J. S. Bell : Physics 1 (1964) 195

Il teorema di Bell

- $\lambda =$ variabile nascosta
- $\rho(\lambda) =$ distribuzione di probabilità
- $A = \pm 1, \quad B = \pm 1, \qquad |A(\hat{\boldsymbol{a}}, \lambda)| \leq 1, \quad |B(\hat{\boldsymbol{b}}, \lambda)| \leq 1$
- $E(\hat{\boldsymbol{a}}, \hat{\boldsymbol{b}}) =$ probabilità congiunta

$$E(\hat{\boldsymbol{a}}, \hat{\boldsymbol{b}}) = \int \mathrm{d}\lambda\, \rho(\lambda)\, A(\hat{\boldsymbol{a}}, \lambda)\, B(\hat{\boldsymbol{b}}, \lambda)$$

$$\begin{aligned} E(\hat{\boldsymbol{a}}, \hat{\boldsymbol{b}}) - E(\hat{\boldsymbol{a}}, \hat{\boldsymbol{b}}') &= \int \mathrm{d}\lambda\, \rho(\lambda) \left[A(\hat{\boldsymbol{a}}, \lambda)\, B(\hat{\boldsymbol{b}}, \lambda) - A(\hat{\boldsymbol{a}}, \lambda)\, B(\hat{\boldsymbol{b}}', \lambda)\right] \\ &= \int \mathrm{d}\lambda\, \rho(\lambda) \left[A(\hat{\boldsymbol{a}}, \lambda)\, B(\hat{\boldsymbol{b}}, \lambda) \left(1 \pm A(\hat{\boldsymbol{a}}', \lambda)\, B(\hat{\boldsymbol{b}}', \lambda)\right)\right] \\ &\quad - \int \mathrm{d}\lambda\, \rho(\lambda) \left[A(\hat{\boldsymbol{a}}, \lambda)\, B(\hat{\boldsymbol{b}}', \lambda) \left(1 \pm A(\hat{\boldsymbol{a}}', \lambda)\, B(\hat{\boldsymbol{b}}, \lambda)\right)\right] \end{aligned}$$

$$\begin{aligned} |E(\hat{\boldsymbol{a}}, \hat{\boldsymbol{b}}) - E(\hat{\boldsymbol{a}}, \hat{\boldsymbol{b}}')| &\leq \int \mathrm{d}\lambda\, \rho(\lambda) \left(1 \pm A(\hat{\boldsymbol{a}}', \lambda)\, B(\hat{\boldsymbol{b}}', \lambda)\right) \\ &\quad + \int \mathrm{d}\lambda\, \rho(\lambda) \left(1 \pm A(\hat{\boldsymbol{a}}', \lambda)\, B(\hat{\boldsymbol{b}}, \lambda)\right) \\ &= 2 \pm \left(E(\hat{\boldsymbol{a}}', \hat{\boldsymbol{b}}') + E(\hat{\boldsymbol{a}}', \hat{\boldsymbol{b}})\right) \end{aligned}$$

$$\boxed{-2 \leq S \leq 2}$$

$$S = E(\hat{\boldsymbol{a}}, \hat{\boldsymbol{b}}) - E(\hat{\boldsymbol{a}}, \hat{\boldsymbol{b}}') + E(\hat{\boldsymbol{a}}', \hat{\boldsymbol{b}}) + E(\hat{\boldsymbol{a}}', \hat{\boldsymbol{b}}')$$

Per particelle a spin $\frac{1}{2}$ in meccanica quantistica è

$$E(\hat{\boldsymbol{a}}, \hat{\boldsymbol{b}}) = c\,\hat{\boldsymbol{a}} \cdot \hat{\boldsymbol{b}} \quad \Longrightarrow \quad \boxed{S_{QM} \to 2\sqrt{2}\,c}$$