

Pressione di radiazione

dalla forza (per unità di volume): $\mathbf{f} = \rho\mathbf{E} + \frac{1}{c}\mathbf{j} \times \mathbf{B}$

$$\begin{aligned}\mathbf{f} &= \frac{1}{4\pi} \left\{ (\nabla \cdot \mathbf{E})\mathbf{E} + \left(\nabla \times \mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \right) \times \mathbf{B} \right\} \\ &= \frac{1}{4\pi} \left\{ (\nabla \cdot \mathbf{E})\mathbf{E} + (\nabla \cdot \mathbf{B})\mathbf{B} + (\nabla \times \mathbf{B}) \times \mathbf{B} - \frac{1}{c} \frac{\partial}{\partial t} (\mathbf{E} \times \mathbf{B}) \right. \\ &\quad \left. + \frac{1}{c} \mathbf{E} \times \frac{\partial \mathbf{B}}{\partial t} \right\} \\ &= \frac{1}{4\pi} \left\{ (\nabla \cdot \mathbf{E})\mathbf{E} + (\nabla \cdot \mathbf{B})\mathbf{B} + (\nabla \times \mathbf{B}) \times \mathbf{B} + (\nabla \times \mathbf{E}) \times \mathbf{E} \right. \\ &\quad \left. - \frac{1}{c} \frac{\partial}{\partial t} (\mathbf{E} \times \mathbf{B}) \right\}\end{aligned}$$

forza media sulla parete \perp all'asse x

$$\begin{aligned}\bar{f}_x &= \frac{1}{4\pi} \left\{ \frac{1}{2} \frac{\partial \bar{E}_x^2}{\partial x} + \frac{1}{2} \frac{\partial \bar{B}_x^2}{\partial x} - \frac{1}{2} \frac{\partial \bar{B}_z^2}{\partial x} - \frac{1}{2} \frac{\partial \bar{B}_y^2}{\partial x} - \frac{1}{2} \frac{\partial \bar{E}_z^2}{\partial x} - \frac{1}{2} \frac{\partial \bar{E}_y^2}{\partial x} \right\} \\ &= \frac{1}{4\pi} \left\{ \frac{\partial \bar{E}_x^2}{\partial x} + \frac{\partial \bar{B}_x^2}{\partial x} - \frac{1}{2} \frac{\partial}{\partial x} (\bar{E}^2 + \bar{B}^2) \right\} \\ &= \frac{1}{4\pi} \frac{\partial}{\partial x} \left\{ \frac{1}{3} (\bar{E}^2 + \bar{B}^2) - \frac{1}{2} (\bar{E}^2 + \bar{B}^2) \right\} \\ &= -\frac{1}{3} \frac{\partial}{\partial x} \left\{ \frac{1}{8\pi} (\bar{E}^2 + \bar{B}^2) \right\} \\ &= -\frac{1}{3} \frac{\partial U}{\partial x}\end{aligned}$$

\Rightarrow pressione

$$p = \int \bar{f}_x dx = \frac{1}{3} U$$