin(\phi_h-\phi_S)} \propto f_{1T}^{\perp} D_1$$

exclusive vector meson (ρ^0) production

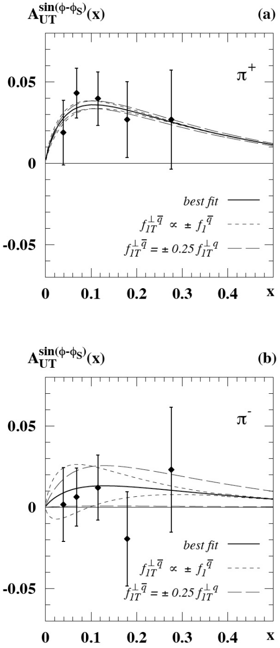


Airapetian *et al.* (Hermes), P.R.L. 94 (2005) 012002

COMPASS charged hadron single-spin asymmetries in SIDIS of high-energy muons on transversely polarized ^{6}LiD

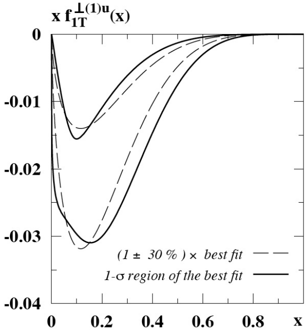


Alexakhin *et al.*, P.R.L. 94 (2005) 202002



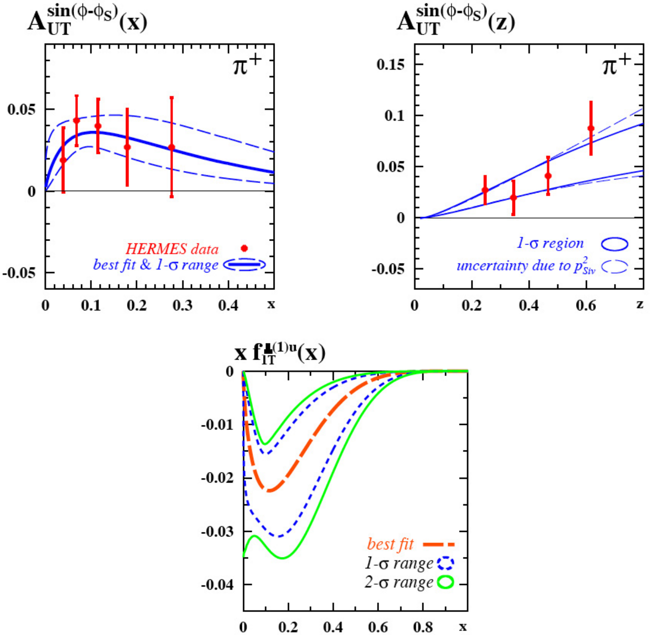
Sivers function from HERMES data with Gaussian ansatz for transverse momenta

$$A_{UT}^{\sin(\phi-\phi_S)} \propto \frac{\sum_a e_a^2 \, x \, f_{1T}^{\perp a}(x) \, D_1^a(z)}{\sum_a e_a^2 \, x f_1^a(x) \, D_1^a(z)}$$

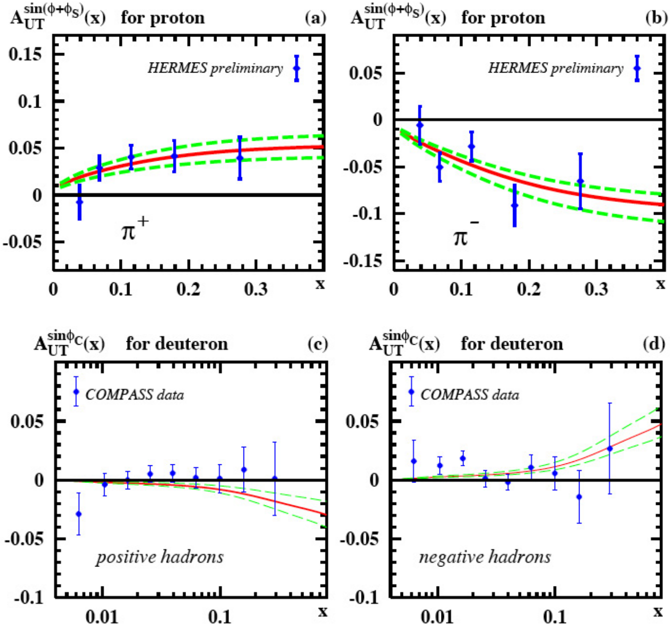


$f_{1T}^{\perp u}(x, k_T^2) = -f_{1T}^{\perp d}(x, k_T^2)$ modulo $1/N_c$ corrections

Collins *et al.*, P.R. D 73 (2006) 014021



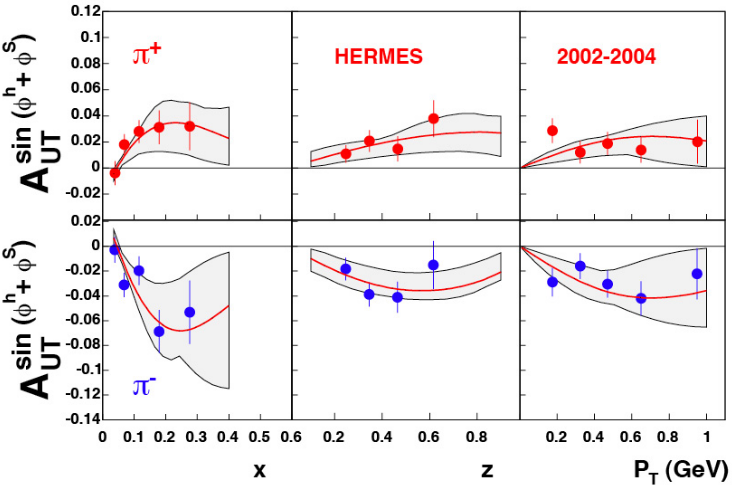
Efremov *et al.*, hep-ph/0702155



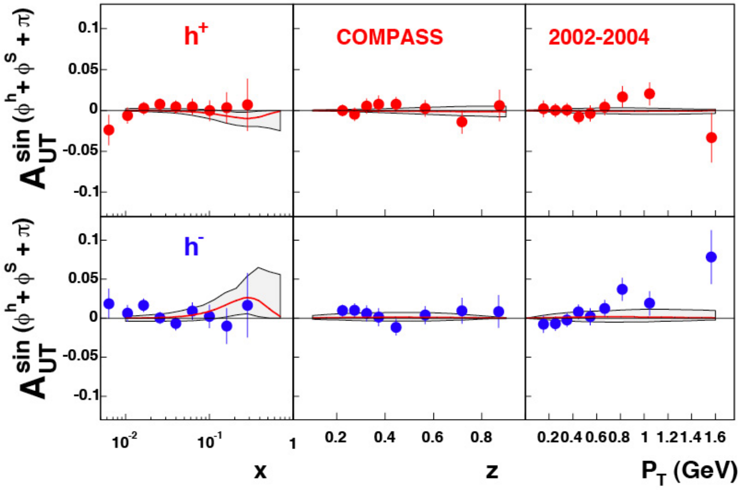
Efremov *et al.*, hep-ph/0702155

global best fit to HERMES and Compass SIDIS and BELLE e^+-e^- at KEK

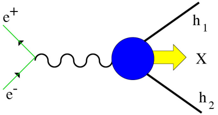
$$h_1(x, \boldsymbol{k}_\perp) \sim [f_1(x) + g_1(x)]\; e^{-\alpha \boldsymbol{k}_\perp^2}, \qquad H_1^\perp(x, \boldsymbol{p}_\perp) \sim D_1(x)\, e^{-\beta \boldsymbol{p}_\perp^2}$$



Anselmino *et al.*, hep-ph/0701006



Anselmino *et al.*, hep-ph/0701006



BELLE $e^+e^- \to h_1 h_2 X$

$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{dz_1\, dz_2\, d^2\boldsymbol{P}_{h_1\perp}\, d^2\boldsymbol{P}_{h_2\perp}\, d\cos\theta} = \sum_{q,s_1,s_2} \frac{d\hat{\sigma}^{e^+e^-\to q(s_1)\,\bar{q}(s_2)}}{d\cos\theta} D_{h_1/q,s_1}(z_1,\boldsymbol{P}_{h_1\perp})\, D_{h_2/\bar{q},s_2}(z_2,\boldsymbol{P}_{h_2\perp})$$

$$D_{h/q,s}(z,\boldsymbol{P}_{h\perp}) = H_1(z,\boldsymbol{P}_{h\perp}) + \frac{\boldsymbol{P}_{h\perp}}{zM_h} H_1^{\perp}(z,\boldsymbol{P}_{h\perp})\, \hat{\boldsymbol{s}}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{P}}_{h\perp})$$

$$\hat{\boldsymbol{s}}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{P}}_{h\perp}) = cos\phi$$

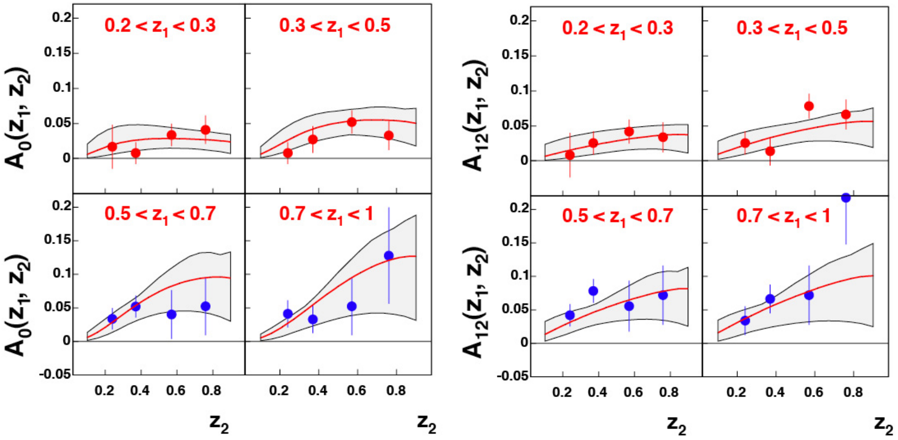
depending on the selected kinematics to detect hadrons one defines

$$A_0(z_1,z_2) = \frac{1}{\pi}\,\frac{z_1\,z_2}{z_1^2+z_2^2}\,\frac{\langle\sin^2\theta_2\rangle}{\langle 1+\cos^2\theta_2\rangle}\,(P_U - P_L)$$

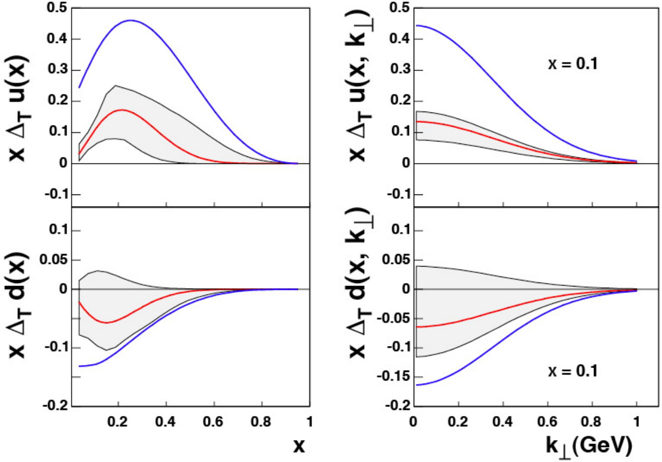
$$A_{12}(z_1,z_2) = \frac{1}{8}\,\frac{\langle\sin^2\theta\rangle}{\langle 1+\cos^2\theta\rangle}\,(P_U - P_L)$$

with P_U (P_L) the contribution of unlike-sign (like-sign) pion production

BELLE $e^+e^- \to h_1 h_2 X$

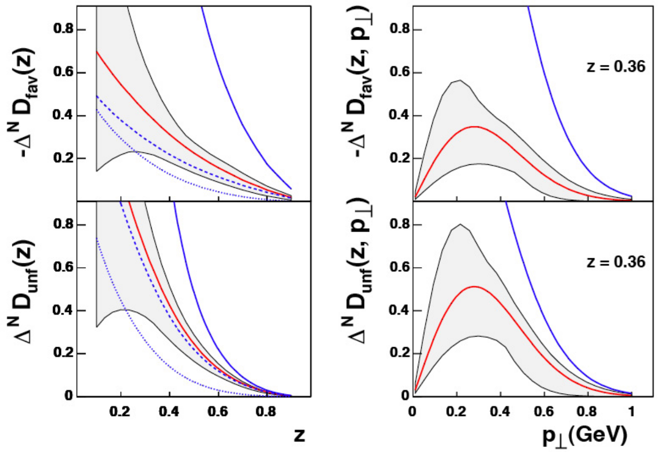


Anselmino *et al.*, hep-ph/0701006



Left (right) panel: integrated (unintegrated) u and d transversity from global best fit. Soffer bound in blue.

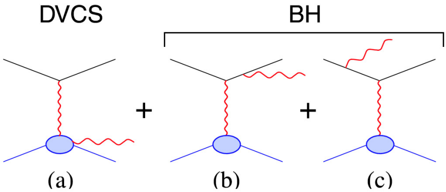
Anselmino *et al.*, hep-ph/0701006



Left (right) panel: integrated (unintegrated) favored ($u \to \pi^+$) and unfavored ($u \to \pi^-$) Collins function from global best fit. Positivity bound in blue.

Anselmino *et al.*, hep-ph/0701006

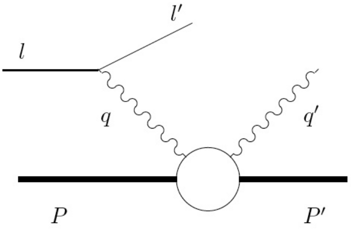
(deeply) virtual Compton scattering (DVCS)



$$\frac{d\sigma}{dx_B\, dy\, d|\Delta^2|\, d\phi\, d\varphi} = \frac{\alpha^3 x_B\, y}{16\pi^2 Q^2 \sqrt{1 + 4x_B^2 M^2/Q^2}} |\mathcal{T}|^2$$

$$|\mathcal{T}|^2 = |\mathcal{T}_{BH}|^2 + |\mathcal{T}_{DVCS}|^2 + \mathcal{T}_{DVCS}\mathcal{T}_{BH}^* + \mathcal{T}_{DVCS}^*\mathcal{T}_{BH}$$

virtual Compton scattering



$$\overline{P}^\mu = \tfrac{1}{2}(P^\mu + P'^\mu)$$

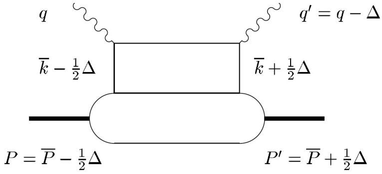
$$\Delta^\mu = P'^\mu - P^\mu = q^\mu - q'^\mu$$

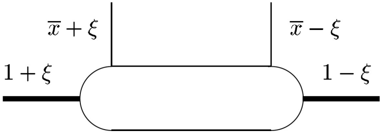
$$t = (P' - P)^2 = \Delta^2$$

$$2\xi = -\frac{\Delta^+}{\overline{P}^+}, \quad \overline{x} = \frac{\overline{k}^+}{\overline{P}^+}$$

$$|\xi| < \sqrt{\frac{-t}{4M^2 - t}}$$

$$\overline{q}^\mu = \tfrac{1}{2}(q^\mu + q'^\mu)$$

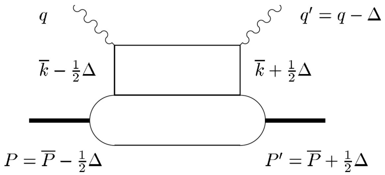




the Compton amplitude

$$\mathcal{T}^{\mu\nu}_{DVCS} = i \int d^4z \, e^{i\overline{q}\cdot z} \langle P'S' | \, T[J^\mu(-\tfrac{1}{2}z) \, J^\nu(\tfrac{1}{2}z)] | PS \rangle$$

to leading order



$$\mathcal{T}^{\mu\nu}_{DVCS} = g_\perp^{\mu\nu} \int_{-1}^{1} d\overline{x} \left(\frac{1}{\overline{x} - \xi + i\epsilon} + \frac{1}{\overline{x} + \xi - i\epsilon} \right) \sum_q e_q^2 \, F^q(\overline{x}, \xi, t)$$

$$+ \, i\varepsilon^{\mu\nu\alpha\beta} n_{+\alpha} n_{-\,\beta} \int_{-1}^{1} d\overline{x} \left(\frac{1}{\overline{x} - \xi + i\epsilon} + \frac{1}{\overline{x} + \xi - i\epsilon} \right) \sum_q e_q^2 \, \tilde{F}^q(\overline{x}, \xi, t)$$

Xiangdong Ji, PRL 78 (1997) 610; PR D 55 (1997) 7114

Radyushkin, PL B 380 (1996) 417; Müller *et al.*, Fortschr. Phys. 42 (1994) 101

generalized parton distributions (GPDs)

- $$F^q(\overline{x},\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{i\overline{x}\overline{P}^+z^-} \langle P',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\,\gamma^+\psi(\tfrac{1}{2}z)\,|P,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2\overline{P}^+}\bar{u}(P',\lambda')\left[H^q(\overline{x},\xi,t)\,\gamma^+ + E^q(\overline{x},\xi,t)\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M}\right]u(P,\lambda),$$

- $$\tilde{F}^q(\overline{x},\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{i\overline{x}\overline{P}^+z^-} \langle P',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\,\gamma^+\gamma_5\,\psi(\tfrac{1}{2}z)\,|P,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2\overline{P}^+}\bar{u}(P',\lambda')\left[\tilde{H}^q(\overline{x},\xi,t)\,\gamma^+\gamma_5 + \tilde{E}^q(\overline{x},\xi,t)\,\frac{\gamma_5\Delta^+}{2M}\right]u(P,\lambda)$$

- $$F_T^q(\overline{x},\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{i\overline{x}\overline{P}^+z^-} \langle P',\lambda'|\,\bar{\psi}(-\tfrac{1}{2}z)\,i\sigma^{+i}\,\gamma_5\,\psi(\tfrac{1}{2}z)\,|P,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0}$$
$$= \frac{1}{2\overline{P}^+}\bar{u}(P',\lambda')\left[H_T^q(\overline{x},\xi,t)\,i\sigma^{+i}\,\gamma_5 + \tilde{H}_T^q(\overline{x},\xi,t)\,\frac{\epsilon^{+j\alpha\beta}\Delta_\alpha\overline{P}_\beta}{M^2}\right.$$
$$\left. + E_T^q(\overline{x},\xi,t)\,\frac{\epsilon^{+j\alpha\beta}\Delta_\alpha\gamma_\beta}{2M} + \tilde{E}_T^q(\overline{x},\xi,t)\,\frac{\epsilon^{+j\alpha\beta}\overline{P}_\alpha\gamma_\beta}{M}\right]u(P,\lambda).$$

Diehl, EPJ C 19 (2001) 485

link to ordinary parton distributions and form factors

- in the forward direction: $P = P' \quad \Rightarrow \quad \xi = 0, t = 0, \quad \bar{x} \to x = k^+/P^+$

$$H^q(x,0,0) = \begin{cases} f_1^q(x), & x > 0 \\ -\bar{f}_1^q(-x), & x < 0 \end{cases}$$

$$\tilde{H}^q(x,0,0) = \begin{cases} g_1^q(x), & x > 0 \\ \bar{g}_1^q(-x), & x < 0 \end{cases}$$

$$H_T^q(x,0,0) = \begin{cases} h_1^q(x), & x > 0 \\ \bar{h}_1^q(-x), & x < 0 \end{cases}$$

- ξ-dependence disappears in first moments

$$\int_{-1}^{1} d\bar{x}\, H^q(\bar{x},\xi,t) = F_1^q(-t), \qquad \int_{-1}^{1} d\bar{x}\, E^q(\bar{x},\xi,t) = F_2^q(-t)$$

$$\int_{-1}^{1} d\bar{x}\, \tilde{H}^q(\bar{x},\xi,t) = G_A^q(-t), \qquad \int_{-1}^{1} d\bar{x}\, \tilde{E}^q(\bar{x},\xi,t) = G_P^q(-t)$$

twist-two operators and polynomiality of GPDs

using Lorentz symmetry, parity and time-reversal invariance

$$\begin{aligned}\langle P'|\mathcal{O}_q^{\mu_1\mu_2\cdots\mu_n}|P\rangle &= \langle P'|\overline{\psi}_q i\mathcal{D}^{(\mu_1}\dots i\mathcal{D}^{\mu_{n-1}}\gamma^{\mu_N)}\psi_q|P\rangle \\ &= \overline{u}(P')\gamma^{(\mu_1}u(P)\sum_{i=0}^{[\frac{n-1}{2}]} A_{qn,2i}(t)\Delta^{\mu_2}\dots\Delta^{\mu_{2i+1}}\overline{P}^{\mu_{2i+2}}\dots\overline{P}^{\mu_n)} \\ &\quad + \overline{u}(P')\frac{\sigma^{(\mu_1\alpha}i\Delta_\alpha}{2M}u(P)\sum_{i=0}^{[\frac{n-1}{2}]} B_{qn,2i}(t)\Delta^{\mu_2}\dots\Delta^{\mu_{2i+1}}\overline{P}^{\mu_{2i+2}}\dots\overline{P}^{\mu_n)} \\ &\quad + C_{qn}(t)\mathrm{Mod}(n+1,2)\frac{1}{M}\overline{u}(P')u(P)\Delta^{(\mu_1}\dots\Delta^{\mu_n)}\end{aligned}$$

in particular

$$\int_{-1}^{1} d\overline{x}\,\overline{x}^n\,H^q(\overline{x},\xi,t) = \sum_{i=0}^{[\frac{n-1}{2}]} A_{qn,2i}(t)\,(2\xi)^{2i} + \mathrm{Mod}(n+1,2)\,C_{qn}(t)\,(2\xi)^n$$

$$\int_{-1}^{1} d\overline{x}\,\overline{x}^n\,E^q(\overline{x},\xi,t) = \sum_{i=0}^{[\frac{n-1}{2}]} B_{qn,2i}(t)\,(2\xi)^{2i} - \mathrm{Mod}(n+1,2)\,C_{qn}(t)\,(2\xi)^n$$

$\Longrightarrow \int_{-1}^{1} d\overline{x}\,\overline{x}^n\,[H^q(\overline{x},\xi,t)+E^q(\overline{x},\xi,t)]$ even polynomial in ξ of degree n

Xiangdong Ji, J. Phys. G: Nucl. Part. Phys. 24 (1998) 1181

Ji's sum rule

- QCD angular momentum as gauge-invariant sum $\boldsymbol{J} = \boldsymbol{J}_q + \boldsymbol{J}_g$

$$J^i_{q,g} = \tfrac{1}{2}\,\epsilon^{ijk} \int d^3x \left(T^{0k}_{q,g}\, x^j - T^{0j}_{q,g}\, x^k \right)$$

- define form factor of quark and gluon energy-momentum tensor

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \overline{u}(P')\left[A_{q,g}(\Delta^2)\gamma^{(\mu}\overline{P}^{\nu)} + B_{q,g}(\Delta^2)\frac{\overline{P}^{(\mu}\, i\,\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + C_{q,g}(\Delta^2)\frac{\Delta^{(\mu}\Delta^{\nu)}}{M}\right]u(P)$$

$$J^i_{q,g} = \tfrac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right]$$

$$\Longrightarrow \qquad \langle P|J_z|P\rangle = \tfrac{1}{2} = J_q + J_g = \tfrac{1}{2}\Delta\Sigma + L_q + J_g$$

$$J_q = \tfrac{1}{2}\int_{-1}^{+1} d\overline{x}\,\overline{x}\left[H^q(\overline{x},\xi,t=0) + E^q(\overline{x},\xi,t=0)\right]$$

Xiangdong Ji, P.R.L. 78 (1997) 610

parton interpretation

in terms of the "good" light-cone components of the field

$$\overline{\psi}_q^c(-\tfrac{1}{2}y)\gamma^+\psi(\tfrac{1}{2}y) = \sqrt{2}\phi_q^{c\,\dagger}(-\tfrac{1}{2}y)\phi_q^c(\tfrac{1}{2}y)$$

with quark field in momentum space

$$\phi_q^c(z^-,\boldsymbol{z}_\perp) = \int \frac{dk^+ d\boldsymbol{k}_\perp}{2k^+(2\pi)^3}\Theta(k^+) \sum_\mu \Big\{ b_q(w) u_+(k,\mu)\, e^{-ik^+z^- + i\boldsymbol{k}_\perp\cdot\boldsymbol{z}_\perp} + d_q^\dagger(w) v_+(k,\mu)\, e^{+ik^+z^- - i\boldsymbol{k}_\perp\cdot\boldsymbol{z}_\perp} \Big\}$$

$$\sum_{c,c'} \int \frac{dy^-}{2\pi}\, e^{ix\overline{P}^+y^-}\, \overline{\psi}(-\tfrac{1}{2}y)\,\gamma^+\,\psi(\tfrac{1}{2}y)$$

$$= 2\sqrt{2}\int \frac{dk'^+ d\boldsymbol{k}'_\perp}{2k'^+(2\pi)^3}\Theta(k'^+) \int \frac{dk^+ d\boldsymbol{k}_\perp}{2k^+(2\pi)^3}\Theta(k^+)$$

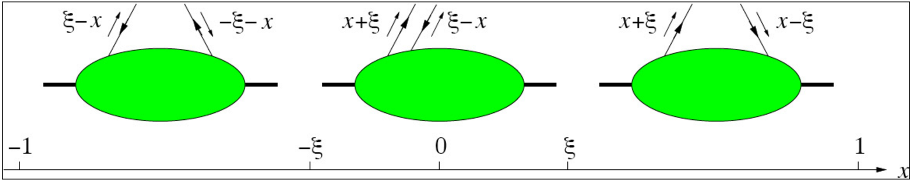
$$\times \sum_{\mu,\mu',c,c'} \delta_{c'c} \Big\{ \delta(2x\overline{P}^+ - k'^+ - k^+)\, b_q^\dagger(w') b_q(w)\, u_+^\dagger(k',\mu') u_+(k,\mu)$$

$$+\, \delta(2x\overline{P}^+ + k'^+ + k^+)\, d_q(w') d_q^\dagger(w)\, v_+^\dagger(k',\mu') v_+(k,\mu)$$

$$+\, \delta(2x\overline{P}^+ + k'^+ - k^+)\, d_q(w') b_q(w)\, v_+^\dagger(k',\mu') u_+(k,\mu)$$

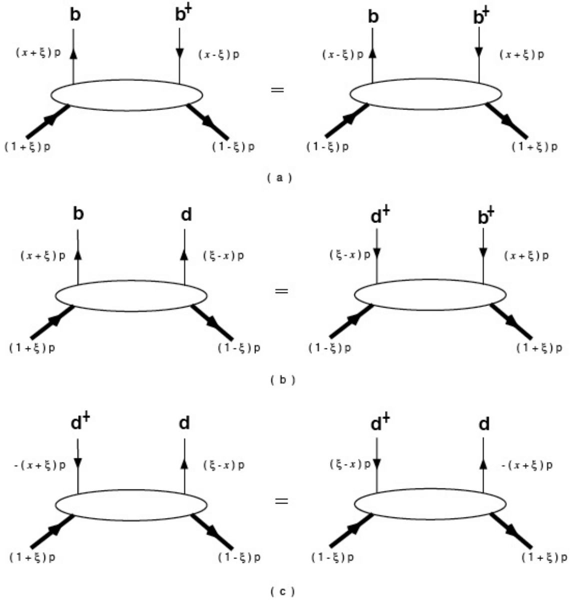
$$+\, \delta(2x\overline{P}^+ - k'^+ + k^+)\, b_q^\dagger(w') d_q^\dagger(w)\, u_+^\dagger(k',\mu') v_+(k,\mu) \Big\}$$

the parton interpretation of GPDs

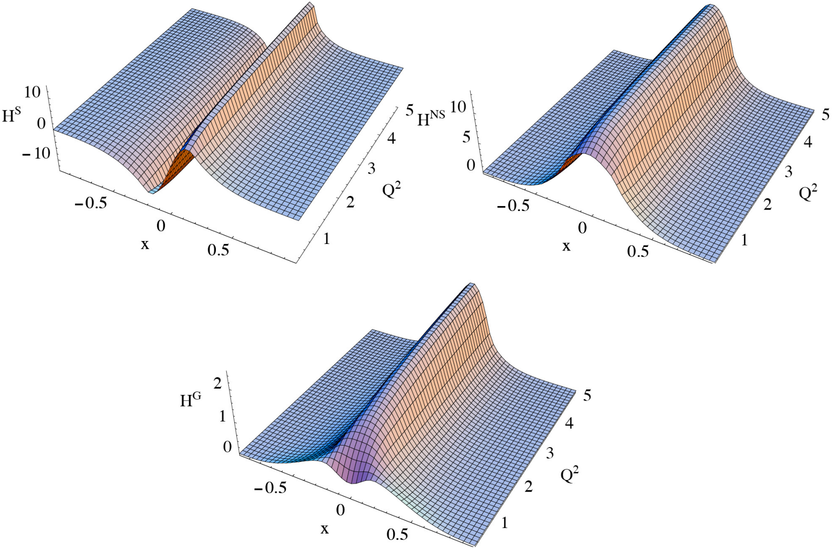


$d^\dagger$ d b d b $b^\dagger$

DGLAP ERBL DGLAP



ξ-symmetry of GPDs ($\xi > 0$): (a) $x > \xi$, (b) $-\xi < x < \xi$, (c) $x < -\xi$



Pasquini, Traini, S.B., P.R. D 71 (2005) 034022

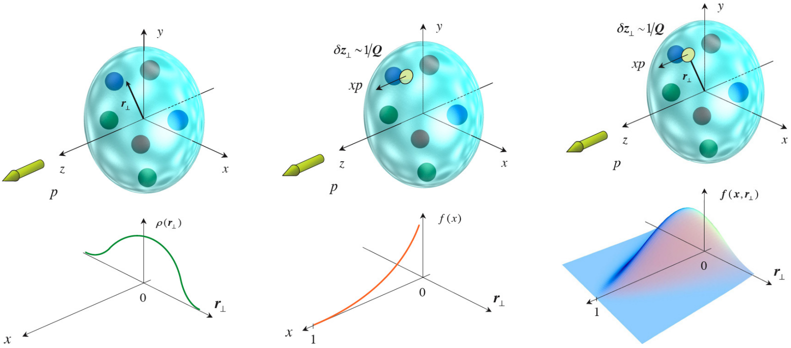
probing transversely localized partons

Burkardt, P.R. D 62 (2000) 071503

in impact parameter space, for purely transverse momentum transfer
($\Delta^+ = 0$, i.e. $\xi = 0$, $\quad t = -\boldsymbol{\Delta}_\perp^2$)

$$q(x, \boldsymbol{b}_\perp) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} \, e^{-i\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp} \, H^q(x, 0, -\boldsymbol{\Delta}_\perp^2)$$

probabilistic interpretation of form factors, parton densities and GPDs at $\xi = 0$



Belitsky, Radyushkin, Phys. Rep. 418 (2005) 1

quark helicity projected out by $\frac{1}{2}\bar{q}\,\gamma^+[1+\lambda\gamma_5]\,q$
density of quark with helicity λ, momentum fraction x and transverse distance $\boldsymbol{b}_\perp$ from center of proton in state $|\Lambda, \boldsymbol{S}\rangle$:

$$\frac{1}{2}\left[F(x,\boldsymbol{b}_\perp)+\lambda\tilde{F}(x,\boldsymbol{b}_\perp)\right] = \frac{1}{2}\left[H(x,\boldsymbol{b}_\perp^2) - S^i\epsilon^{ij}b^j\frac{1}{m}\frac{\partial}{\partial \boldsymbol{b}_\perp^2}E(x,\boldsymbol{b}_\perp^2)+\lambda\Lambda\tilde{H}(x,\boldsymbol{b}_\perp^2)\right]$$

proton polarization along $\hat{x}$ $\qquad q(x,\boldsymbol{b}_\perp)$

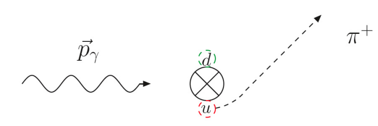
$\longleftarrow \quad q_X(x,\boldsymbol{b}_\perp) \quad \longrightarrow$

N.B. $$d_y^q \equiv \int dx \int d^2\boldsymbol{b}_\perp\, b_y\, q_X(x,\boldsymbol{b}_\perp) = \int dx\, E^q(x,0,0) = \frac{\kappa^q}{2M},$$

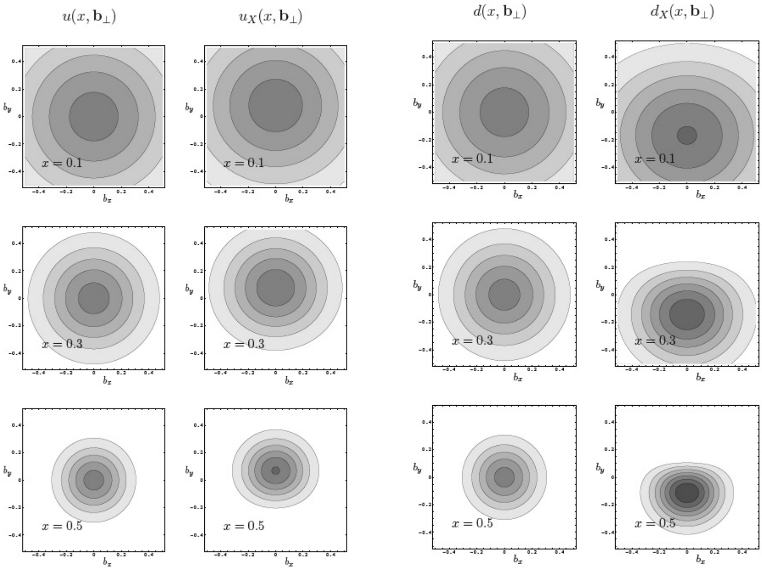
$$\kappa_u = 2\kappa_p + \kappa_n = 1.673, \qquad \kappa_d = 2\kappa_n + \kappa_p = -2.033$$

distortion + FSI

$\Longrightarrow$ Sivers effect



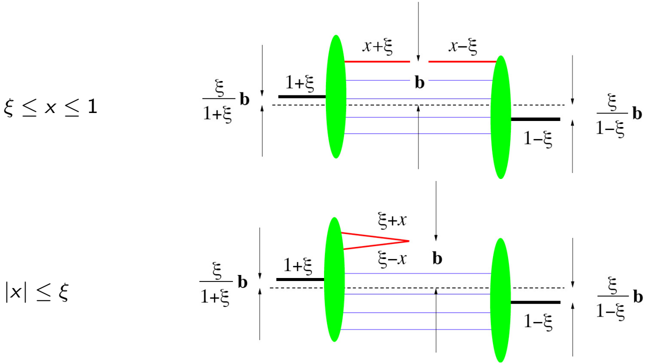
Burkardt, Int. J. Mod Phys. A 18 (2003) 173



Burkardt, Int. J. Mod Phys. A 18 (2003) 173

representation of a GPD in impact parameter space for $\xi \neq 0$

- in a frame with large P^+ the proton is seen as a bunch of partons
- the center-of-momentum of the initial and final proton are differently displaced by a finite longitudinal momentum transfer
- GPDs probe the active partons at transverse position $\mathbf{b}$



Diehl, EPJ C 25 (2002) 223; (E) 31 (2003) 277

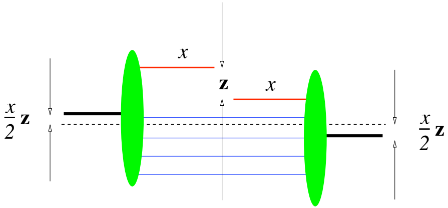
impact parameter representation of an unintegrated parton distribution

$$f_1(x, \boldsymbol{k}_T) = \int \frac{d^2\boldsymbol{z}\, dz^-}{16\pi^3}\, e^{ixp^+z^- - i\boldsymbol{k}_T\cdot\boldsymbol{z}}\, \langle p,\lambda|\, \bar{q}(0, -\tfrac{1}{2}z^-, -\tfrac{1}{2}\boldsymbol{z})\, \gamma^+ q(0, \tfrac{1}{2}z^-, \tfrac{1}{2}\boldsymbol{z})\, |p,\lambda\rangle$$

$$f_1(x, \boldsymbol{z}) = \int d^2\boldsymbol{k}_T\; e^{i\boldsymbol{k}_T\cdot\boldsymbol{z}}\, f_1(x, \boldsymbol{k}_T)$$

$\boldsymbol{z}$ is the Fourier conjugate variable to the transverse momentum $\boldsymbol{k}_T$ of the struck parton

unintegrated parton distributions describe correlation in transverse position of a single quark



in contrast to GPDs, the struck quark now has different transverse location *relative* to spectator partons in the initial and the final state, in addition to overall shift of proton center of momentum dictated again by Lorentz invariance

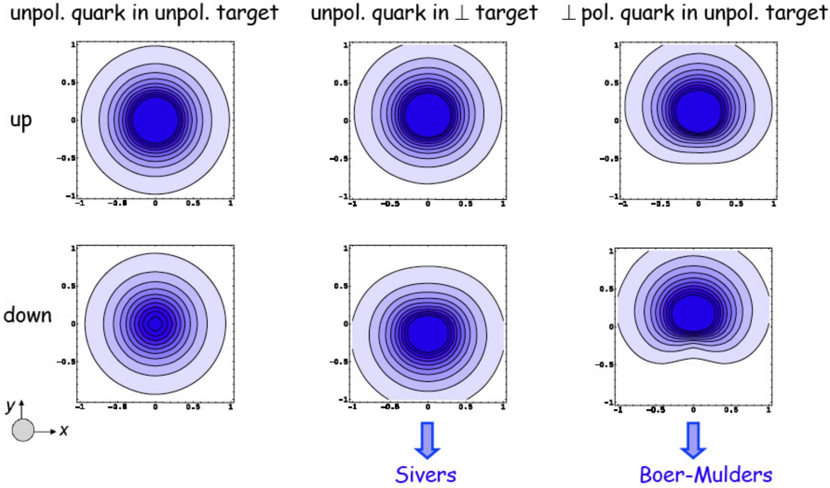
Diehl, EPJ C 25 (2002) 223; (E) 31 (2003) 277

spin density in the transverse plane and GPDs

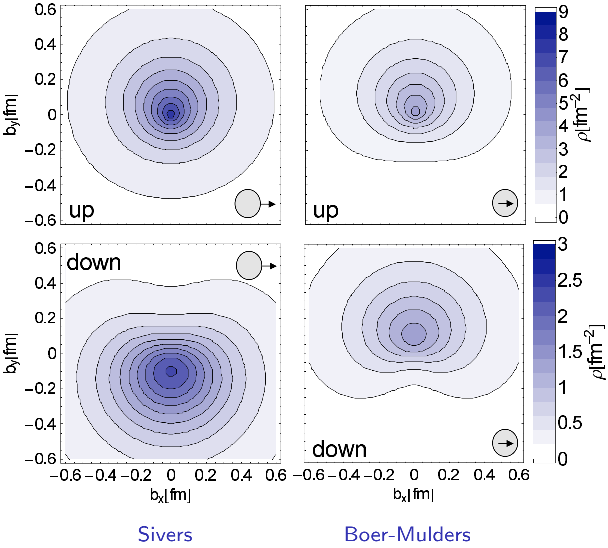
quarks with transverse polarization $\boldsymbol{s}$ projected out by $\frac{1}{2}\bar{q}\,\gamma^{+}[1+(\boldsymbol{s\gamma})\gamma_5]\,q$
probability to find a quark with momentum fraction x and transverse spin $\boldsymbol{s}_\perp$ at distance $\boldsymbol{b}$ from the center-of-momentum of the nucleon with transverse spin $\boldsymbol{S}_\perp$:

$$
\begin{aligned}
\rho(x,\boldsymbol{b},\boldsymbol{s}_\perp,\boldsymbol{S}_\perp) &= \tfrac{1}{2}\left[F(x,\boldsymbol{b}) + s^i\,F_T^i(x,\boldsymbol{b},\boldsymbol{s}_\perp,\boldsymbol{S}_\perp)\right] \\
&= \tfrac{1}{2}\Bigg[H + s^i S^i\left(H_T - \frac{1}{4M^2}\Delta_b\tilde{H}_T\right) && \text{monopole} \\
&\qquad - S^i\varepsilon^{ij}b^j\,\frac{1}{M}E' - s^i\varepsilon^{ij}b^j\,\frac{1}{M}\left(E_T' + 2\tilde{H}_T'\right) && \text{dipole} \\
&\qquad + s^i(2b^ib^j - b^2\delta_{ij})S^j\,\frac{1}{M^2}\tilde{H}_T''\Bigg] && \text{quadrupole}
\end{aligned}
$$

Spin densities



B. Pasquini



Göckeler *et al.*, hep-lat/0612032

useful literature besides that already cited

- M. Anselmino, A. Efremov, E. Leader: Phys. Rep. 261 (1995) 1–124; (E) 281 (1997) 399–400, hep-ph/9501369.
- V. Barone, A. Drago, P.G. Ratcliffe: Phys. Rep. 359 (2002) 1–168, hep-ph/0104283.
- A.V. Belitsky, A.V. Radyushkin: Phys. Rep. 418 (2005) 1–387, hep-ph/0504030.
- M. Diehl: Phys. Rep. 388 (2003) 41–277, hep-ph/0307382.
- M. Diehl, Th. Feldmann, R. Jakob, P. Kroll: Nucl. Phys. B 596 (2001) 33–65; (E) Nucl. Phys. B 605 (2001) 647, hep-ph/0009255.
- K. Goeke, M.V. Polyakov, M. Vanderhaeghen: Prog. Part. Nucl. Phys. 47 (2001) 401–515, hep-ph/0106012.
- R.L. Jaffe, Xiangdong Ji: Phys. Rev. Lett. 67 (1991) 552–555; Nucl. Phys. B 375 (1992) 527–560.
- Xiangdong Ji: Ann. Rev. Nucl. Part. Sci. 54 (2004) 413–450.
- P.J. Mulders: *Transverse momentum dependence in structure functions in hard scattering processes*, Lecture notes, Amsterdam, 2001.