Overview of the lectures

1. Introduction
2. Inclusive and semi-inclusive DIS (structure functions)

   Basics of collinear PDFs at tree level (definition, gauge link)
3. Basics of collinear PDFs (interpretation)

   Basics of TMDs at tree level (definition, gauge link, interpretation)
4. Basics of factorization

   Basics of TMD evolution
5. Phenomenology of SIDIS integrated over azimuth

- Phenomenology of SIDIS with azimuthal dependence

Unpolarized SIDIS

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}$$

$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}\right\}$$

Done last time

Polarized SIDIS

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}$$

$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\,\Bigg\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

$$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+ S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$

$$+ S_T\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+ \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$$

$$+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] + S_T\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+ \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}$$

TMDs and their probabilistic interpretation

quark pol.

nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

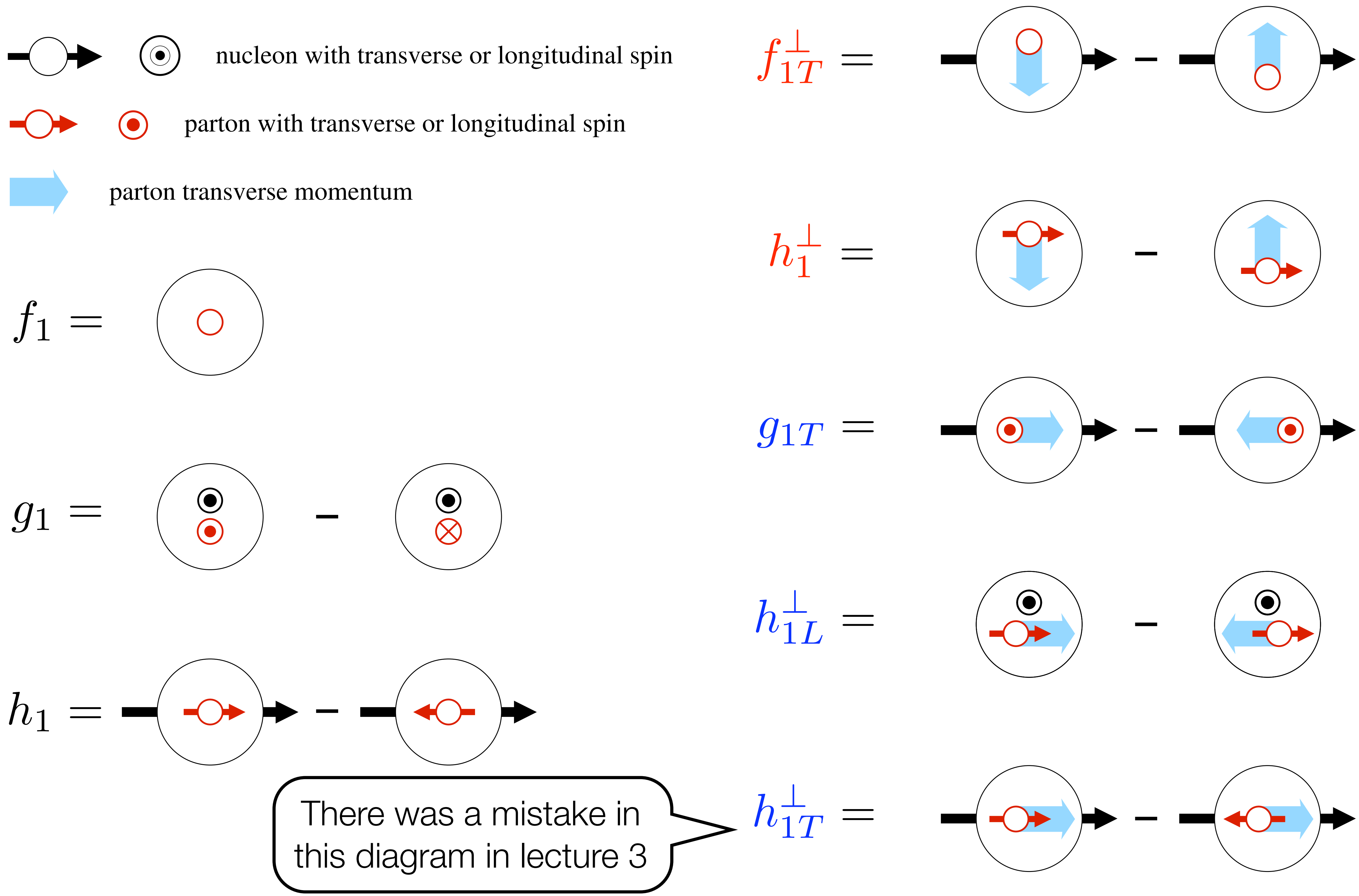
Twist-2 TMDs

TMDs in black survive transverse-momentum integration

TMDs in red are T-odd

TMDs and their probabilistic interpretation

Proton goes out of the screen/ photon goes into the screen



Sivers

Asymmetry

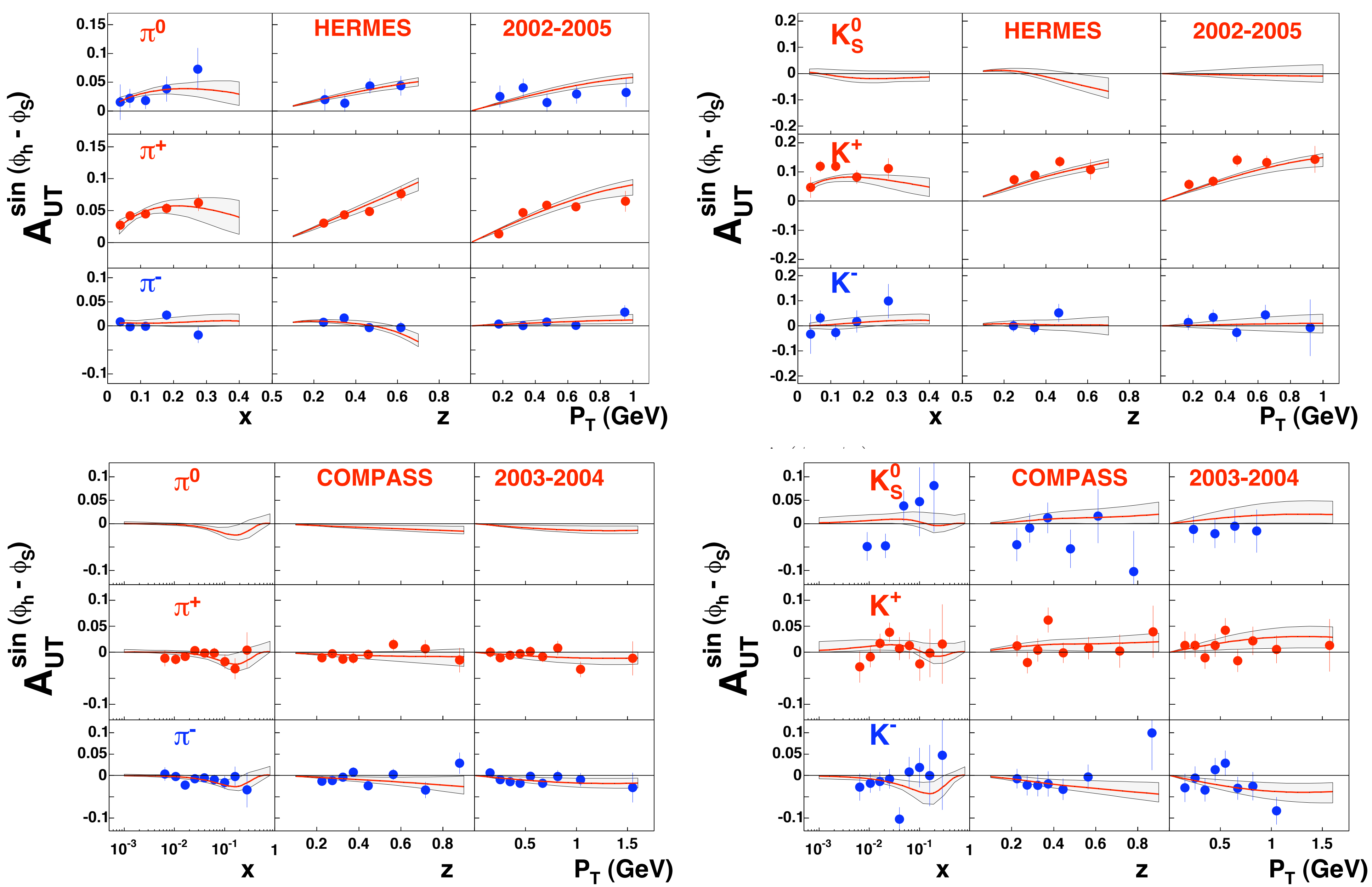
$$F_{UT}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right]$$

Gaussian ansatz

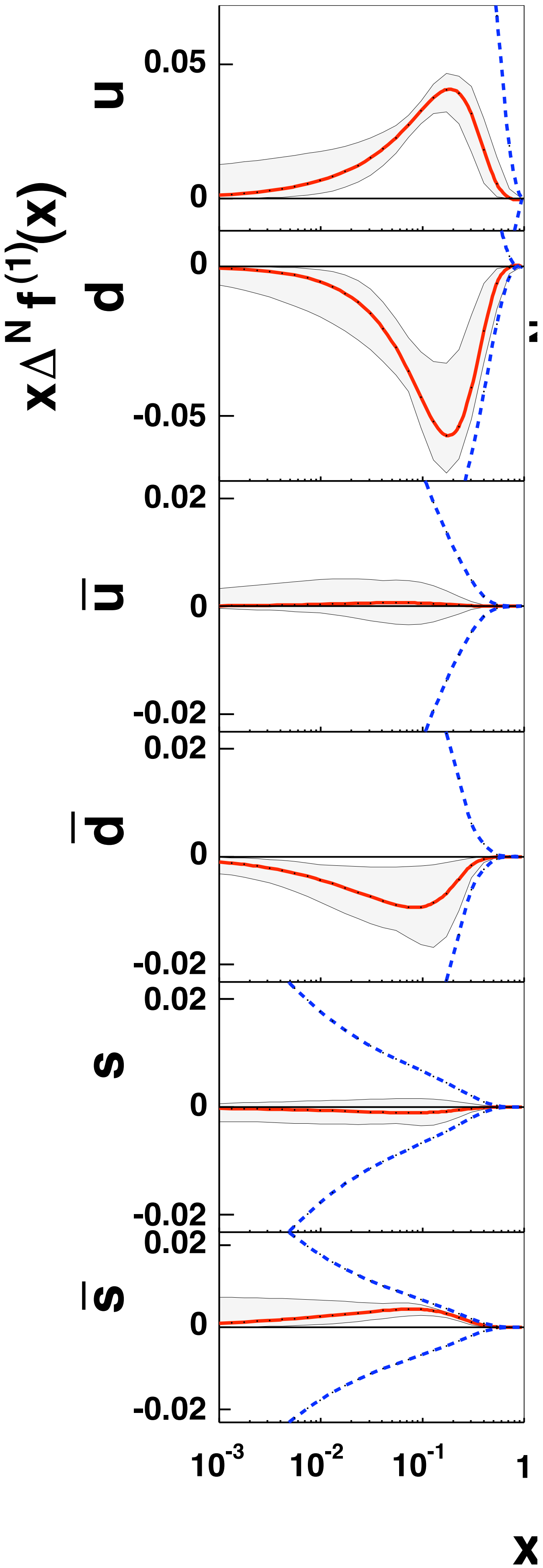
$$f_{1T}^{\perp a}(x, p_T^2) = \frac{f_{1T}^{\perp a}(x)}{\pi\rho_a^2} e^{-\boldsymbol{p}_T^2/\rho_a^2}, \qquad D_1^a(z, k_T^2) = \frac{D_1^a(z)}{\pi\sigma_a^2} e^{-z^2\boldsymbol{k}_T^2/\sigma_a^2}$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = x\sum_a e_a^2 \frac{|P_{h\perp}|}{M} f_{1T}^{\perp a}(x) D_1^a(z) \frac{z\rho_a^2}{\pi(z^2\rho_a^2+\sigma_a^2)^2}\, e^{-\boldsymbol{P}_{h\perp}^2/(z^2\rho_a^2+\sigma_a^2)}$$

Data



HERMES
2002-2005
COMPASS
2003-2004
π^0
π^+
π^-
K^0_S
K^+
K^-
$A_{UT}^{\sin(\phi_h - \phi_S)}$
x
z
P_T (GeV)

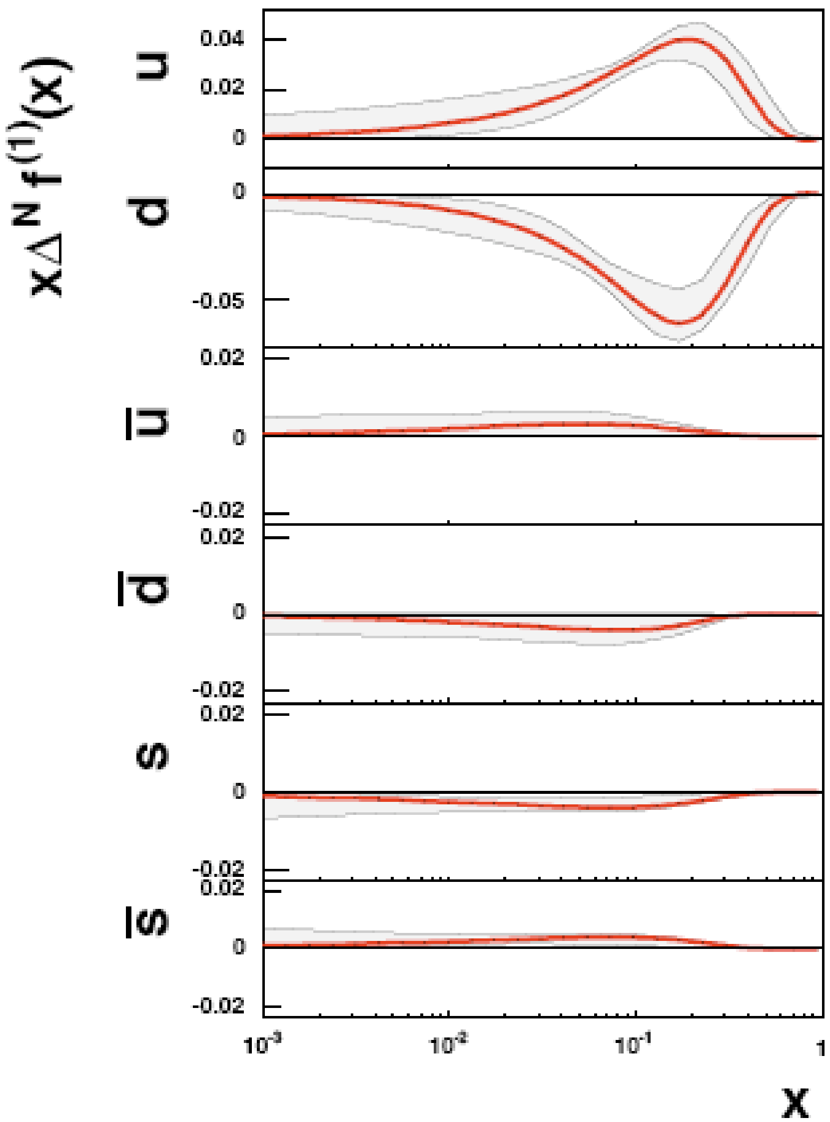


Sivers functions

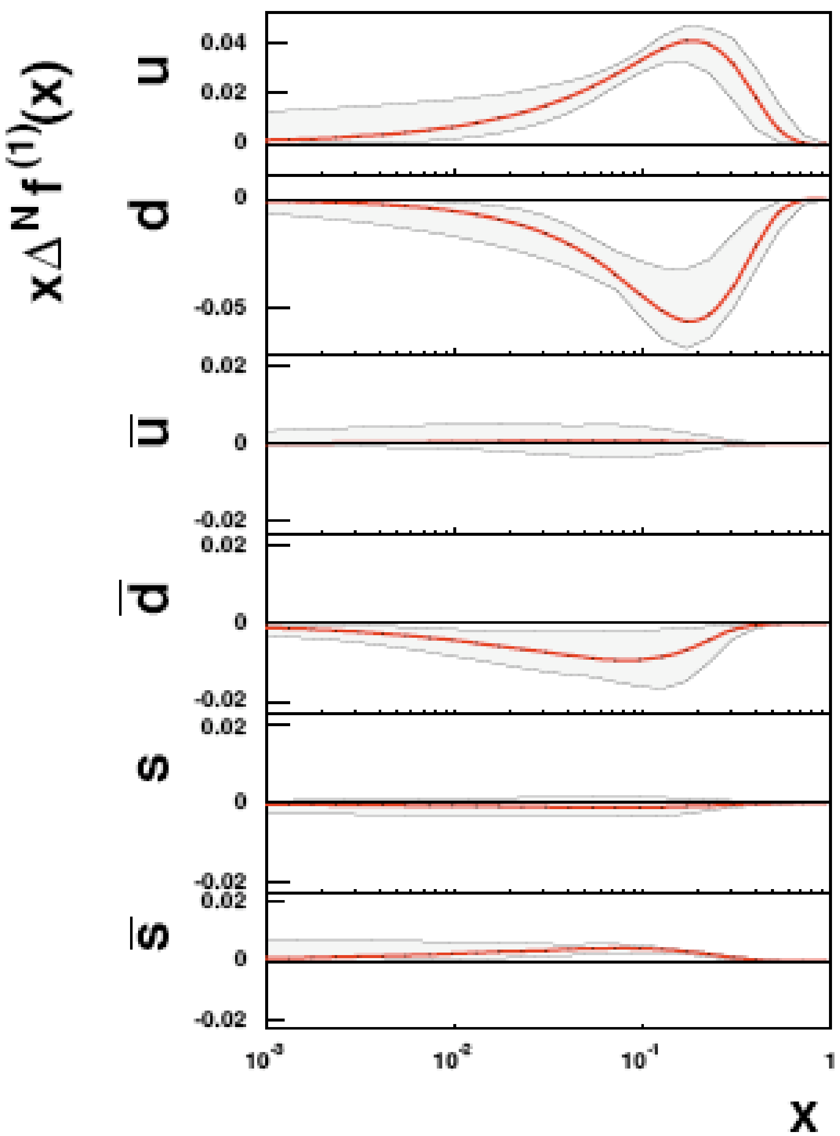
- Data from HERMES, COMPASS
- 96 data points (some correlations -- cf. 467 points for Δq fits)
- no sys errors
- $\chi^2 \approx 1.0$
- Statistical uncertainty only ($\Delta\chi^2 \approx 17$)

Sivers function: Torino

“Symmetric sea”



Free fit



Anselmino et al., 0805.2677

Sivers function: Bochum

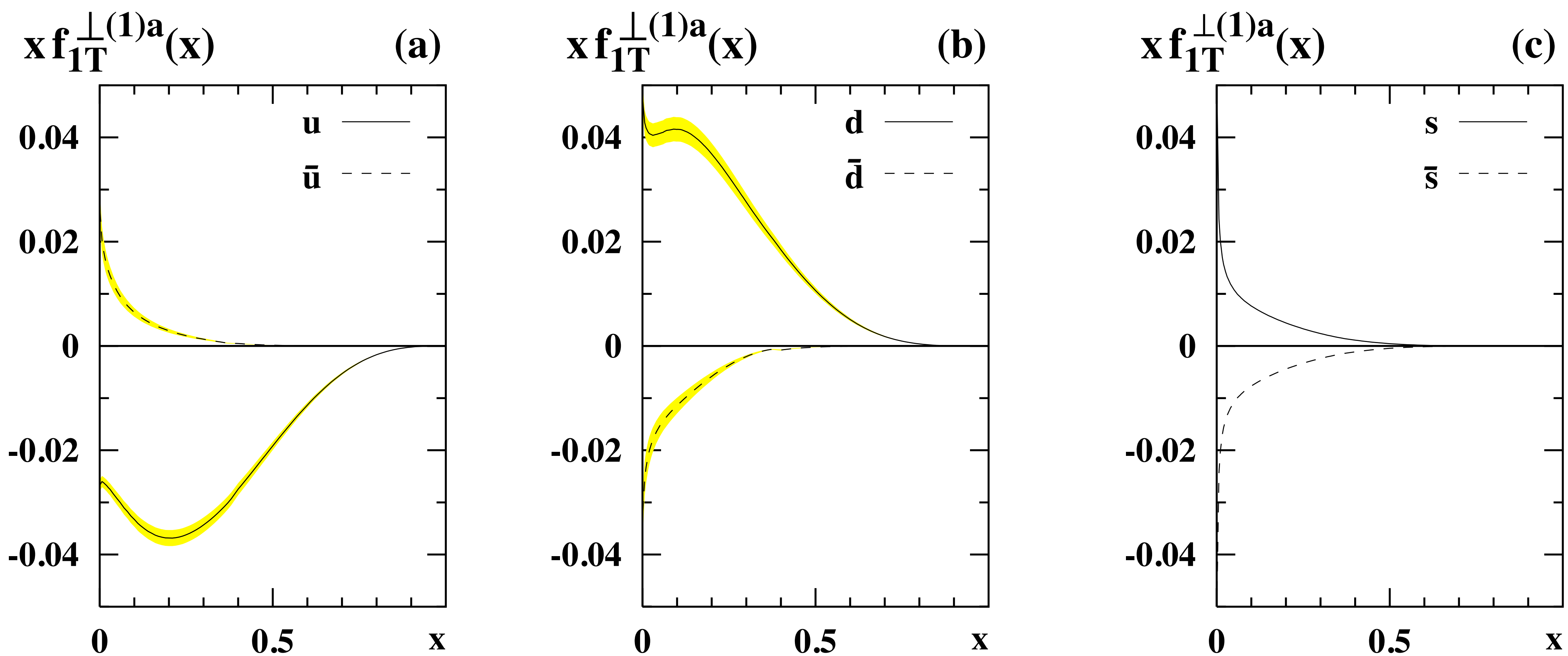


FIGURE 7. The $x f_{1T}^{\perp(1)a}(x)$ vs. x as extracted from preliminary HERMES and COMPASS data [10, 11]. (a) The flavours u and $\bar{u}$. (b) The flavours d and $\bar{d}$. (c) The flavours s and $\bar{s}$ that were fixed to $\pm$ positivity bounds (17) for reasons explained in Sec. 7, see also Eqs. (18, 19). The shaded areas in (a) and (b) show the respective 1-σ-uncertainties.

Arnold et al. , 0805.2137

Relation to anomalous magnetic moment

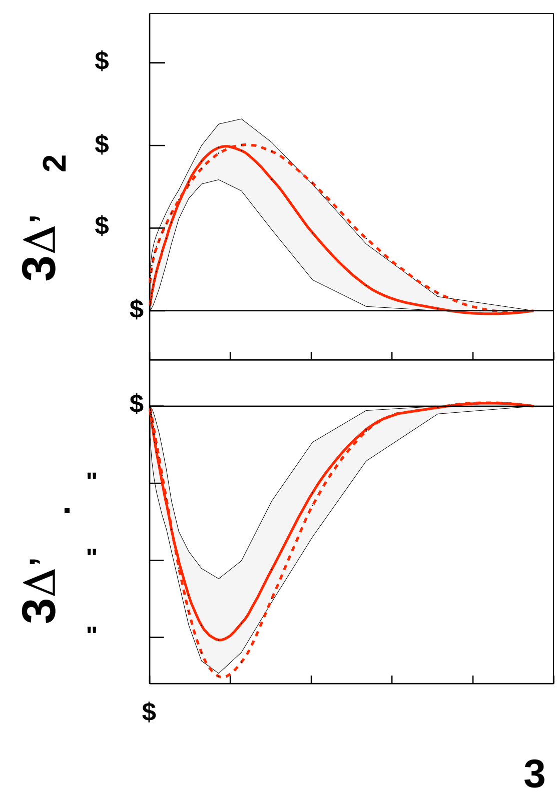
Model statement

$$(1-x) f_{1T}^{\perp q}(x) = -\frac{3}{2} M C_F \alpha_S \, E^q(x,0,0)$$

$$\int_0^1 dx (1-x) f_{1T}^{\perp q}(x) = -\frac{3}{2} M C_F \alpha_S \, \kappa^q$$

Burkardt, Hwang, PRD69 (04)
Lu, Schmidt, PRD75 (07)
A.B., F. Conti, M. Radici, arXiv:0807.0323

$$k^u = 1.67$$
$$k^d = -2.03$$



Anselmino et al., 0805.2677,
Arnold et al. , 0805.2137

The relation is not general

A simple assumption

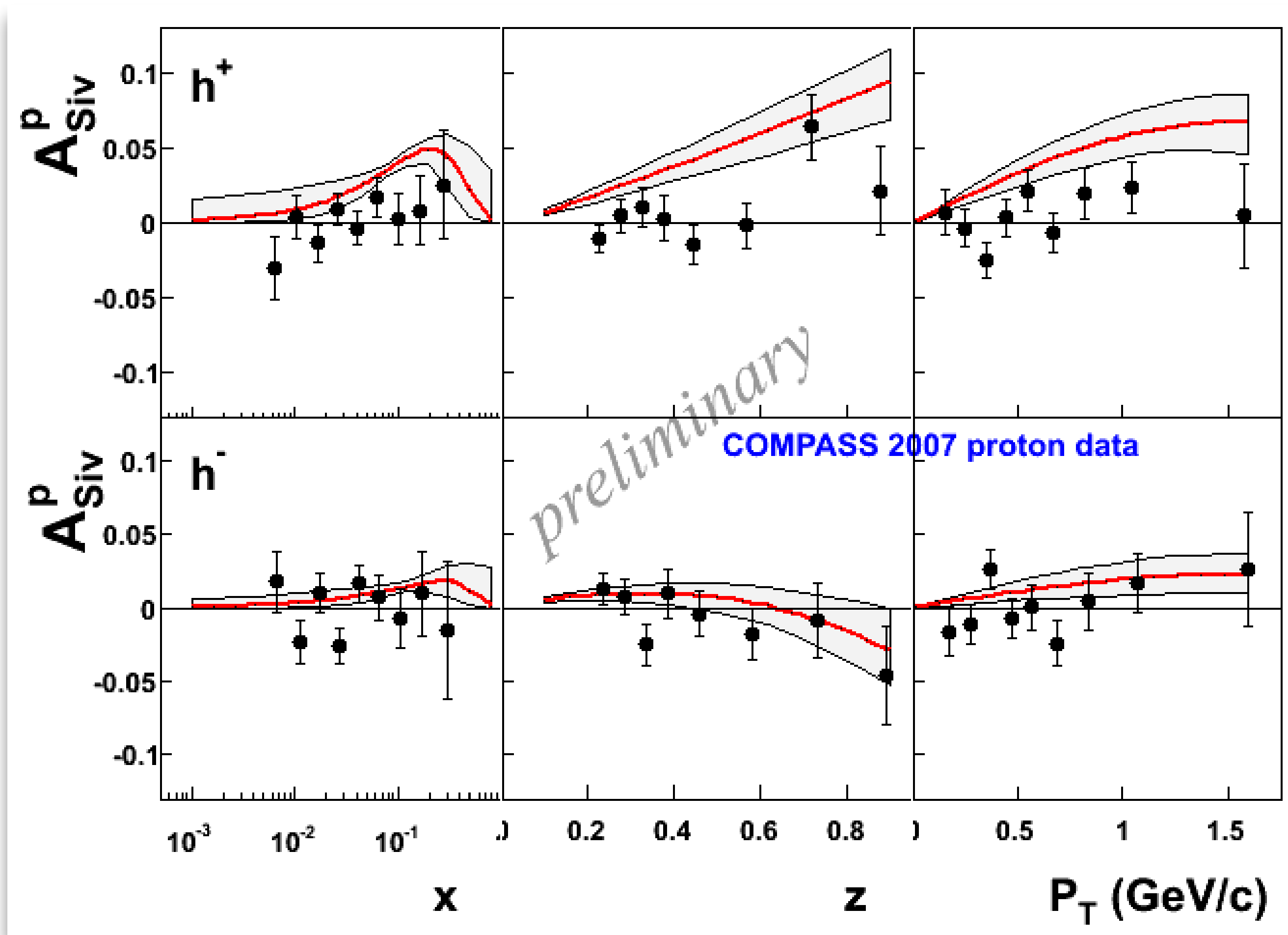
$$f_{1T}^{\perp q}(x) = -f(x)\, E^q(x,0,0)$$
$$f_{1T}^{\perp g}(x) = -f'(x)\, E^g(x,0,0)$$

$$\frac{E^a(x,0,0)}{E^u(x,0,0)} = \frac{f_{1T}^{\perp a}(x)}{f_{1T}^{\perp u}(x)} = \frac{A_a}{A_u}\,\frac{f_1^a(x)}{f_1^u(x)},$$

$$\frac{A_d}{A_u} = -1.8 \pm 0.2, \quad \frac{A_{\bar{u}}}{A_u} = -1.1 \pm 0.1, \quad \frac{A_{\bar{d}}}{A_u} = 1.3 \pm 0.2, \quad \frac{A_s}{A_u} = -\frac{A_{\bar{s}}}{A_u} = -4.8.$$

AB, arXiv:0902.2712 [hep-ph]
using fit of Arnold et al. , 0805.2137

Sivers: COMPASS proton



data: S. Levorato, Transversity 08
prediction: Anselmino et al., 0805.2677

The question of evolution

Back to $F_{UU,T}$, "Leading Log" only, b space

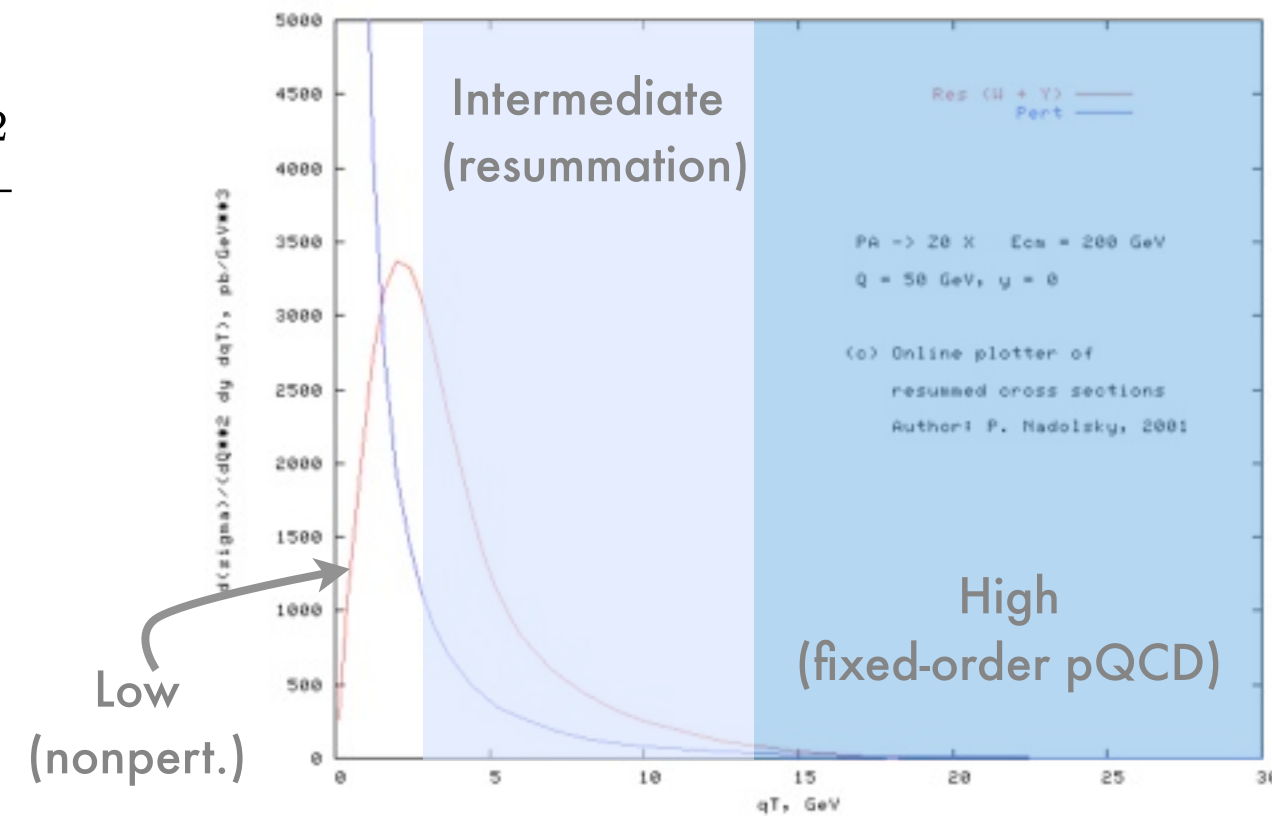
Collins, Soper, Sterman, NPB250 (85)

$$F_{UU,T}(x,z,b,Q^2) = x\sum_a e_a^2\, f_1^a\Big(x;\frac{b^0}{b}\Big)\, D_1^a\Big(z;\frac{b^0}{b}\Big)\, e^{-S} e^{-S_{NP}}$$

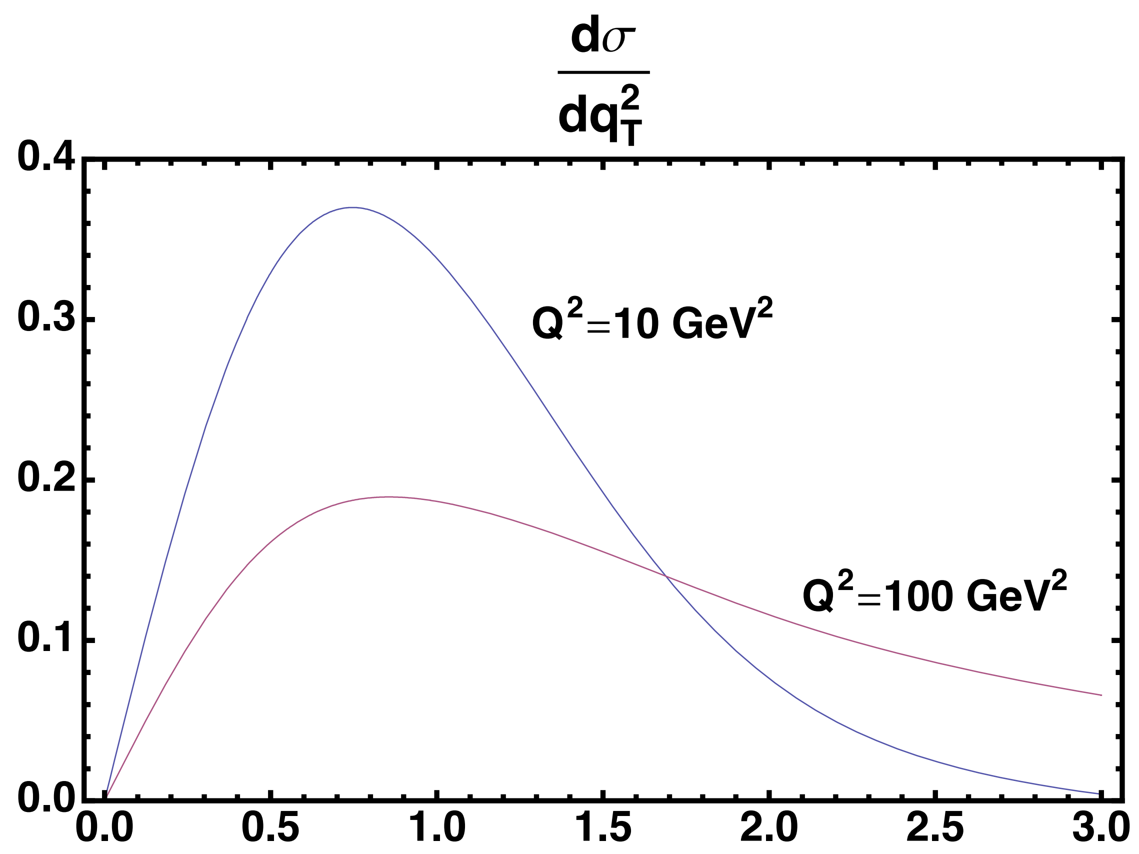
collinear PDFs

nonperturbative part of TMD

$$S(b,Q^2) = \int_{b_0^2/b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\,\frac{\alpha_S(\mu^2)}{2\pi}\, 2C_F \log\frac{Q^2}{\mu^2}$$



Leading-log evolution



Evolution of Sivers function

$$f_1^{\rm NS}(x,p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{1}{\boldsymbol{p}_T^2}\left[\left(\frac{L(\eta^{-1})}{2} - C_F\right) f_1^{\rm NS}(x) + \left(P_{qq}\otimes f_1^{\rm NS}\right)\right]$$

$$\frac{\boldsymbol{p}_T^2}{2M^2}\, f_{1T}^{\perp{\rm NS}}(x,p_T^2) = \frac{\alpha_s}{2\pi^2}\,\frac{M}{\boldsymbol{p}_T^2}\left[\left(\frac{L(\eta^{-1})}{2} - C_F\right) f_{1T}^{\perp(1){\rm NS}}(x) + \ldots\right]$$

$$F_{UU,T} = \frac{1}{q_T^2}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x e_a^2\left[f_1^a(x)\, D_1^a(z)\, L\left(\frac{Q^2}{q_T^2}\right) + \ldots\right]$$

$$\frac{q_T}{M}\, F_{UT,T}^{\sin(\phi_h-\phi_s)} = \frac{1}{q_T^2}\,\frac{\alpha_s}{2\pi^2 z^2}\sum_a x e_a^2\left[-f_{1T}^{\perp(1)a}(x)\, D_1^a(z)\, L\left(\frac{Q^2}{q_T^2}\right) + \ldots\right]$$

Idilbi, Ji, Ma, Yuan, PRD70 (04)
Boer, NPB 806 (09)

Evolution of trans. moment of Sivers function

Kang, Qiu, PRD79 (09)
Vogelsang, Yuan, arXiv:0904.0410 [hep-ph]

$$\frac{\partial f_1^{\mathrm{NS}}(x,\mu^2)}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi}\int_x^1 \frac{d\xi}{\xi} f_1^{\mathrm{NS}}(\xi,\mu^2) P_{qq}(z)\Big|_{z=x/\xi}$$

$$\begin{aligned}\frac{\partial \mathcal{T}_{q,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} &= \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Big\{ P_{qq}(z)\,\mathcal{T}_{q,F}(\xi,\xi,\mu_F) \\ &\quad + \frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[\mathcal{T}_{q,F}(\xi,x,\mu_F) - \mathcal{T}_{q,F}(\xi,\xi,\mu_F)\right] + z\,\mathcal{T}_{q,F}(\xi,x,\mu_F)\right] \\ &\quad + \frac{C_A}{2}\left[\mathcal{T}_{\Delta q,F}(x,\xi,\mu_F)\right]\Big\},\end{aligned}$$

$$T_F(x,x) \equiv 2M f_{1T}^{\perp(1)}(x)$$

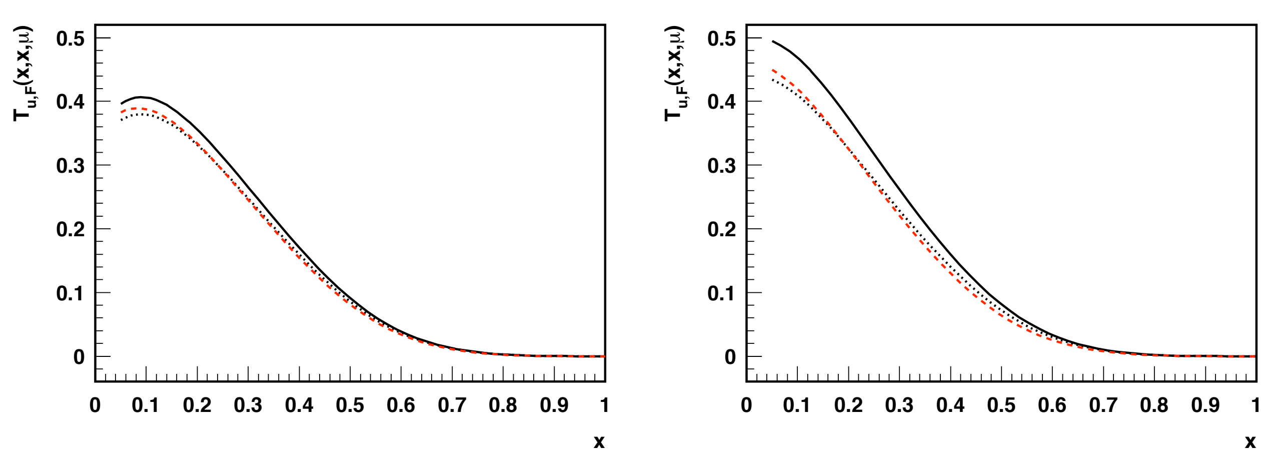


FIG. 12: Twist-3 up-quark-gluon correlation $T_{u,F}(x,x,\mu_F)$ as a function of x at $\mu_F = 4$ GeV (left) and $\mu_F = 10$ GeV (right). The factorization scale dependence is a solution of the flavor non-singlet evolution equation in Eq. (99). Solid and dotted curves correspond to $\sigma = 1/4$ and $1/8$, while the dashed curve is obtained by keeping only the DGLAP evolution kernel $P_{qq}(z)$ in Eq. (99).

Conclusions about Sivers

- Several limits to present analysis, but things are moving
- There is a framework to study evolution
- One of the problems is also the knowledge of the kT dependence of fragmentation functions (help from BELLE soon?)
- Connections with orbital angular momentum?

Transversity

Asymmetry

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right]$$

Collins asymmetries

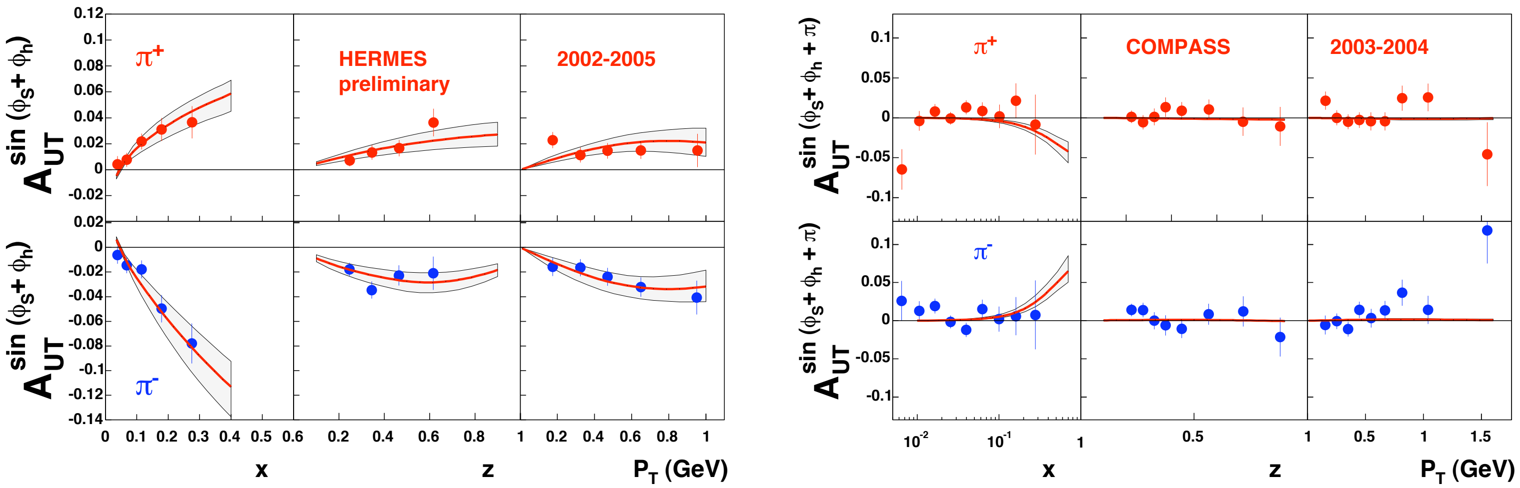


Figure 2: Fits of HERMES [4] and COMPASS [5] data. The shaded area corresponds to the uncertainty in the parameter values, see Ref. [3].

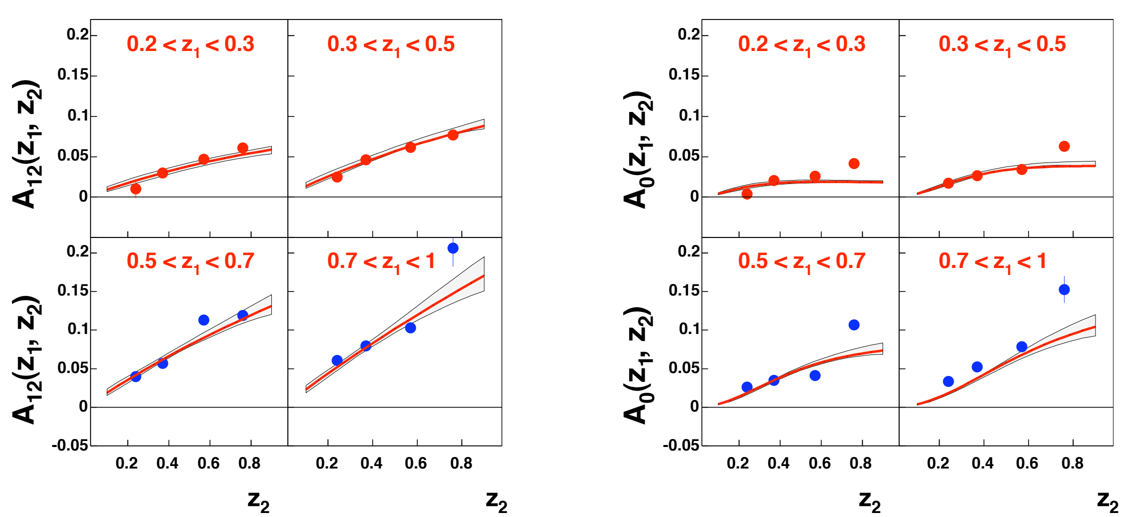


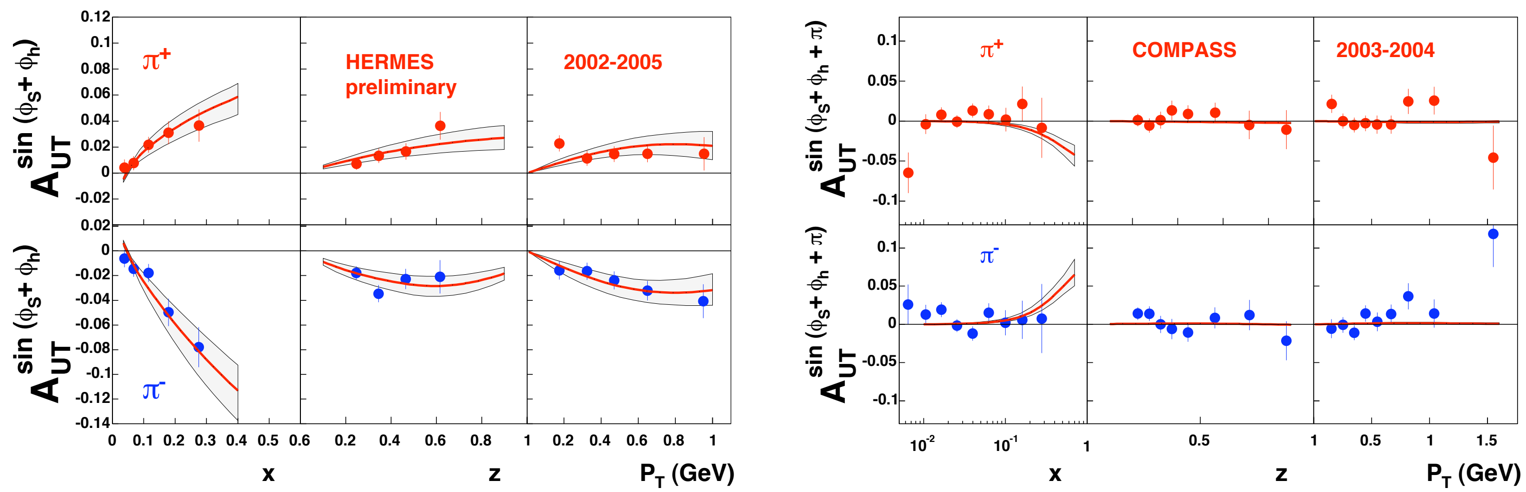
Figure 3: Left panel: fit of the BELLE [6] data on the A_{12} asymmetry ($\cos(\varphi_1+\varphi_2)$ method). Right panel: predictions for the A_0 BELLE asymmetry ($\cos(2\,\varphi_0)$ method).

Data:
HERMES, arXiv:0706.2242
COMPASS, PLB673 (09)

Data: BELLE, PRD78 (08)

Analysis: Anselmino et al., arXiv:0807.0173

Collins asymmetries



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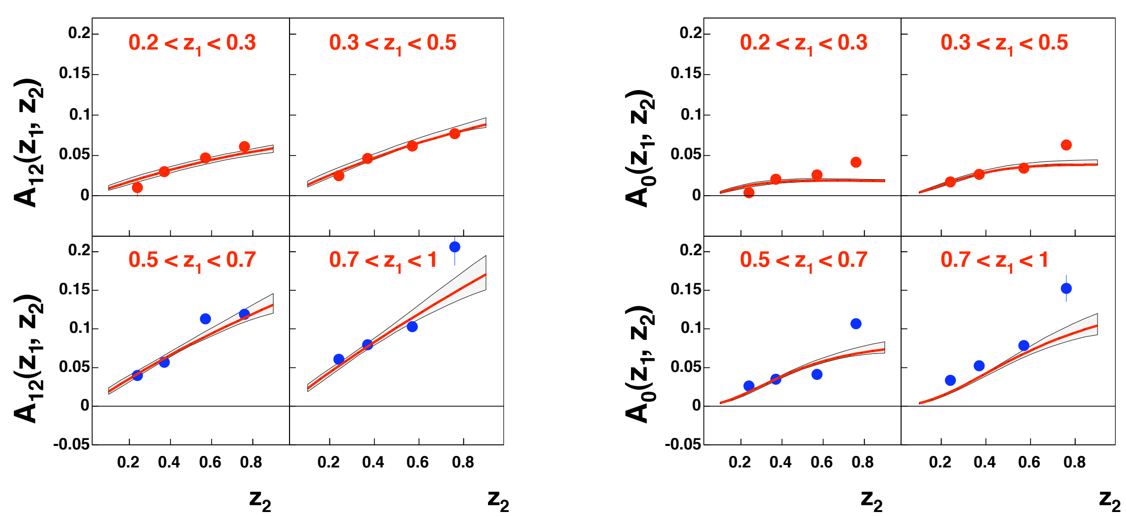

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Anselimino et al., arXiv:0807.0173

Transversity and Collins

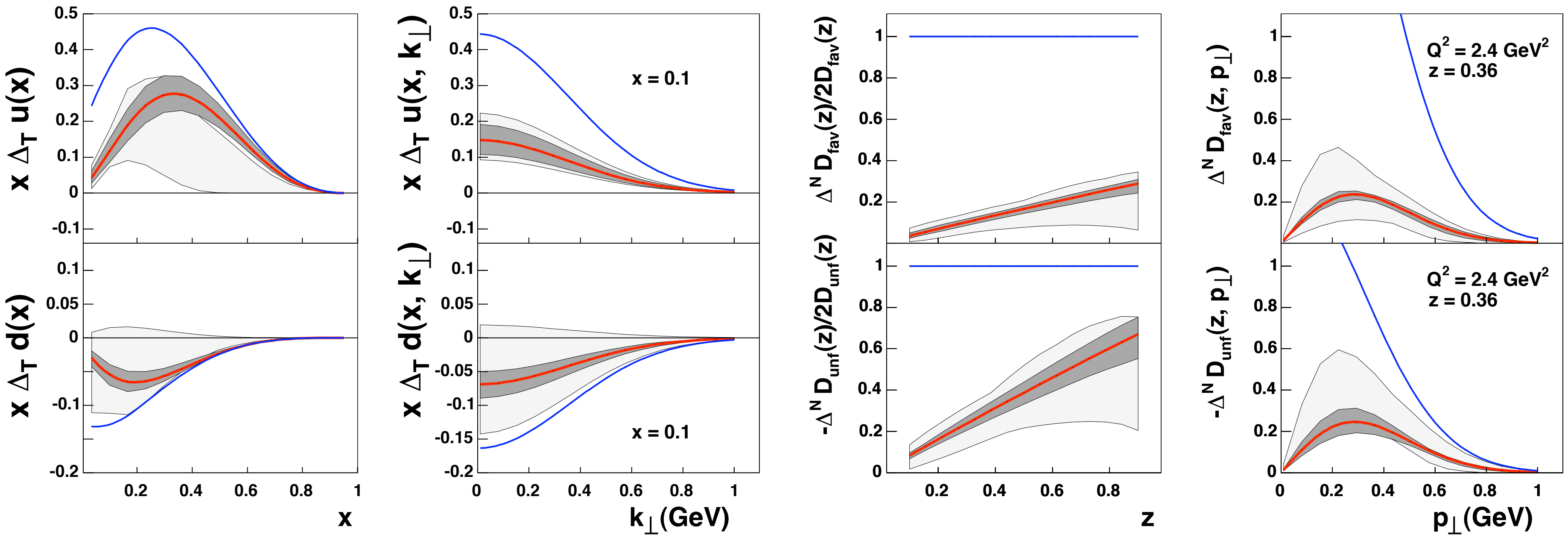
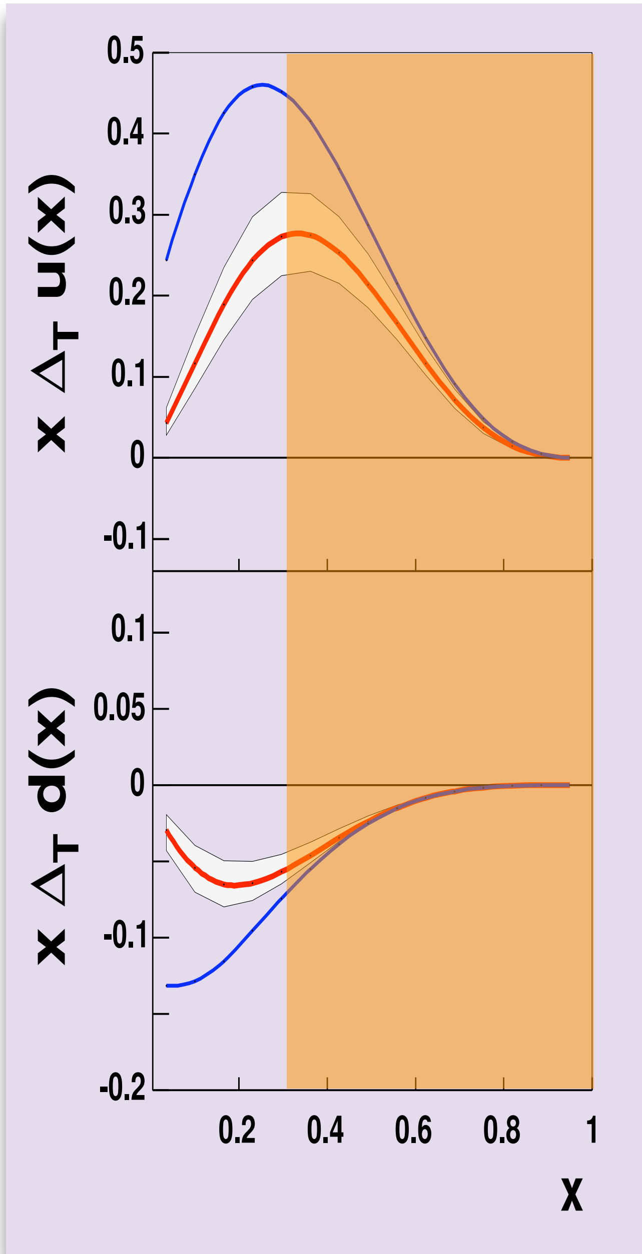


Figure 1: Left panel: the transversity distribution functions for u and d flavours as determined by our global fit; we also show the Soffer bound (highest or lowest lines) and the (wider) bands of our previous extraction [3]. Right panel: favoured and unfavoured Collins fragmentation functions as determined by our global fit; we also show the positivity bound and the (wider) bands as obtained in Ref. [3].

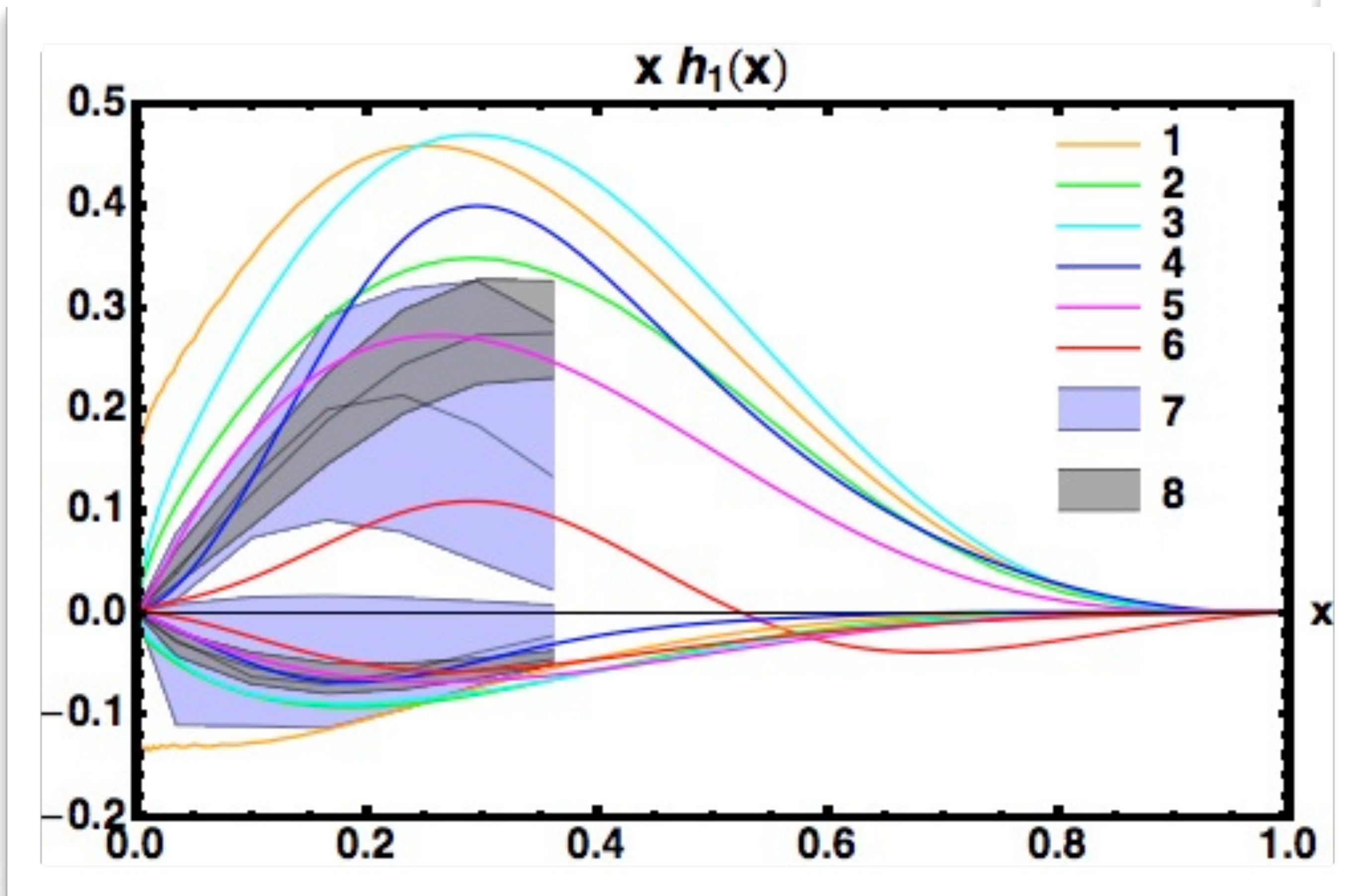
Transversity

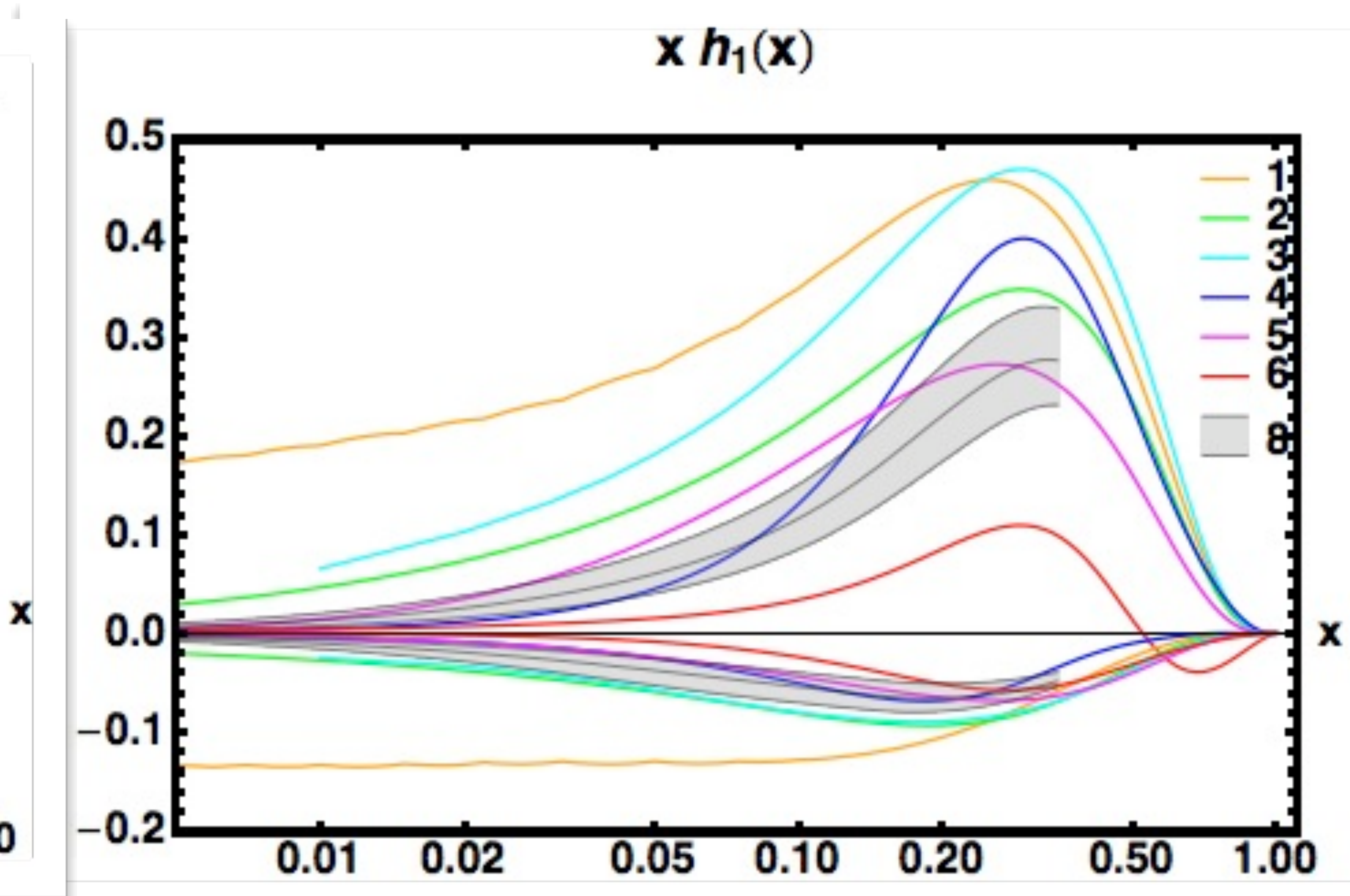


- Data from HERMES, COMPASS, BELLE
- 96 data points (some correlations -- cf. 467 points for Δq fits)
- no sys errors taken into account
- $\chi^2 \approx 1.4$
- Statistical uncertainty only ($\Delta\chi^2 \approx 17$)

A. Prokudin, talk at DIS08 (extraction by Anselmino et al.)

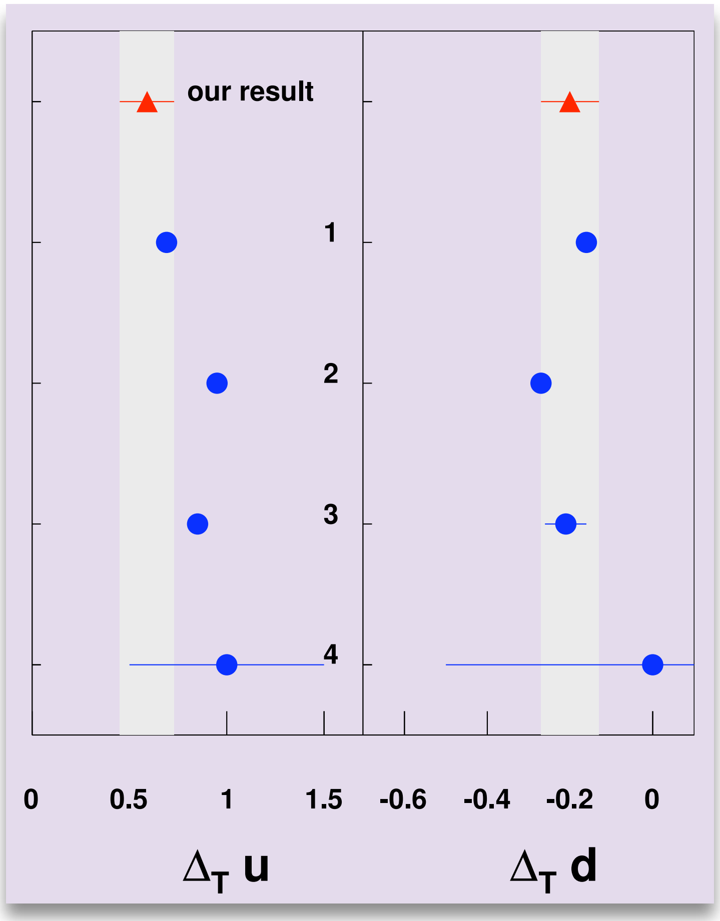
Comparison with models





[1] Soffer et al. PRD 65 (02)
[2] Korotkov et al. EPJC 18 (01)
[3] Schweitzer et al., PRD 64 (01)
[4] Wakamatsu, PLB 509 (01)
[5] Pasquini et al., PRD 72 (05)
[6] Bacchetta, Conti, Radici, PRD 78 (08)
[7] Anselmino et al., PRD 75 (07)
[8] Anselmino et al., arXiv:0807.0173

Tensor charge



[our result] Anselmino et al. DIS 08

[1] Diquark spectator model,
Cloet, Bentz, Thomas, PLB 659 (08)

[2] Chiral quark soliton model,
Wakamatsu, PLB 653 (07)

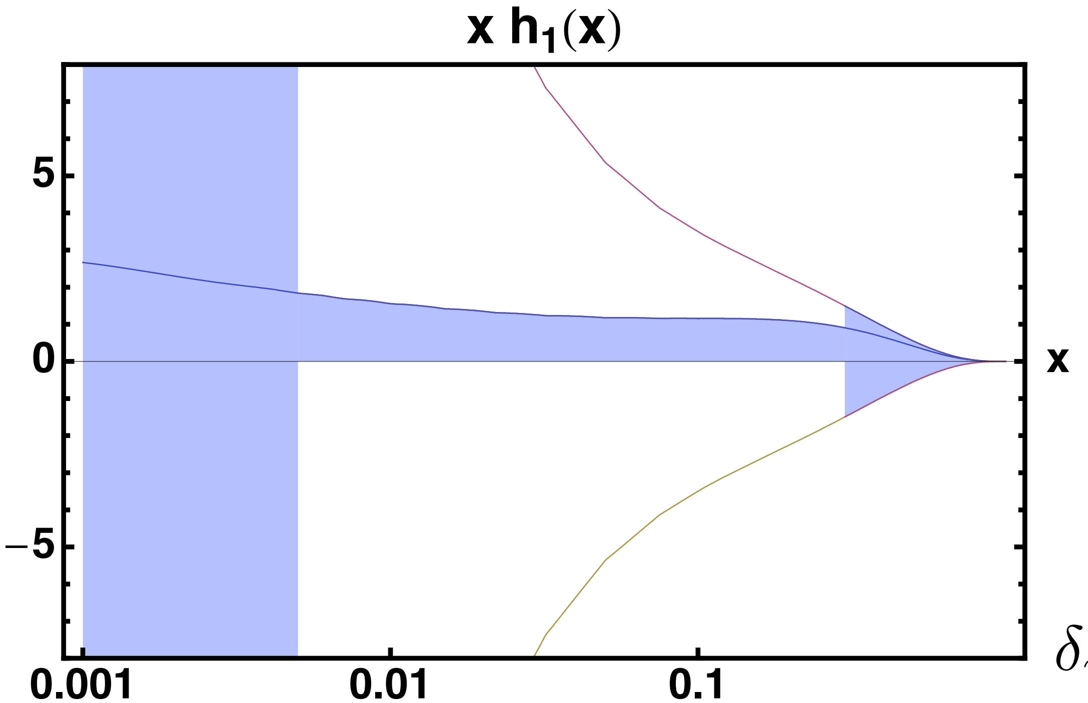
[3] Lattice QCD,
Goekeler et al. PLB 627 (05)

[4] QCD sum rules,
He, Ji, PRD 52 (95)

The first x-moments of the transversity distribution – related to the tensor charge, and defined as $\Delta_T q \equiv \int_0^1 \mathrm{d}x \Delta_T q(x)$ – are found to be $\Delta_T u = 0.59^{+0.14}_{-0.13}$, $\Delta_T d = -0.20^{+0.05}_{-0.07}$ at $Q^2 = 0.8$ GeV^2.

Anselmino et al., arXiv:0807.0173

Tensor charge: extremes



$$\delta_u\Big|_{0.005}^{1} = 0.33^{+0.11}_{-0.08} \pm 0.15$$

$$\delta_u\Big|_{0.005}^{0.3} = 0.33^{+0.11}_{-0.08}$$

$$\delta_u\Big|_{0.001}^{1} = 0.33^{+0.11}_{-0.08}(\mathrm{stat}) \pm 0.15(\mathrm{sys,\ high}\ x) \pm 0.14(\mathrm{sys,\ low}\ x)$$

$$\delta_d\Big|_{0.001}^{1} = -0.14^{+0.04}_{-0.06}(\mathrm{stat}) \pm 0.02(\mathrm{sys,\ high}\ x) \pm 0.12(\mathrm{sys,\ low}\ x)$$

Collins asymmetry, *b* space analysis

D. Boer, NPB 806 (08)

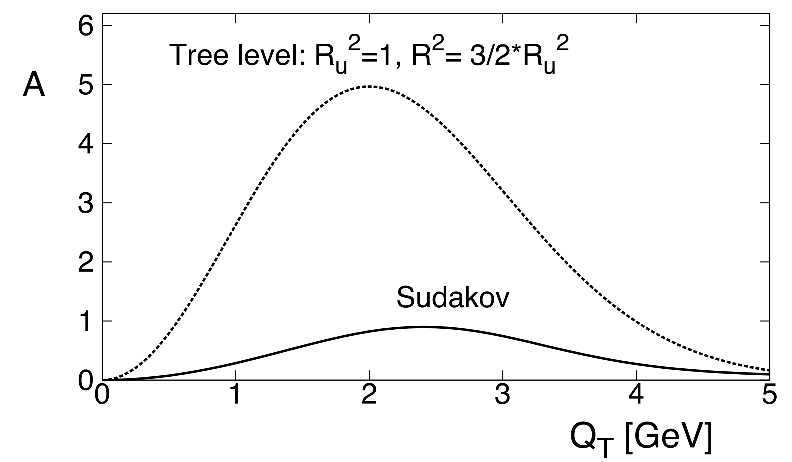


FIG. 6: The asymmetry factor $\mathcal{A}(Q_T)$ at $Q = 10\,\mathrm{GeV}$ (solid curve) and the tree level quantity $\mathcal{A}^{(0)}(Q_T)$ using $R_u^2 = 1\,\mathrm{GeV}^{-2}$ and $R^2/R_u^2 = 3/2$. Both factors are given in units of M^2.

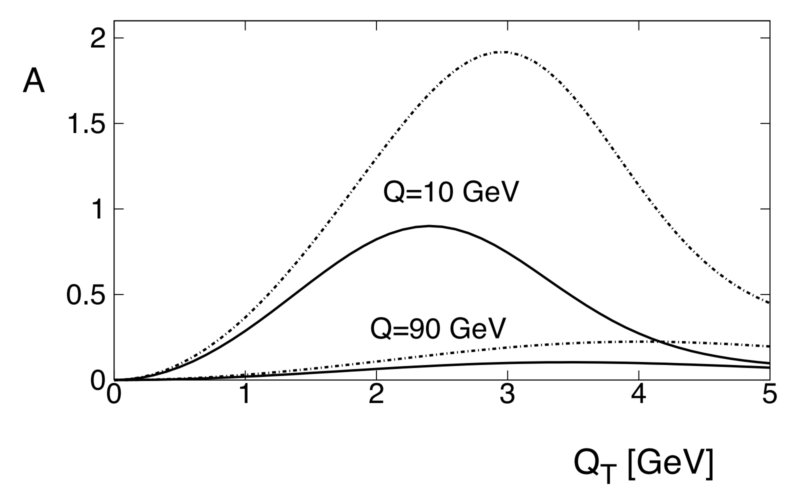


FIG. 5: The asymmetry factor $\mathcal{A}(Q_T)$ (in units of M^2) at $Q = 10\,\mathrm{GeV}$ and $Q = 90\,\mathrm{GeV}$. The solid curves are obtained with the method explained in the text; the dashed-dotted curves are from the earlier analysis of Ref. [64].

Cos 2 φ

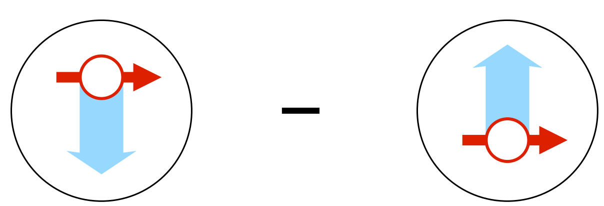
TMD convolution (low transverse momentum)

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h}\, h_1^{\perp} H_1^{\perp}\right],$$

Boer-Mulders function

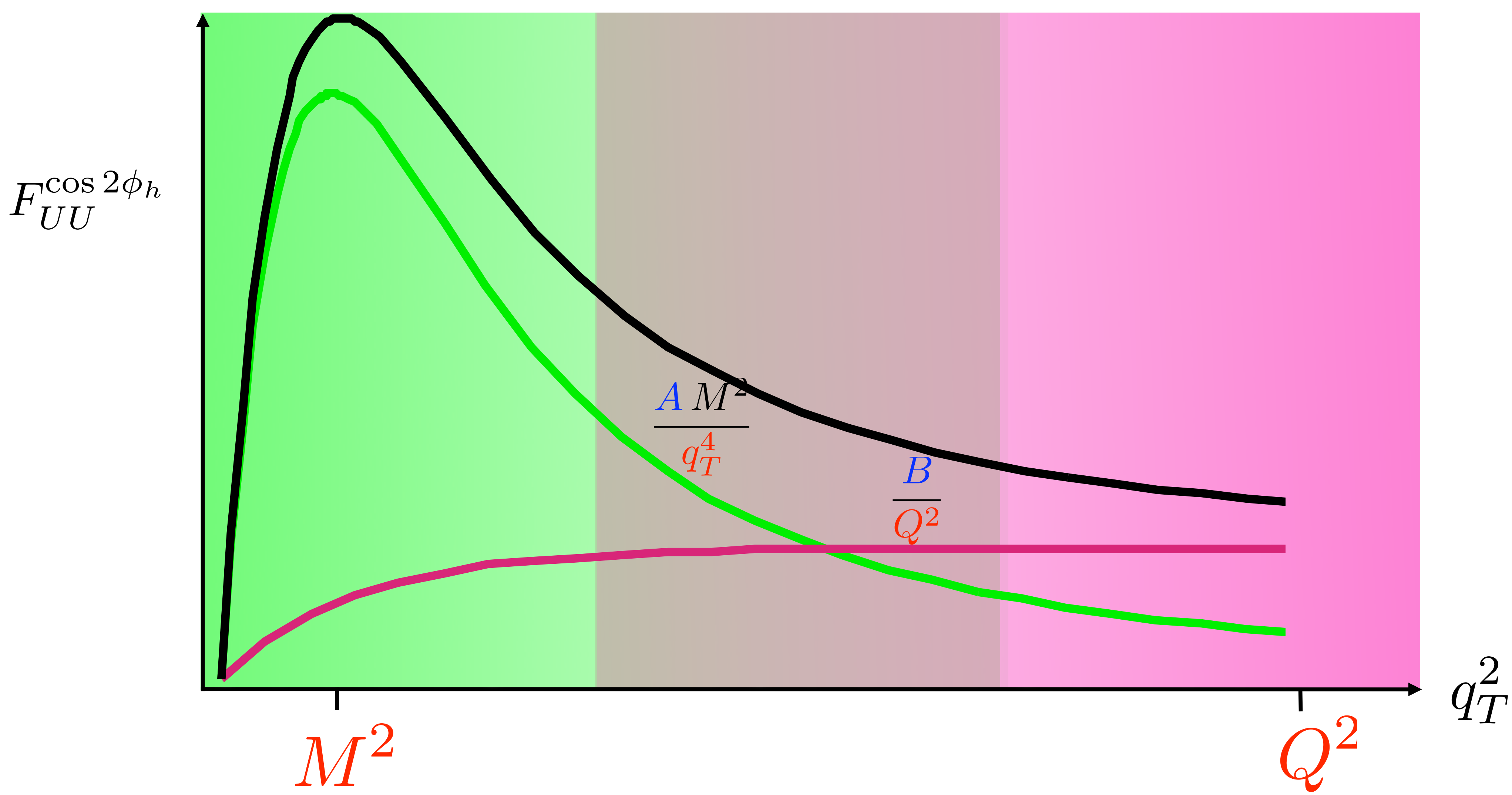
Collins function

$h_1^{\perp} =$



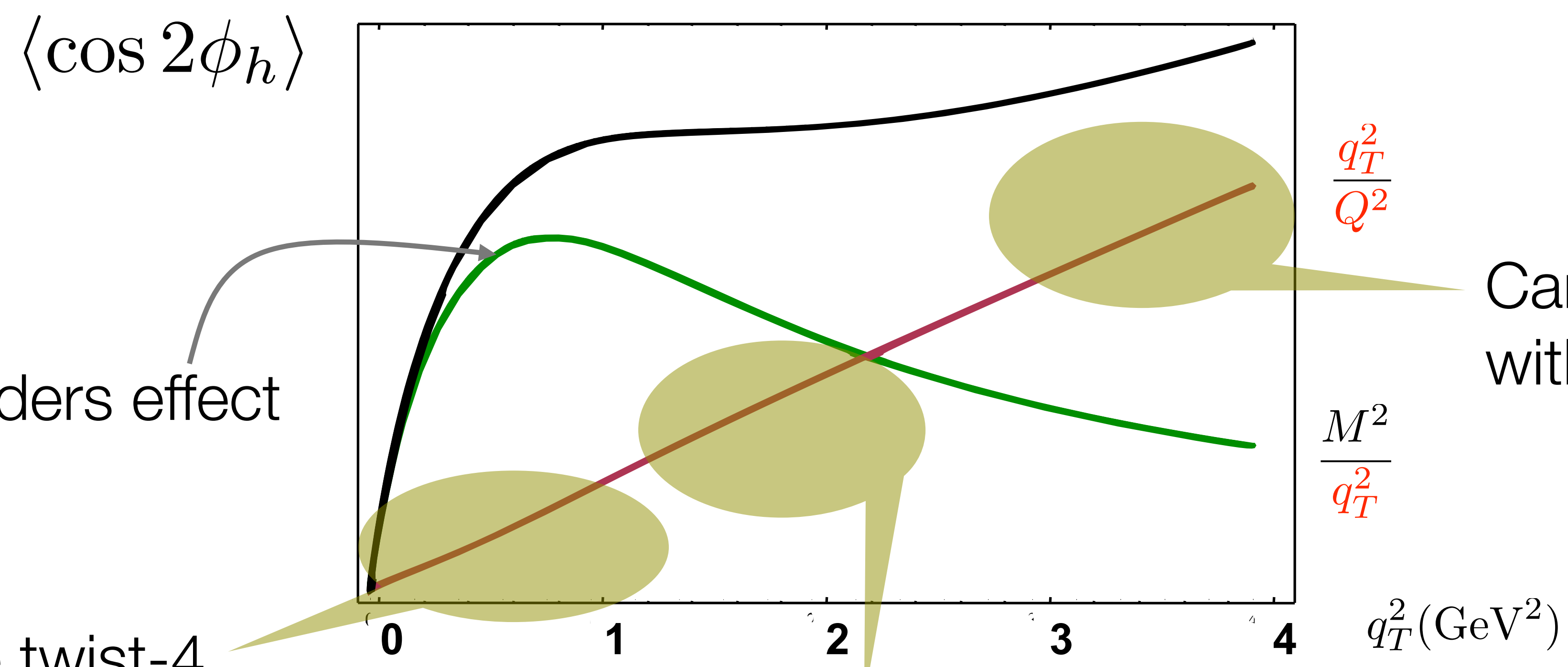
Expected mismatch at high trans. momentum

The leading terms in the two expansions CANNOT and MUST not match!



Two distinct mechanisms are involved

Cos 2 φ asymmetry



Boer-Mulders effect

Can be calculated with pQCD

Nonperturbative twist-4
no factorization, only models
(Cahn twist-4?)

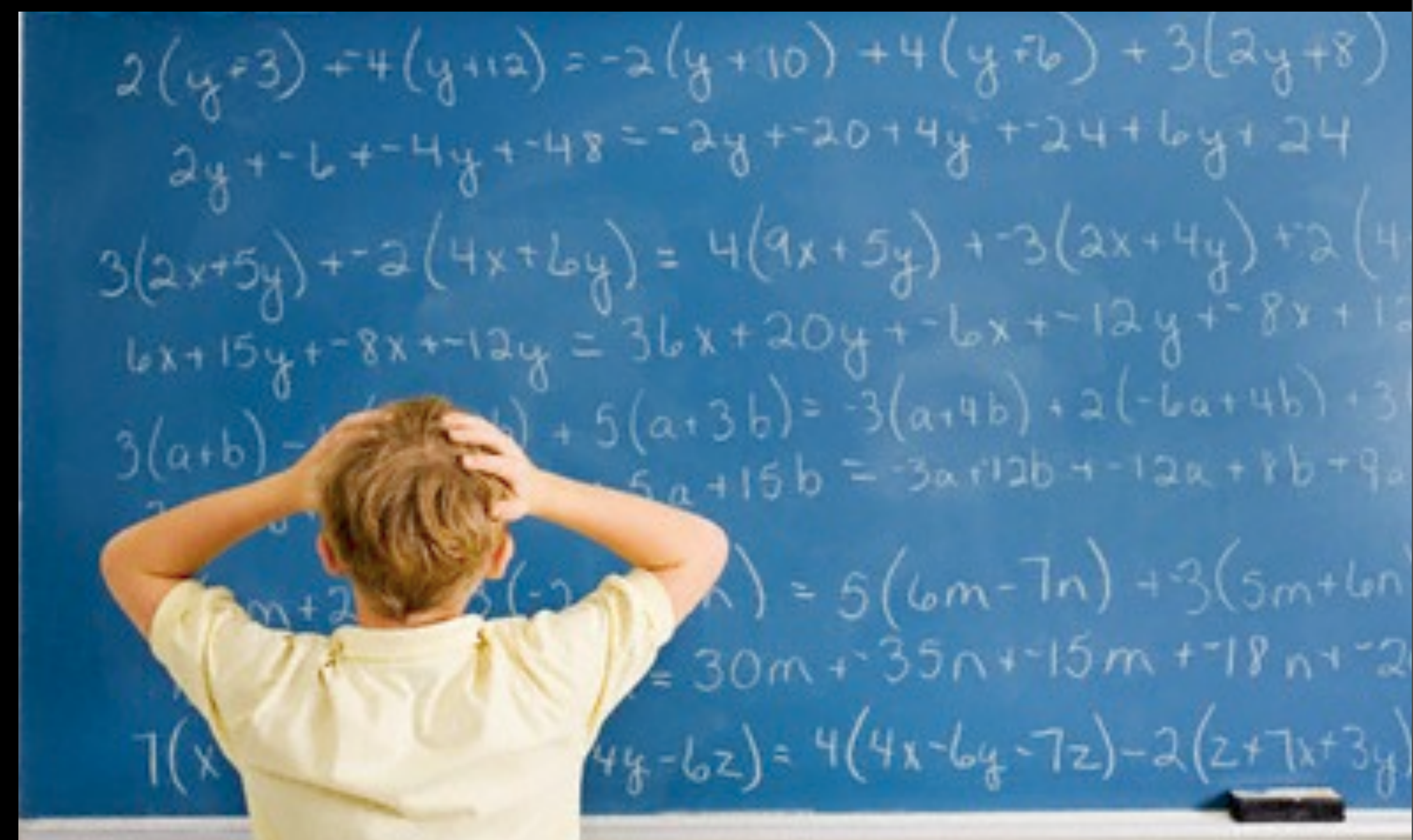
needs resummation, not simply extend pQCD

see also Berger, Qiu, Rodriguez-Pedraza, PRD76 (07)

Similarly for Drell-Yan Boer-Mulders measurement
and Belle Collins measurement

Complications of a full-fledged analysis

- Isolate Boer-Mulders effect
- Take into account convolutions (usually Gaussian)
- Take properly into account experimental errors
- Take into account Sudakov form factors (probably working in *b* space)



Nevertheless...

Boer-Mulders extraction (?) in *pD* scattering

Zhang, Lu, Ma, Schmidt, PRD77 (08)

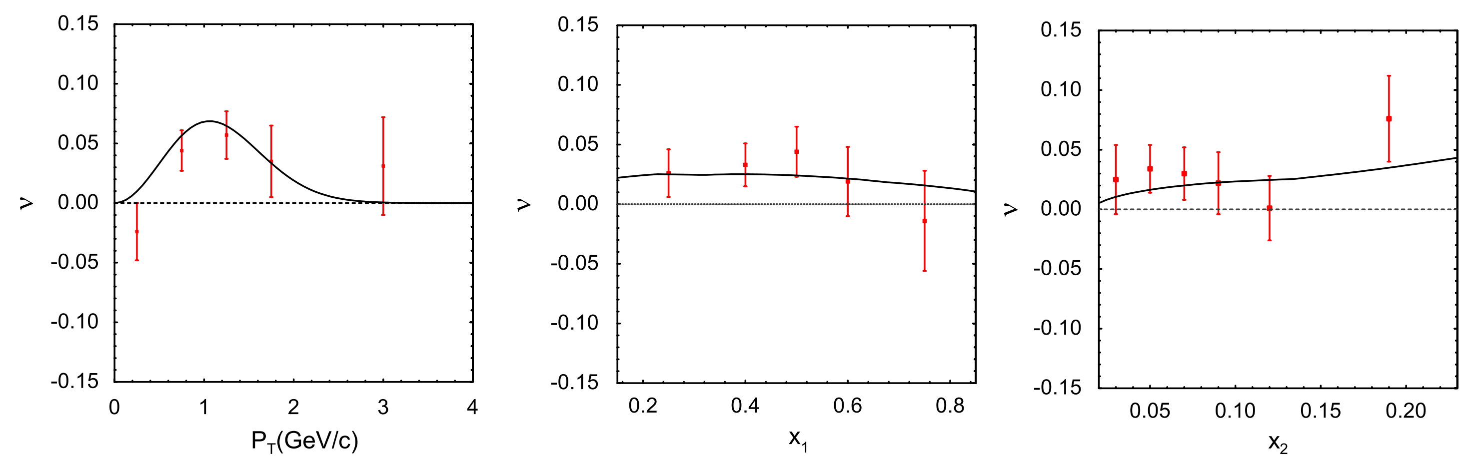


FIG. 1 (color online). Fits to the p_T, x_1, x_2-dependent $\cos 2\phi$ asymmetries ν_{pD} for Drell-Yan processes. Data are from the FNAL E866/NuSea collaboration.

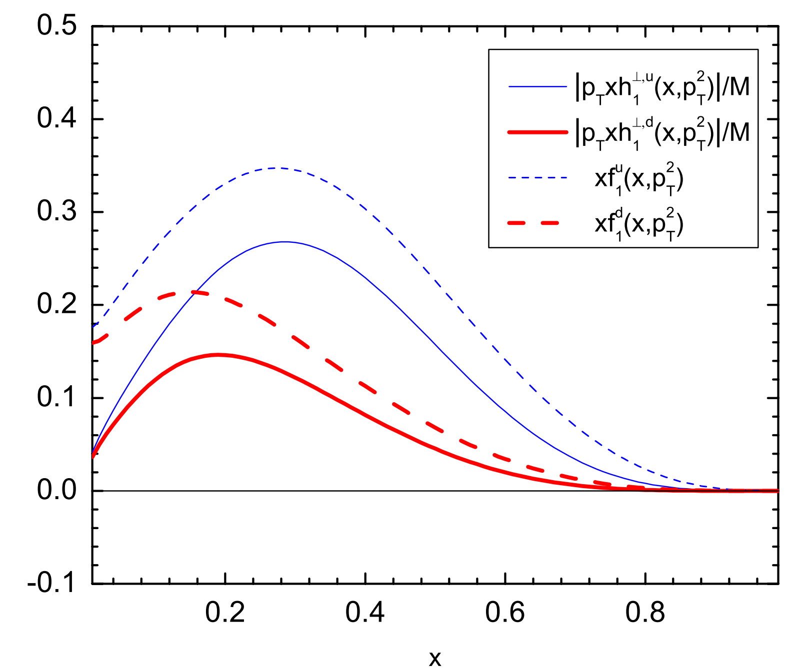


FIG. 2 (color online). Comparison of $|p_T x h_1^\perp(x, \mathbf{p}_T^2)|/M$ and $x f_1(x, \mathbf{p}_T^2)$ for u and d quarks at $p_T = 0.45$ GeV and $Q = 1$ GeV. Here f_1 is a combination of valence and sea quark distributions.

- modulo overall factor
- no errors

A different study

$$h_1^{\perp u} = 1.80 f_{1T}^{\perp u}, \qquad h_1^{\perp d} = -0.94 f_{1T}^{\perp d}$$

Barone, Prokudin, Ma, PRD78 (08)

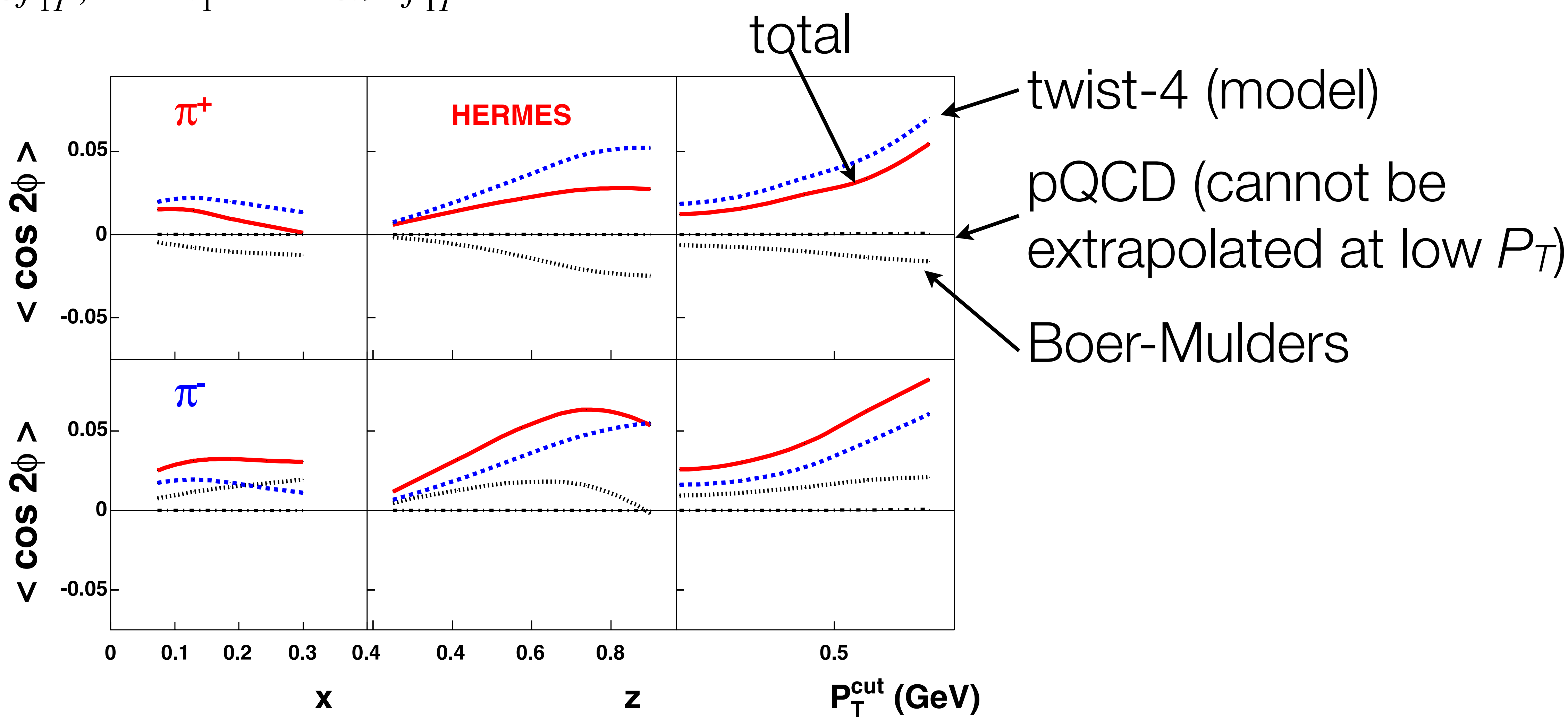


FIG. 5 (color online). Our prediction for the $\cos 2\phi$ asymmetry at HERMES. The dot-dashed line is the $\mathcal{O}(\alpha_s)$ QCD contribution, the dotted line is the Boer-Mulder contribution, the dashed line is the higher-twist Cahn contribution. The continuous line is the resulting asymmetry taking all contributions into account.

Ways out

- Perturbative corrections are flavor independent: use proper ratios or difference to cancel them
    - unlikely that it works for nonperturbative twist-4

- Exploit the relation, analogous to Lam-Tung in Drell-Yan

$$F_{UU}^{\cos 2\phi_h}{}_{\text{pert.}} = 2F_{UU,L}{}_{\text{pert.}}$$

    - may work also for nonperturbative twist-4

Cos φ

Convolution

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\Big)\Big]$$

$$xf^{\perp} = x\tilde{f}^{\perp} + f_1$$

$$xh = x\tilde{h} + \frac{p_T^2}{M^2}\,h_1^{\perp}.$$

pure twist-3 part

twist-2 part

No reason to believe that twist-3 part is small!
(see Wandzura-Wilzcek breaking)

Nevertheless...

Neglecting pure twist-3 and also T-odd...

$$F_{UU}^{\cos\phi_h} \approx \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\,f_1 D_1\right].$$

see Cahn, PLB78 (89)
Anselmino et al., PRD71 (05)

What do we know about the different pieces?

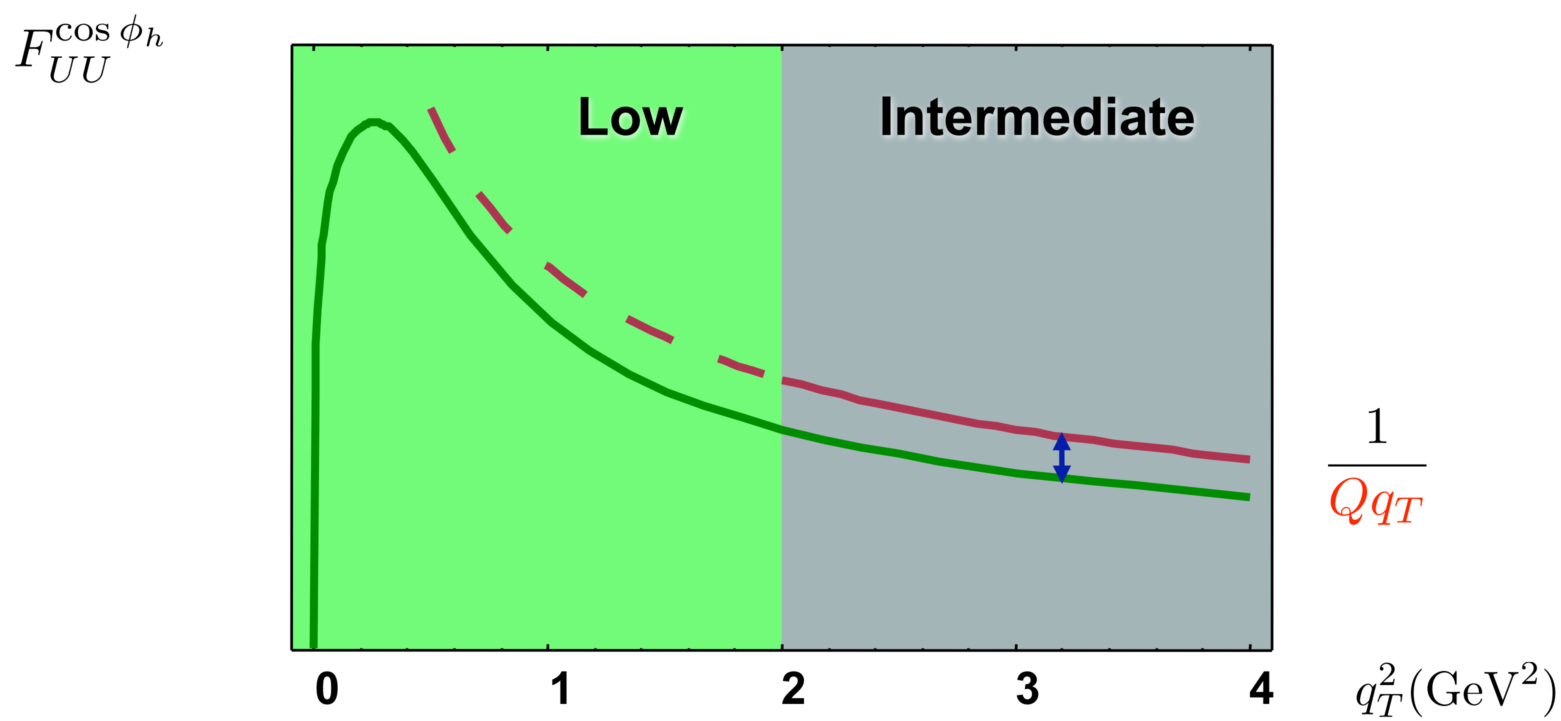
$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\,\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\,\frac{\tilde{H}}{z}\Big)\Big]$$

- vanishes if integrated over x
- peculiar behavior of Collins function

- vanishes if integrated over z

- dominant at high transverse momentum
- we know f_1 and D_1 pretty well

Unexpected mismatch



Unexpected mismatch: same power behavior, but they don't match

Problems with the formalism at low transverse momentum!

Conclusion on unp. azimuthal modulations

- They are a bit messy
- More difficult at the moment to extract information
- Teach us to be aware of complications

“Education is what remains after one has forgotten everything he learned”

one of Anatoly’s favorite quotes

Thank you!