Preliminary plan

1. Introduction
2. Inclusive and semi-inclusive DIS (structure functions)

   Basics of collinear PDFs at tree level (definition, gauge link)
3. Basics of collinear PDFs (interpretation)

   Basics of TMDs at tree level (definition, gauge link, interpretation)
4. Basics of factorization

   Basics of TMD evolution

- Phenomenology of unpolarized SIDIS
- Phenomenology of polarized SIDIS

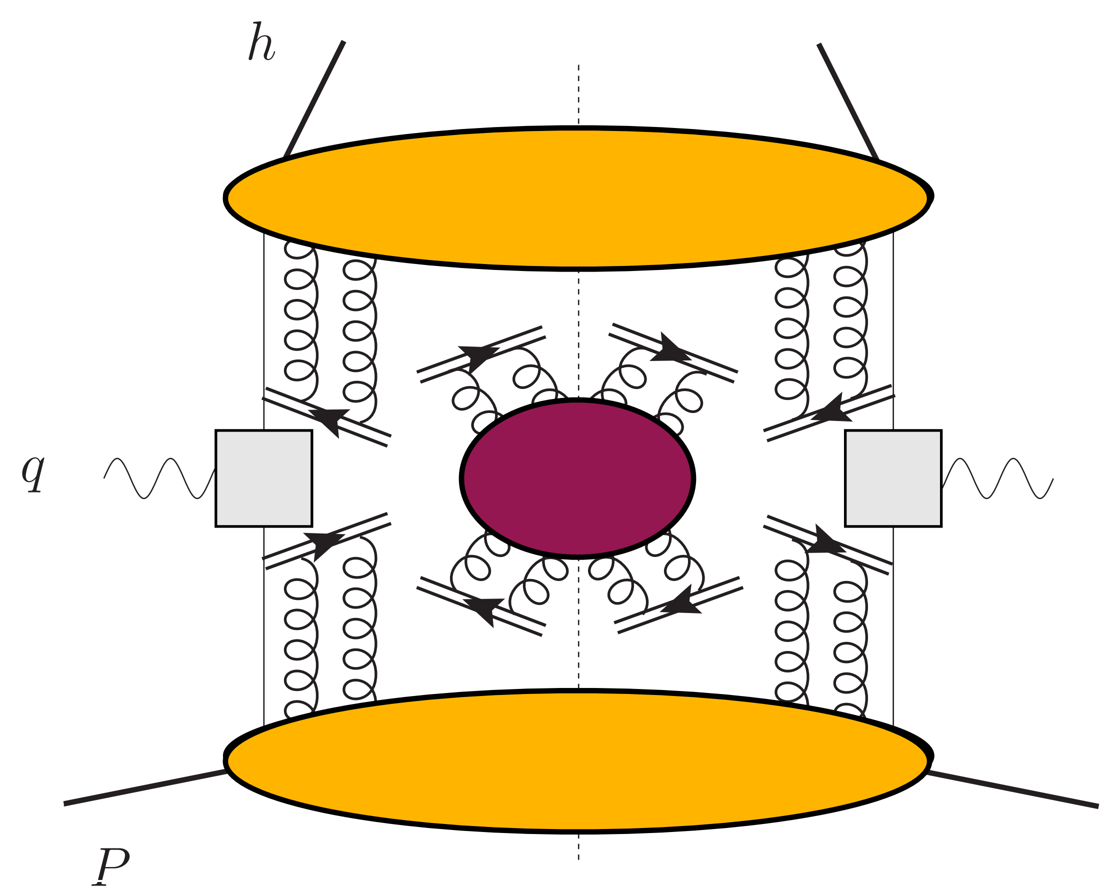
Next lecture

- May 27, 12:00 PM in F113

Quick review of last lecture

TMD factorization

Collins, Soper, NPB 193 (81)
Ji, Ma, Yuan, PRD 71 (05)



$$F_{UU,T}(x, z, P_{h\perp}^2, Q^2) = \mathcal{C}'\big[f_1 D_1\big]$$

$$= H(Q^2, \mu^2, \zeta, \zeta_h) \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T - \boldsymbol{P}_{h\perp}/z\big)$$

$$x \sum_a e_a^2\, f_1^a(x, p_T^2, \mu^2, \zeta)\, D_1^a(z, k_T^2, \mu^2, \zeta_h)\, U(l_T^2, \mu^2, \zeta\zeta_h)$$

Hard part

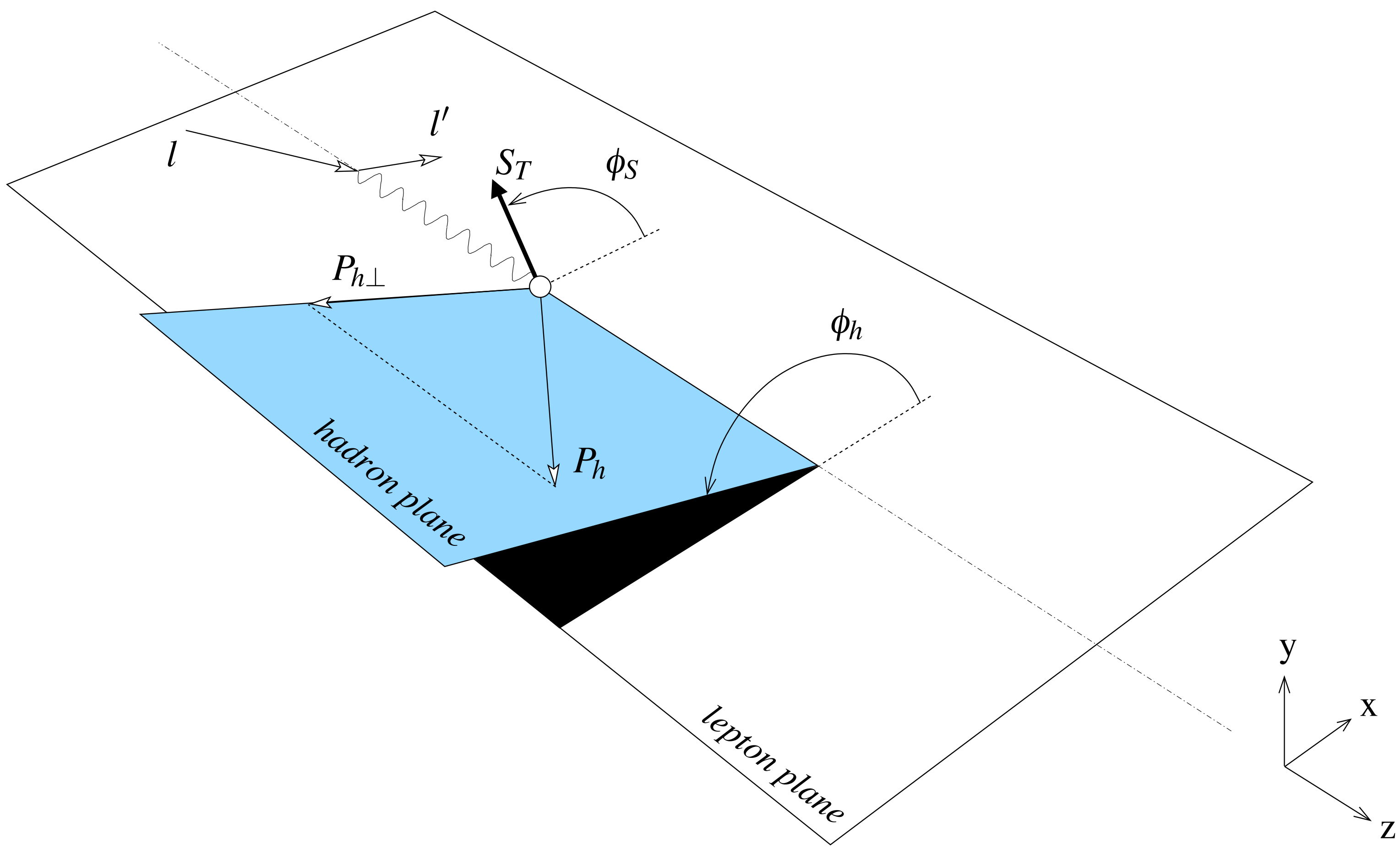
TMD PDF

TMD FF

Soft factor

High and low transverse momentum

SIDIS once again



$$Q = \text{photon virtuality}$$

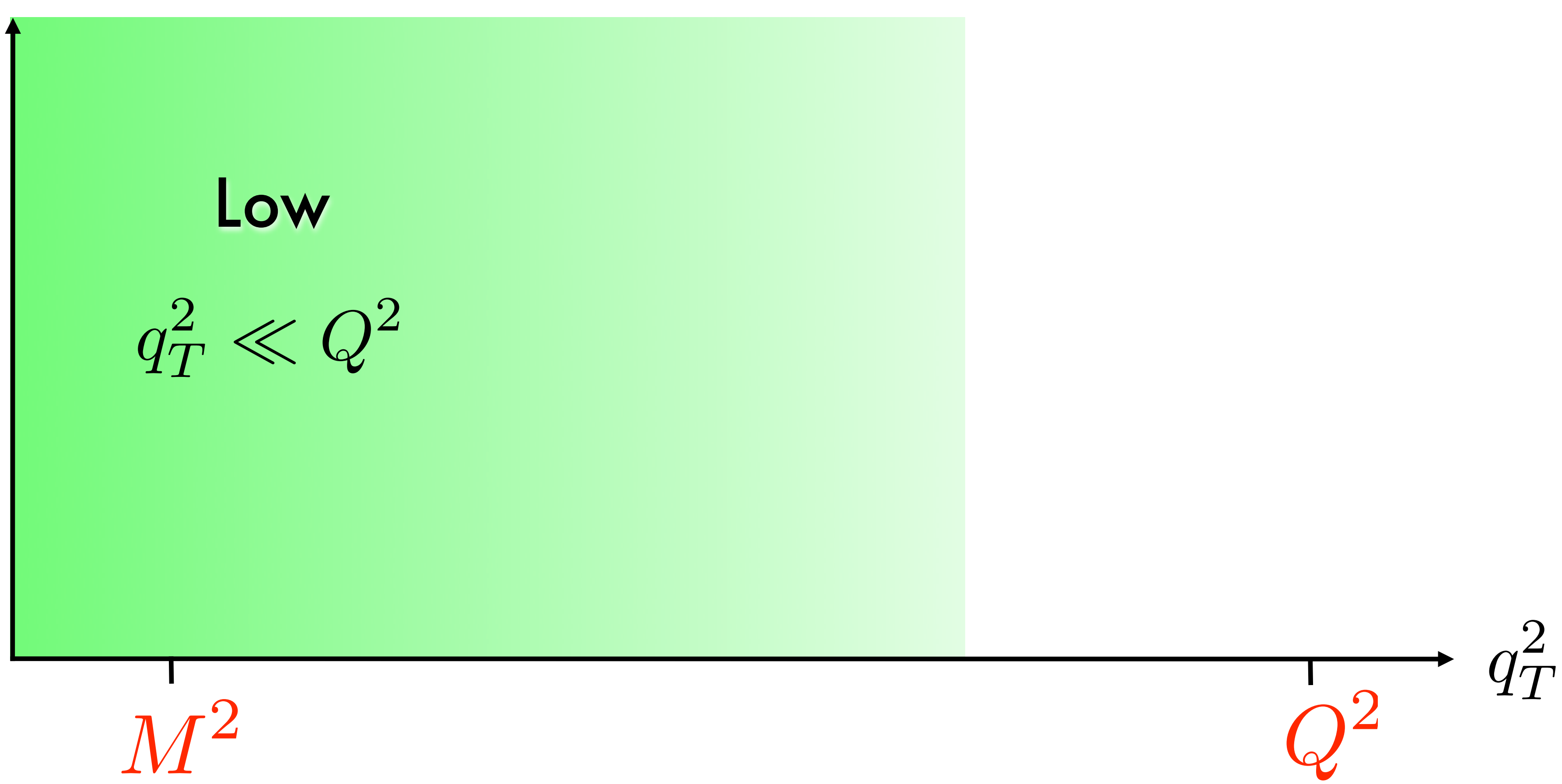
$$M = \text{hadron mass}$$

$$P_{h\perp} = \text{hadron transverse momentum}$$

$$q_T^2 \approx P_{h\perp}^2 / z^2$$

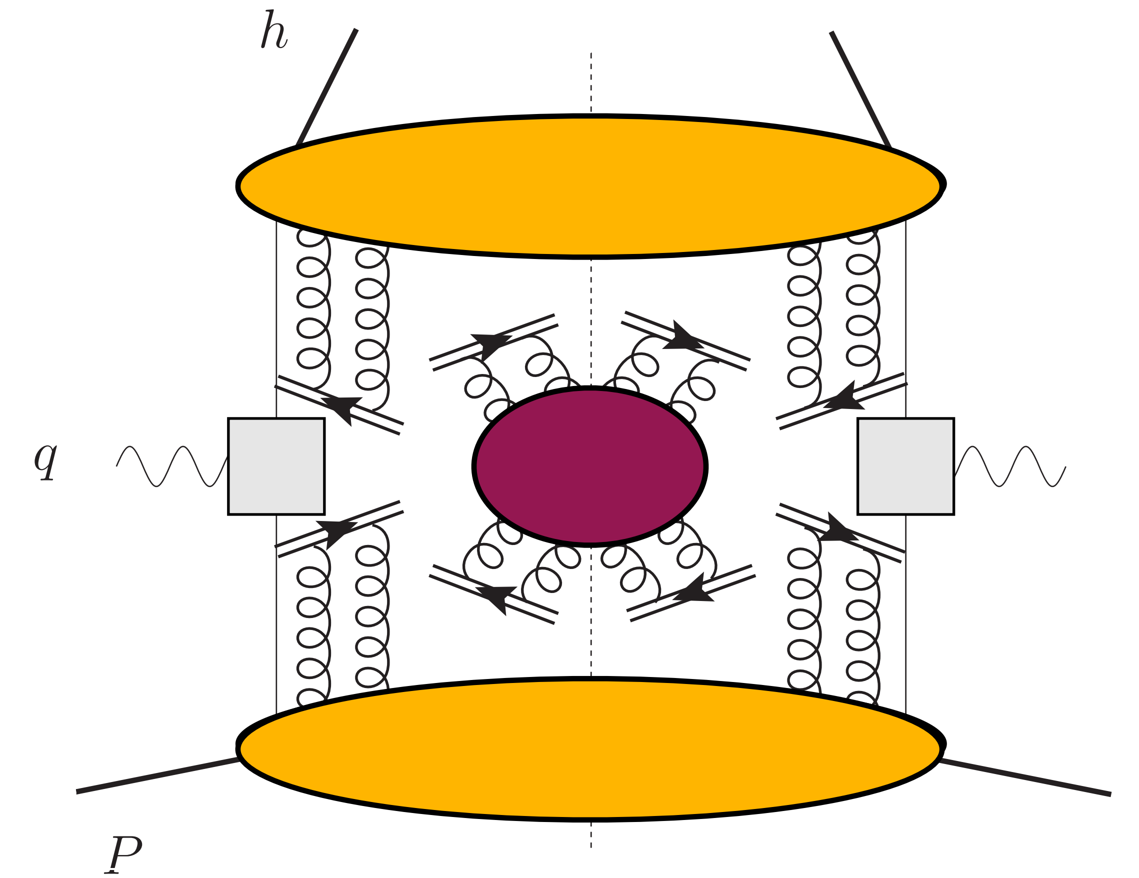
Low and high transverse momentum

AB, D. Boer, M. Diehl, P.J. Mulders, JHEP 08 (08)



TMD factorization

Collins, Soper, NPB 193 (81)
Ji, Ma, Yuan, PRD 71 (05)

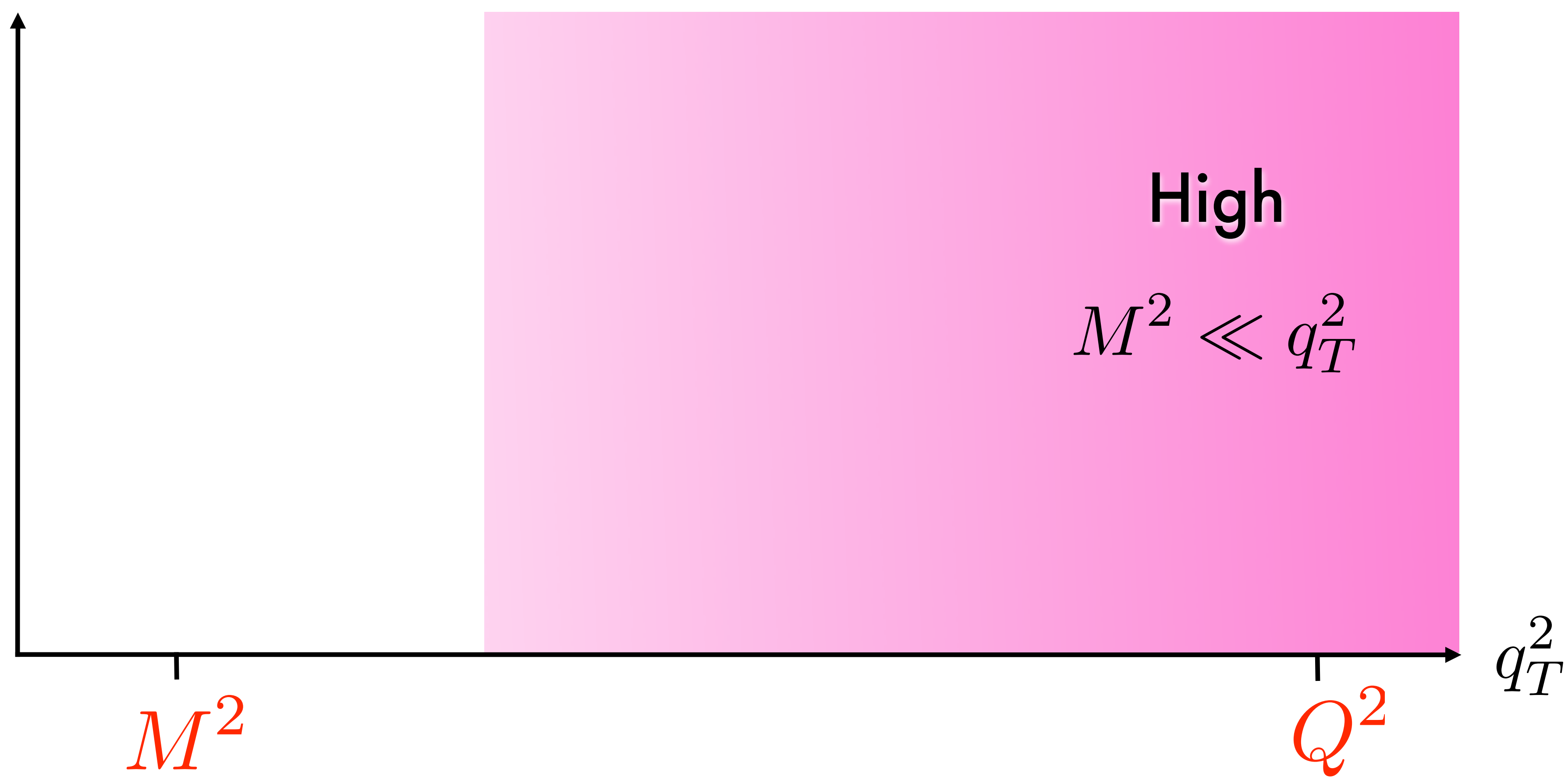


$$F_{UU,T}(x, z, P_{h\perp}^2, Q^2) = \mathcal{C}'\left[f_1 D_1\right]$$

$$= H(Q^2, \mu^2, \zeta, \zeta_h) \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T - \boldsymbol{P}_{h\perp}/z\big)$$

$$x \sum_a e_a^2\, f_1^a(x, p_T^2, \mu^2, \zeta)\, D_1^a(z, k_T^2, \mu^2, \zeta_h)\, U(l_T^2, \mu^2, \zeta\zeta_h)$$

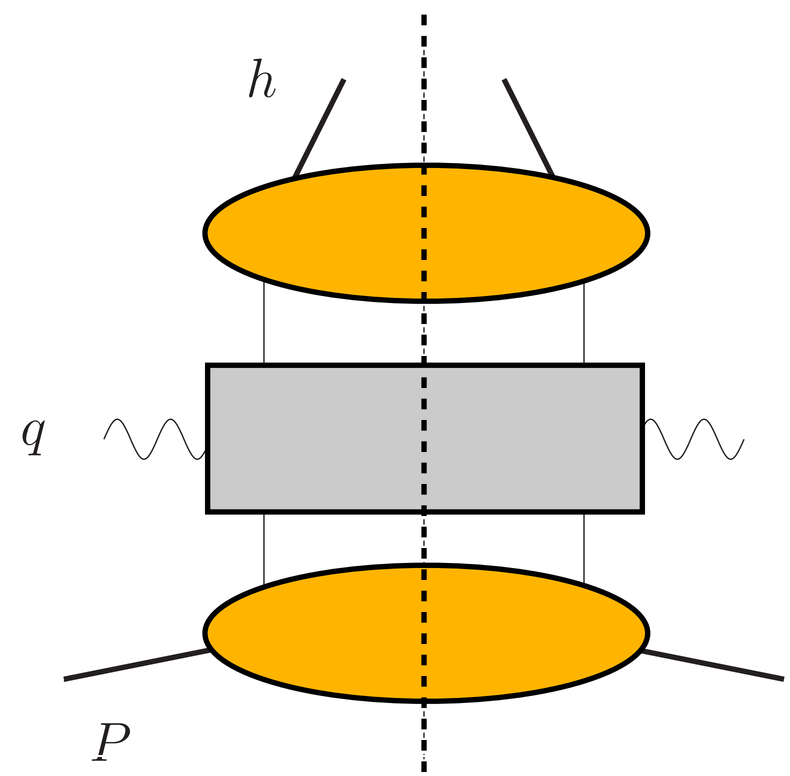
Low and high transverse momentum



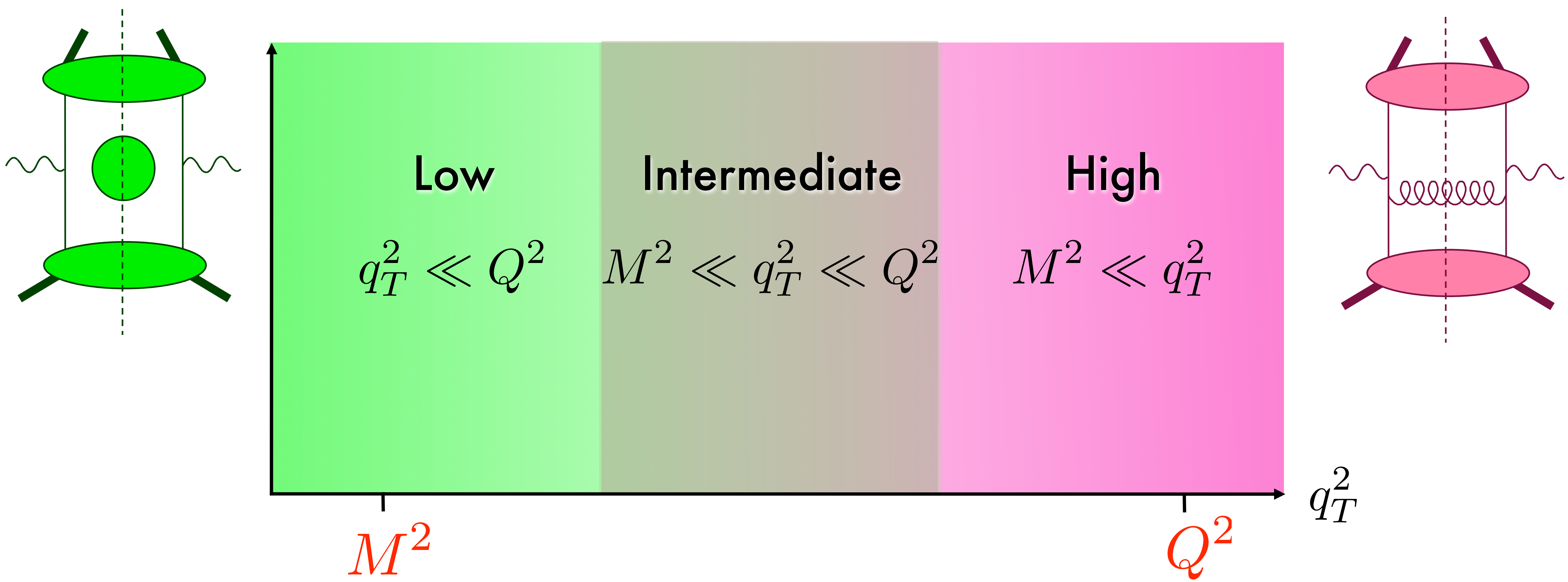
Collinear factorization

$$F(x,z,Q^2) = \frac{1}{Q^2 z^2}\, x \sum_{a,b} \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}}\, \delta\Big(\frac{P_{h\perp}^2}{Q^2 z^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\Big)$$

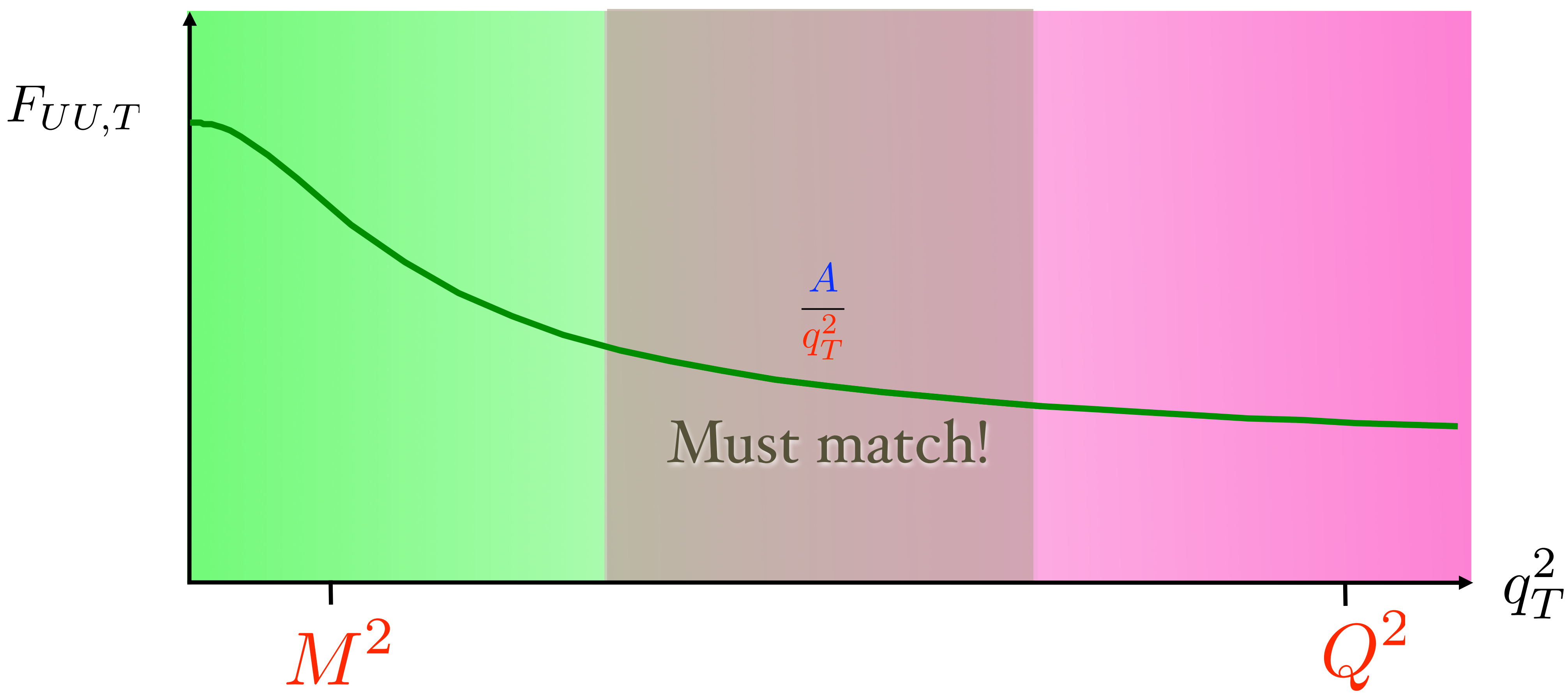
$$\times\ f^a\Big(\frac{x}{\hat{x}}, \mu_F^2\Big)\, D^b\Big(\frac{z}{\hat{z}}, \mu_F^2\Big)\ H'_{ab}\Big(\hat{x}, \hat{z}, \ln\frac{\mu_F^2}{Q^2}\Big)$$



Low and high transverse momentum



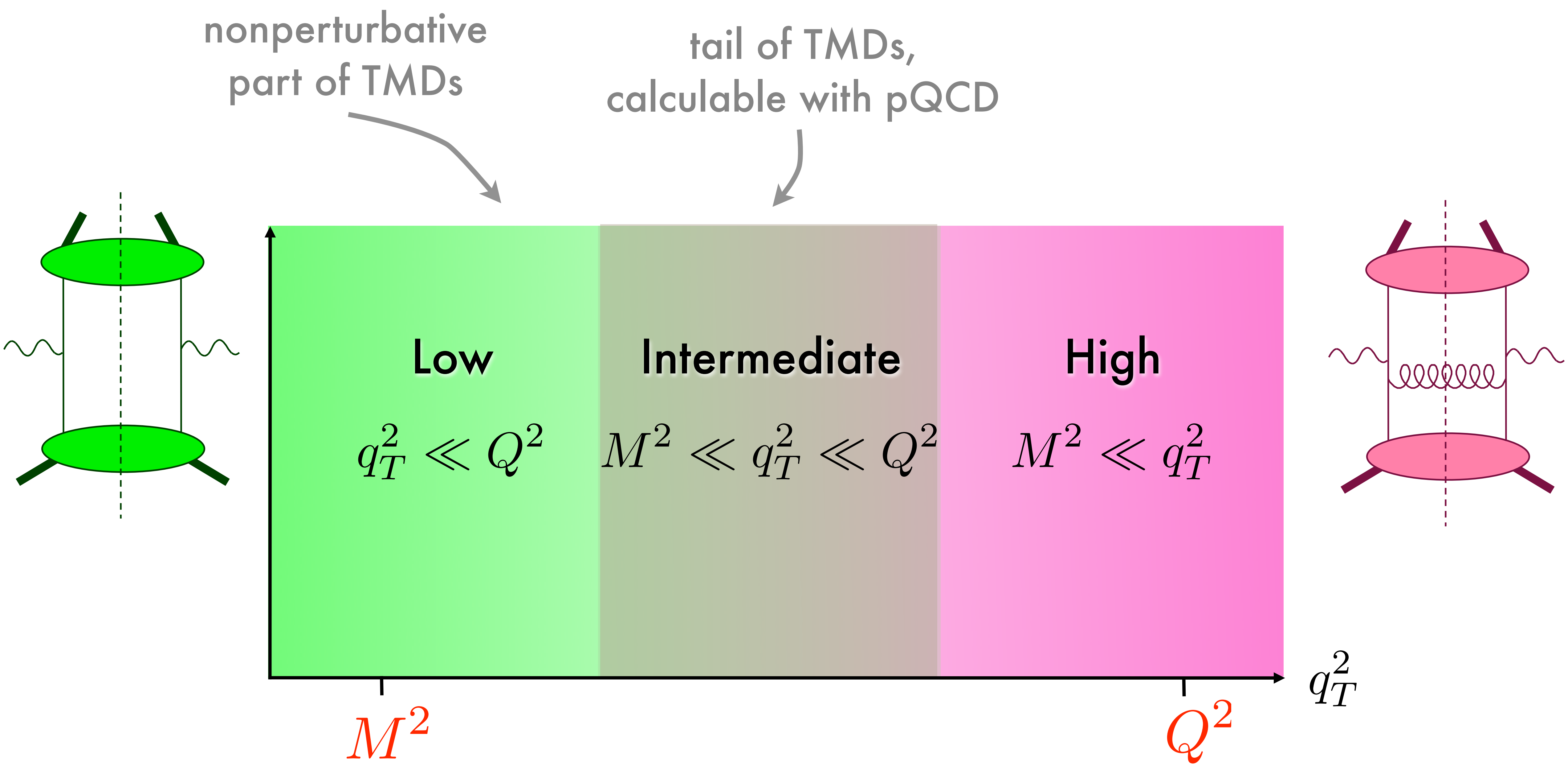
Matching



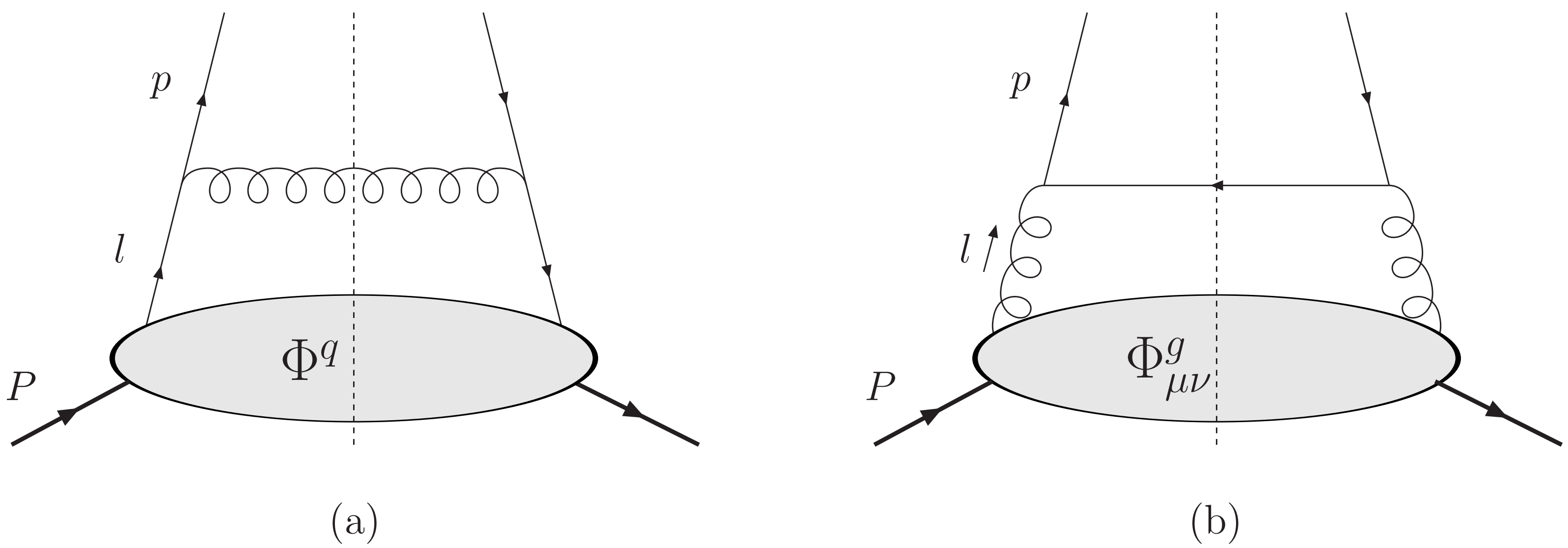
The leading high-q_T part is just the "tail" of the leading low-q_T part

Collins, Soper, Sterman, NPB250 (85)

Low and high transverse momentum



Perturbative corrections to TMDs



$$f_1^q(x, p_T^2) = \frac{\alpha_s}{2\pi^2} \frac{1}{\boldsymbol{p}_T^2} \left[\frac{L(\eta^{-1})}{2} f_1^q(x) - C_F f_1^q(x) + \left(P_{qq} \otimes f_1^q + P_{qg} \otimes f_1^g \right)(x) \right],$$

$$F_{UU,T} = \frac{1}{q_T^2} \frac{\alpha_s}{2\pi^2 z^2} \sum_a x e_a^2 \left[f_1^a(x)\, D_1^a(z)\, L\left(\frac{Q^2}{q_T^2}\right) + f_1^a(x) \left(D_1^a \otimes P_{qq} + D_1^g \otimes P_{gq} \right)(z) \right.$$
$$\left. + \left(P_{qq} \otimes f_1^a + P_{qg} \otimes f_1^g \right)(x)\, D_1^a(z) \right]$$

Large log,
needs resummation

DGLAP splitting
functions

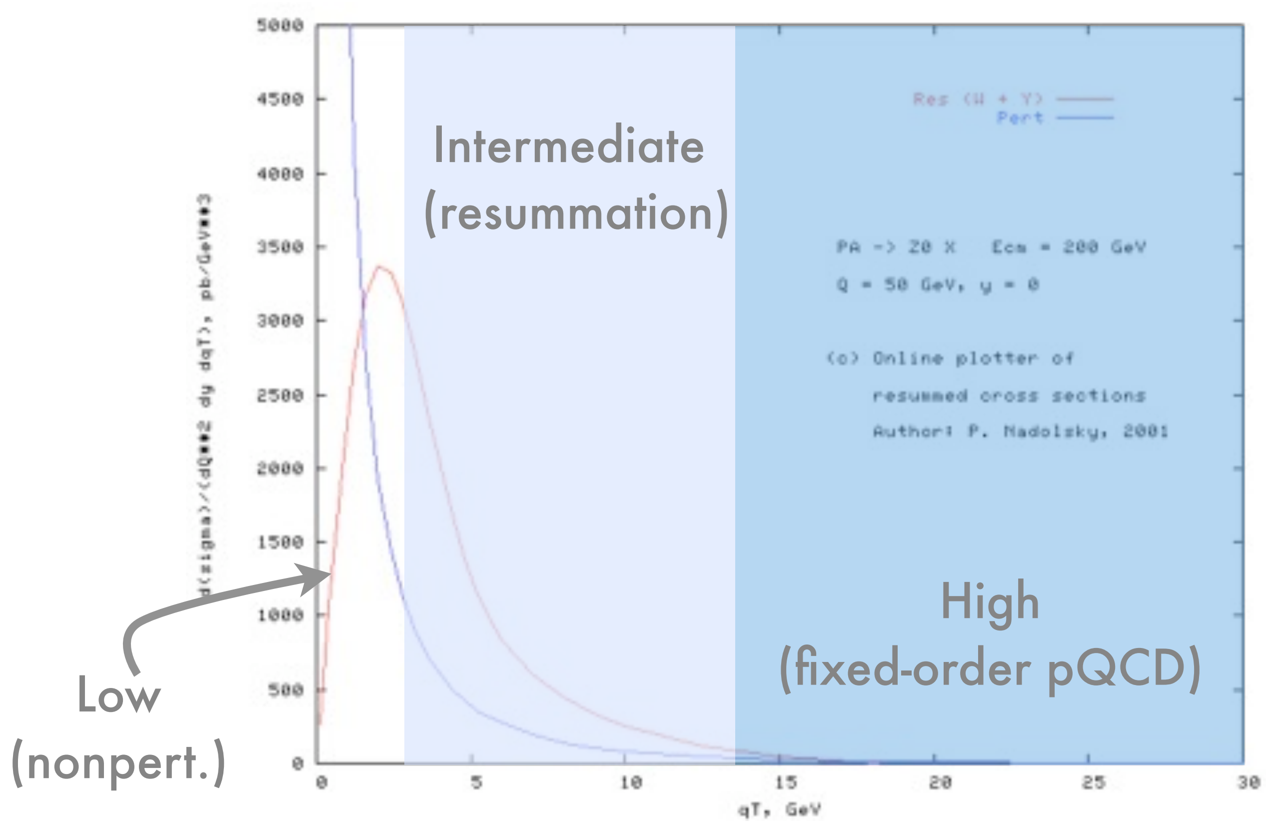
where $L\left(\frac{Q^2}{q_T^2}\right) = 2C_F \ln \frac{Q^2}{q_T^2} - 3C_F$

TMD factorization: *b* space

Sudakov form factor

$$F_{UU,T}(x,z,b,Q^2) = x\sum_a e_a^2 \left[(f_1^i \otimes \mathcal{C}_{ia})\,(\mathcal{C}_{aj} \otimes D_1^j)\, e^{-S} e^{-S_{NP}} \right]$$

collinear PDF and FF — calculable with pQCD — nonperturbative part of TMDs



$$F_{UU,T}(x,z,q_T^2,Q^2) = x\sum_a e_a^2 \frac{d}{dq_T^2} \left[(f_1^i \otimes \mathcal{C}_{ia})\,(\mathcal{C}_{aj} \otimes D_1^j)\, e^{-S} \left(1 - e^{-S_{NP}}\right) \right]$$

Leading-log formula

Ellis, Veseli, NPB 511 (98)

$$F_{UU,T}(x,z,q_T^2,Q^2) = x\sum_a e_a^2\,\frac{d}{dq_T^2}\Big[f_1^a(x;[q_T^2])\,D_1^a(z;[q_T^2])\,e^{-S}\big(1-e^{-S_{NP}}\big)\Big]$$

$$S(q_T^2,Q^2) = -\int_{q_T^2}^{Q^2}\frac{d\mu^2}{\mu^2}\,\frac{\alpha_S(\mu^2)}{2\pi}\,2C_F\log\frac{Q^2}{\mu^2}$$

$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_0\log(\mu^2/\Lambda^2)}$$

Part 5: Unpolarized Phenomenolgy

Experimental access

Drell-Yan

$$\frac{d\sigma}{dq_T^2} \sim \sum_q e_q^2\, f_1^q(x, p_T^2) \otimes f_1^{\bar{q}}(\bar{x}, \bar{p}_T^2)$$

Semi-inclusive DIS

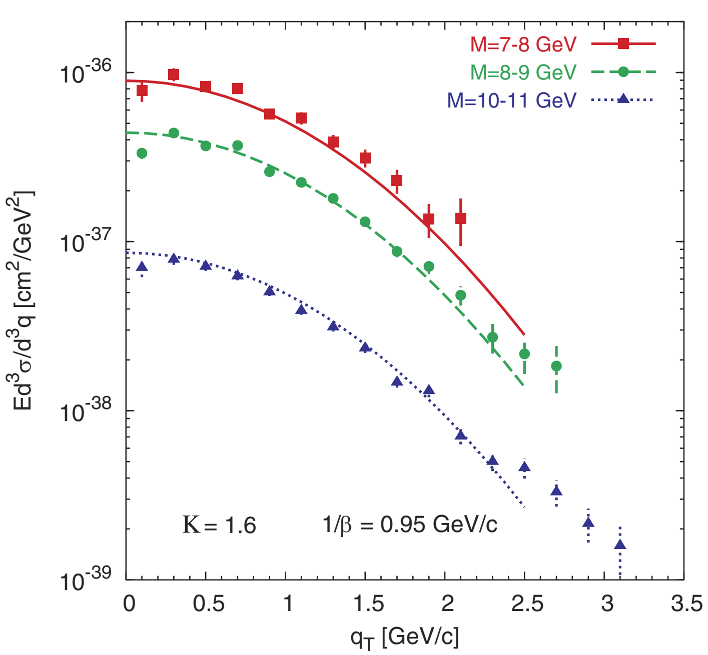
$$\frac{d\sigma}{dq_T^2} \sim \sum_q e_q^2\, f_1^q(x, p_T^2) \otimes D_1^q(z, k_T^2)$$

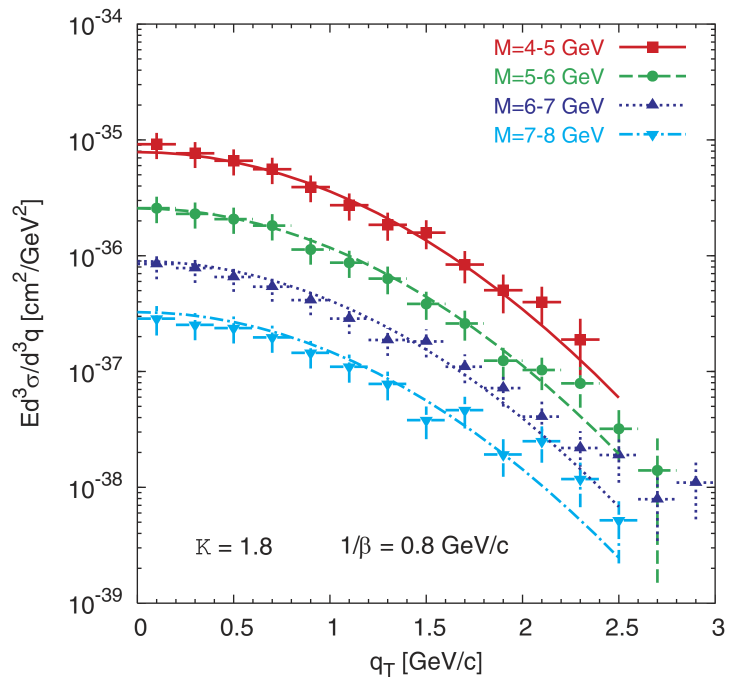
electron-positron annihilation

$$\frac{d\sigma}{dq_T^2} \sim \sum_q e_q^2\, D_1^q(z, k_T^2) \otimes D_1^{\bar{q}}(\bar{z}, \bar{k}_T^2)$$

Some studies in Drell-Yan

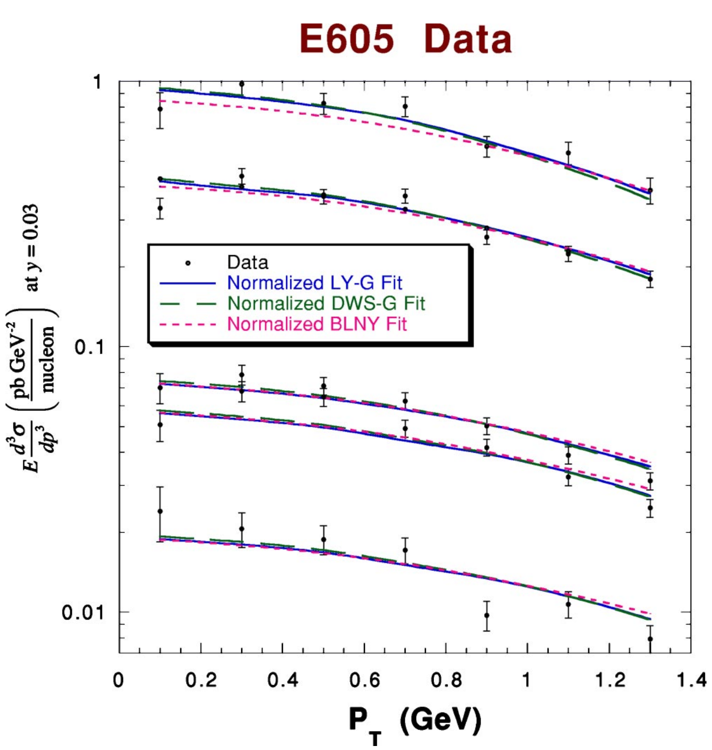
Available studies

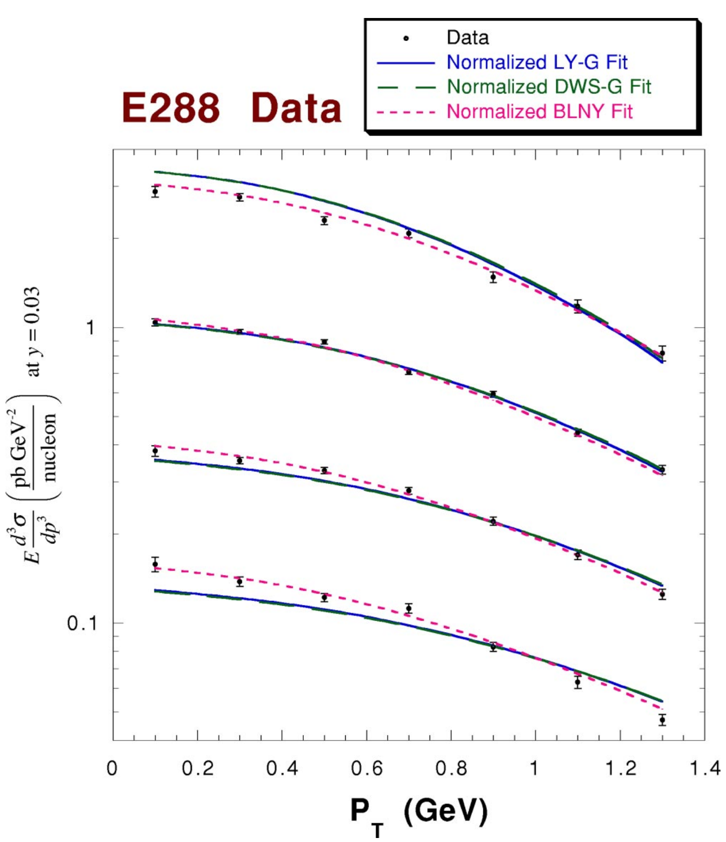




Gaussians

D'Alesio, Murgia, PRD70 (04)

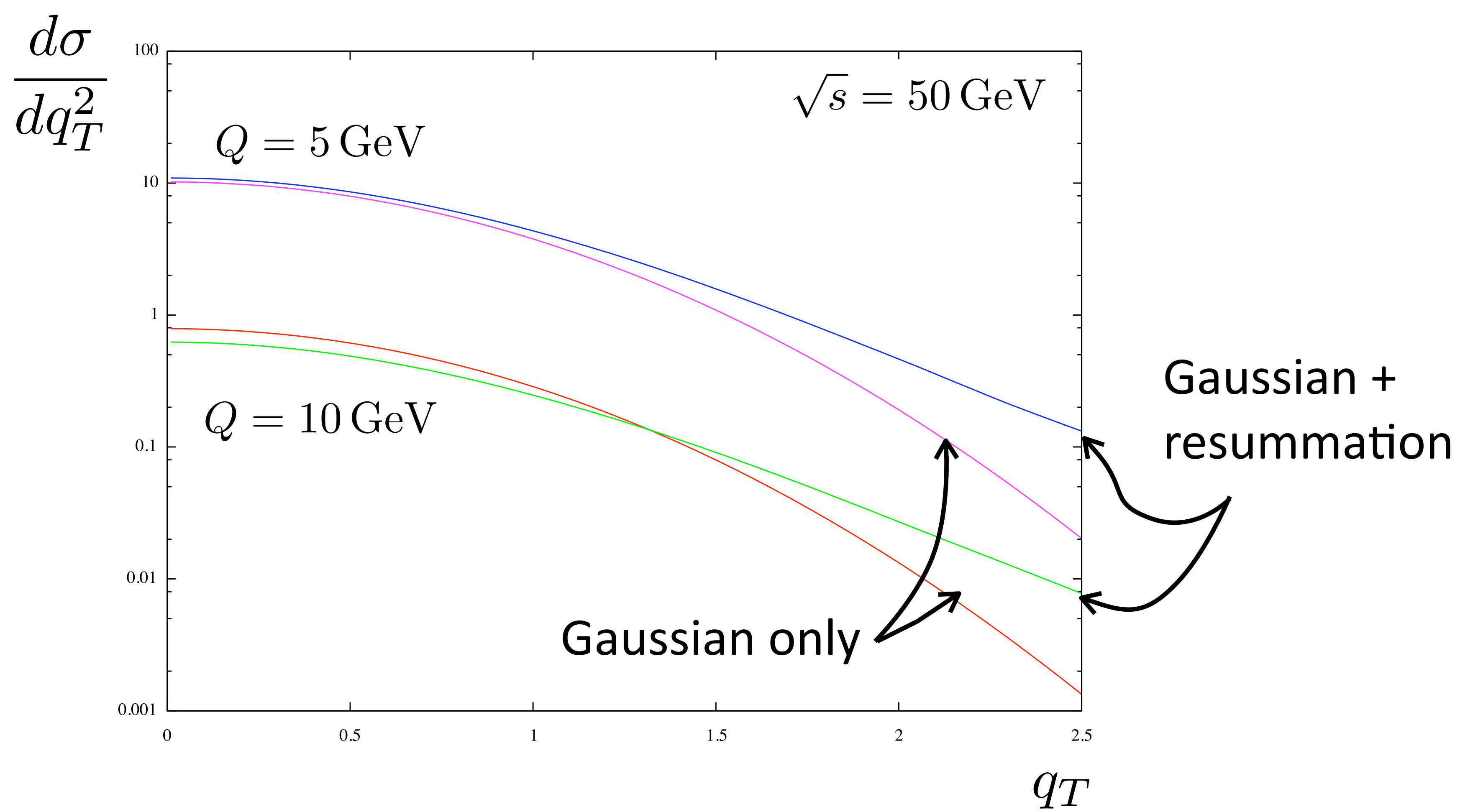




Gaussians
+ kT resummation

Landry, Brock, Nadolsky, Yuan, PRD67 (03)

Example of resummation effects

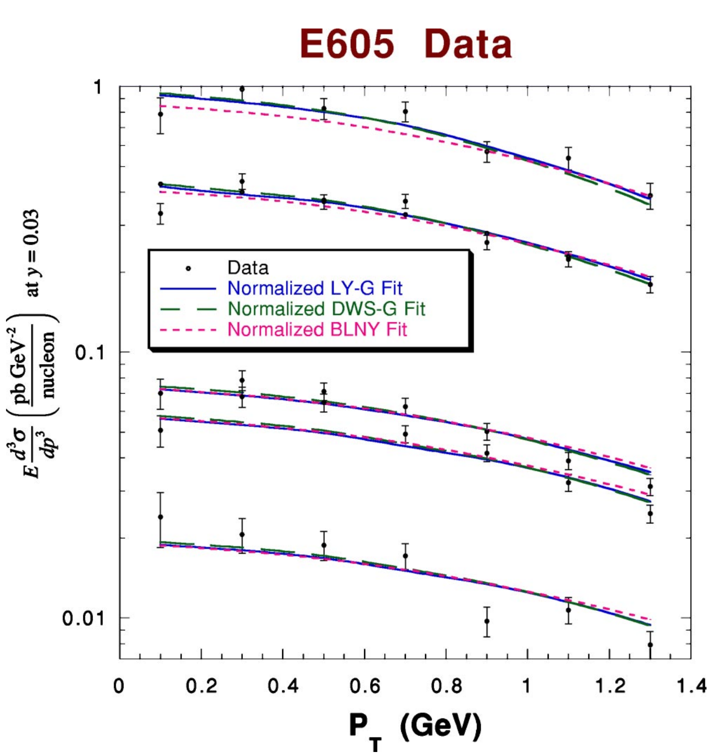


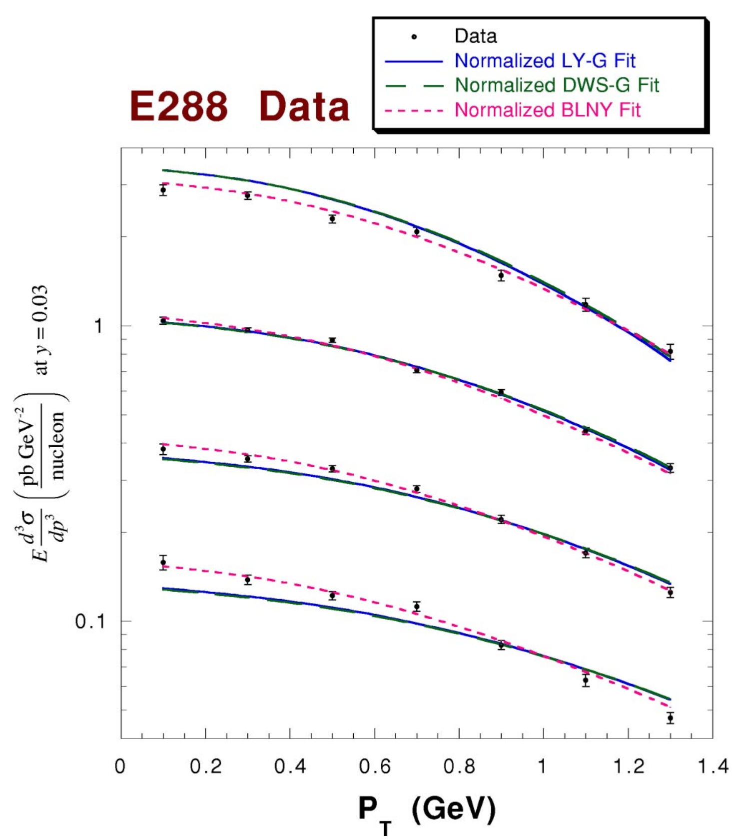
Nonperturbative part

- In b space

$$S_{NP} = -\frac{b^2}{\langle b^2 \rangle}$$

$$\frac{1}{\langle b^2 \rangle} = 0.21 + 0.68 \log\left(\frac{Q}{3.2}\right) - 0.13 \log\left(100 x_A x_B\right) \qquad b_{\max} = 0.5 \text{ GeV}^{-1}$$





111 data points (Drell-Yan)

Brock, Landry, Nadolsky, Yuan, PRD67 (03)

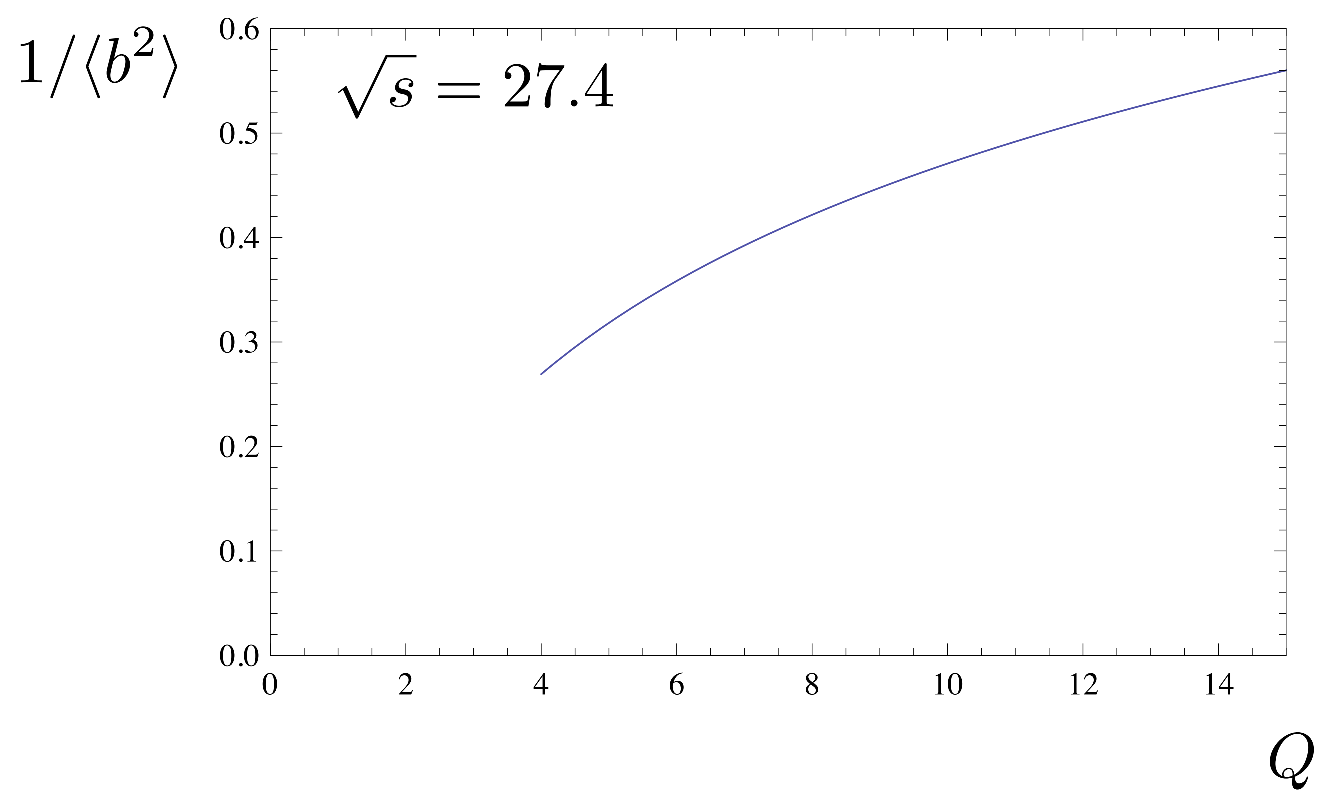
Nonperturbative part

- In b space

Kulesza, Stirling, JHEP 12 (03)

$$S_{NP} = -\frac{b^2}{\langle b^2 \rangle}$$

$$\frac{1}{\langle b^2 \rangle} = 0.12 + 0.22 \log\left(\frac{Q}{3.2}\right) + 0.29 \log\left(\frac{\sqrt{s}}{19.4}\right)$$



Note: there should be a factor 4 between $1/b$ and k_T

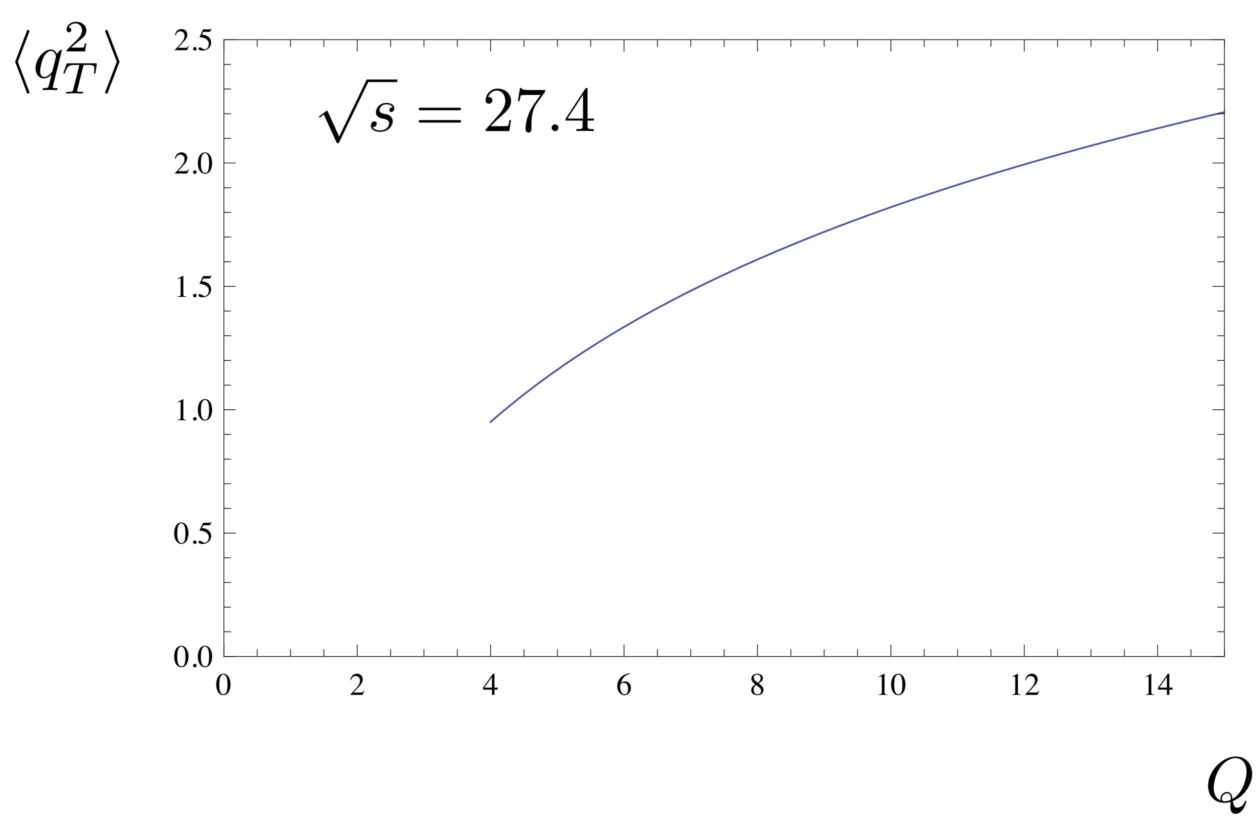
Nonperturbative part

- In k_T space

Kulesza, Stirling, JHEP 12 (03)

$$S_{NP} = -\frac{q_T^2}{\langle q_T^2 \rangle}$$

$$\langle q_T^2 \rangle = 0.20 + 0.95\ \log\left(\frac{Q}{3.2}\right) + 1.56\ \log\left(\frac{\sqrt{s}}{19.4}\right)$$



Unpolarized SIDIS

Unpolarized SIDIS

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}$$
$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}\right\}$$

Azimuth-independent pieces

Convolution

$$F_{UU,T} = \sum_a e_a^2 f_1^a \otimes D_1^a, \qquad\qquad F_{UU,L} = \mathcal{O}\left(\frac{M^2}{Q^2}, \frac{P_{h\perp}^2}{Q^2}\right)$$

$$f \otimes D = x_B \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, f^a(x_B, p_T^2)\, D^a(z, k_T^2)$$

$$f \otimes D = x_B \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z + \boldsymbol{l}_T\big)\, f^a(x_B, p_T^2)\, D^a(z, k_T^2)\, U(l_T^2)$$

Does not make a big difference if Gaussians are used

Fragmentation functions

For the "favored" functions

$$D_1^{u\to\pi^+} = D_1^{\bar{d}\to\pi^+} = D_1^{d\to\pi^-} = D_1^{\bar{u}\to\pi^-}, \equiv D_1^{\mathrm{f}}$$
$$D_1^{u\to K^+} = D_1^{\bar{u}\to K^-}, \equiv D_1^{\mathrm{fd}}$$
$$D_1^{\bar{s}\to K^+} = D_1^{s\to K^-} \equiv D_1^{\mathrm{f}'}$$

for the "unfavored" functions

$$D_1^{\bar{u}\to\pi^+} = D_1^{d\to\pi^+} = D_1^{\bar{d}\to\pi^-} = D_1^{u\to\pi^-} \equiv D_1^{\mathrm{d}},$$
$$D_1^{s\to\pi^+} = D_1^{\bar{s}\to\pi^+} = D_1^{s\to\pi^-} = D_1^{\bar{s}\to\pi^-} \equiv D_1^{\mathrm{df}},$$
$$D_1^{\bar{u}\to K^+} = D_1^{\bar{d}\to K^+} = D_1^{d\to K^+} = D_1^{\bar{d}\to K^-} = D_1^{d\to K^-} = D_1^{u\to K^-} \equiv D_1^{\mathrm{dd}},$$
$$D_1^{s\to K^+} = D_1^{\bar{s}\to K^-} \equiv D_1^{\mathrm{d}'}.$$

Various combinations

$$F_{UU,T}^{p/\pi^+}(x,z,P_{h\perp}^2) = \Big(4\,f_1^u + f_1^{\bar d}\Big)\otimes D_1^{\rm f} + \Big(4\,f_1^{\bar u} + f_1^{d}\Big)\otimes D_1^{\rm d} + \Big(f_1^s + f_1^{\bar s}\Big)\otimes D_1^{\rm df},$$

$$F_{UU,T}^{p/\pi^-}(x,z,P_{h\perp}^2) = \Big(4\,f_1^{\bar u} + f_1^{d}\Big)\otimes D_1^{\rm f} + \Big(4\,f_1^{u} + f_1^{\bar d}\Big)\otimes D_1^{\rm d} + \Big(f_1^s + f_1^{\bar s}\Big)\otimes D_1^{\rm df},$$

$$F_{UU,T}^{n/\pi^+}(x,z,P_{h\perp}^2) = \Big(4\,f_1^{d} + f_1^{\bar u}\Big)\otimes D_1^{\rm f} + \Big(4\,f_1^{\bar d} + f_1^{u}\Big)\otimes D_1^{\rm d} + \Big(f_1^s + f_1^{\bar s}\Big)\otimes D_1^{\rm df}$$

$$F_{UU,T}^{n/\pi^-}(x,z,P_{h\perp}^2) = \Big(4\,f_1^{\bar d} + f_1^{u}\Big)\otimes D_1^{\rm f} + \Big(4\,f_1^{d} + f_1^{\bar u}\Big)\otimes D_1^{\rm d} + \Big(f_1^s + f_1^{\bar s}\Big)\otimes D_1^{\rm df},$$

$$F_{UU,T}^{p/K^+}(x,z,P_{h\perp}^2) = 4\,f_1^u\otimes D_1^{\rm fd} + \Big(4\,f_1^{\bar u} + f_1^{d} + f_1^{\bar d}\Big)\otimes D_1^{\rm dd} + f_1^{\bar s}\otimes D_1^{\rm f'} + f_1^{s}\otimes D_1^{\rm d'},$$

$$F_{UU,T}^{p/K^-}(x,z,P_{h\perp}^2) = 4\,f_1^{\bar u}\otimes D_1^{\rm fd} + \Big(4\,f_1^{u} + f_1^{d} + f_1^{\bar d}\Big)\otimes D_1^{\rm dd} + f_1^{s}\otimes D_1^{\rm f'} + f_1^{\bar s}\otimes D_1^{\rm d'},$$

$$F_{UU,T}^{n/K^+}(x,z,P_{h\perp}^2) = 4\,f_1^{d}\otimes D_1^{\rm fd} + \Big(4\,f_1^{\bar d} + f_1^{u} + f_1^{\bar u}\Big)\otimes D_1^{\rm dd} + f_1^{\bar s}\otimes D_1^{\rm f'} + f_1^{s}\otimes D_1^{\rm d'},$$

$$F_{UU,T}^{n/K^-}(x,z,P_{h\perp}^2) = 4\,f_1^{\bar d}\otimes D_1^{\rm fd} + \Big(4\,f_1^{d} + f_1^{u} + f_1^{\bar u}\Big)\otimes D_1^{\rm dd} + f_1^{s}\otimes D_1^{\rm f'} + f_1^{\bar s}\otimes D_1^{\rm d'}$$

Valence and pions only

$$F_{UU,T}^{p/\pi^+}(x, z, P_{h\perp}^2) = 4\, f_1^u \otimes D_1^{\mathrm{f}} + f_1^d \otimes D_1^{\mathrm{d}},$$
$$F_{UU,T}^{p/\pi^-}(x, z, P_{h\perp}^2) = f_1^d \otimes D_1^{\mathrm{f}} + 4\, f_1^u \otimes D_1^{\mathrm{d}},$$
$$F_{UU,T}^{n/\pi^+}(x, z, P_{h\perp}^2) = 4\, f_1^d \otimes D_1^{\mathrm{f}} + f_1^u \otimes D_1^{\mathrm{d}},$$
$$F_{UU,T}^{n/\pi^-}(x, z, P_{h\perp}^2) = f_1^u \otimes D_1^{\mathrm{f}} + 4\, f_1^d \otimes D_1^{\mathrm{d}}$$

Gaussian ansatz

$$f_1^a(x,p_T^2) = \frac{f_1^a(x)}{\pi\rho_a^2} e^{-\boldsymbol{p}_T^2/\rho_a^2}, \qquad D_1^a(z,k_T^2) = \frac{D_1^a(z)}{\pi\sigma_a^2} e^{-z^2\boldsymbol{k}_T^2/\sigma_a^2}$$

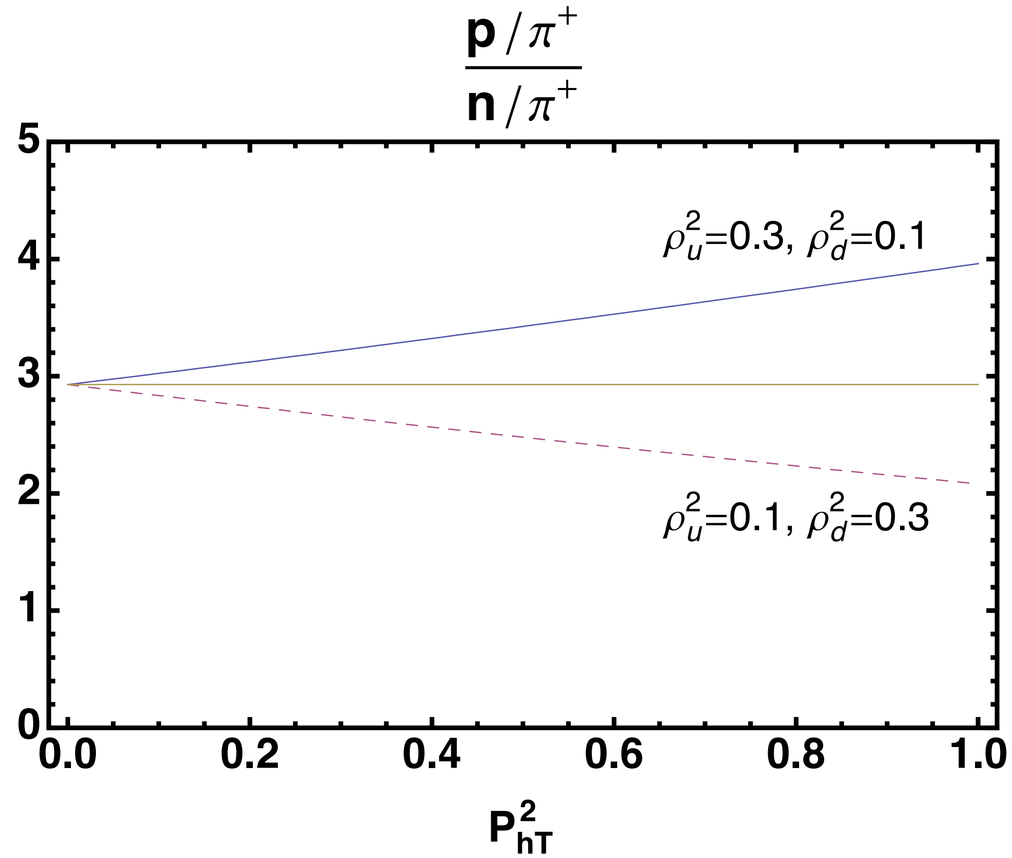
$$f_1^a \otimes D_1^a = \frac{1}{\pi(z^2\rho_a^2+\sigma_a^2)}\, e^{-\boldsymbol{P}_{h\perp}^2/(z^2\rho_a^2+\sigma_a^2)}$$

With Gaussian soft factor

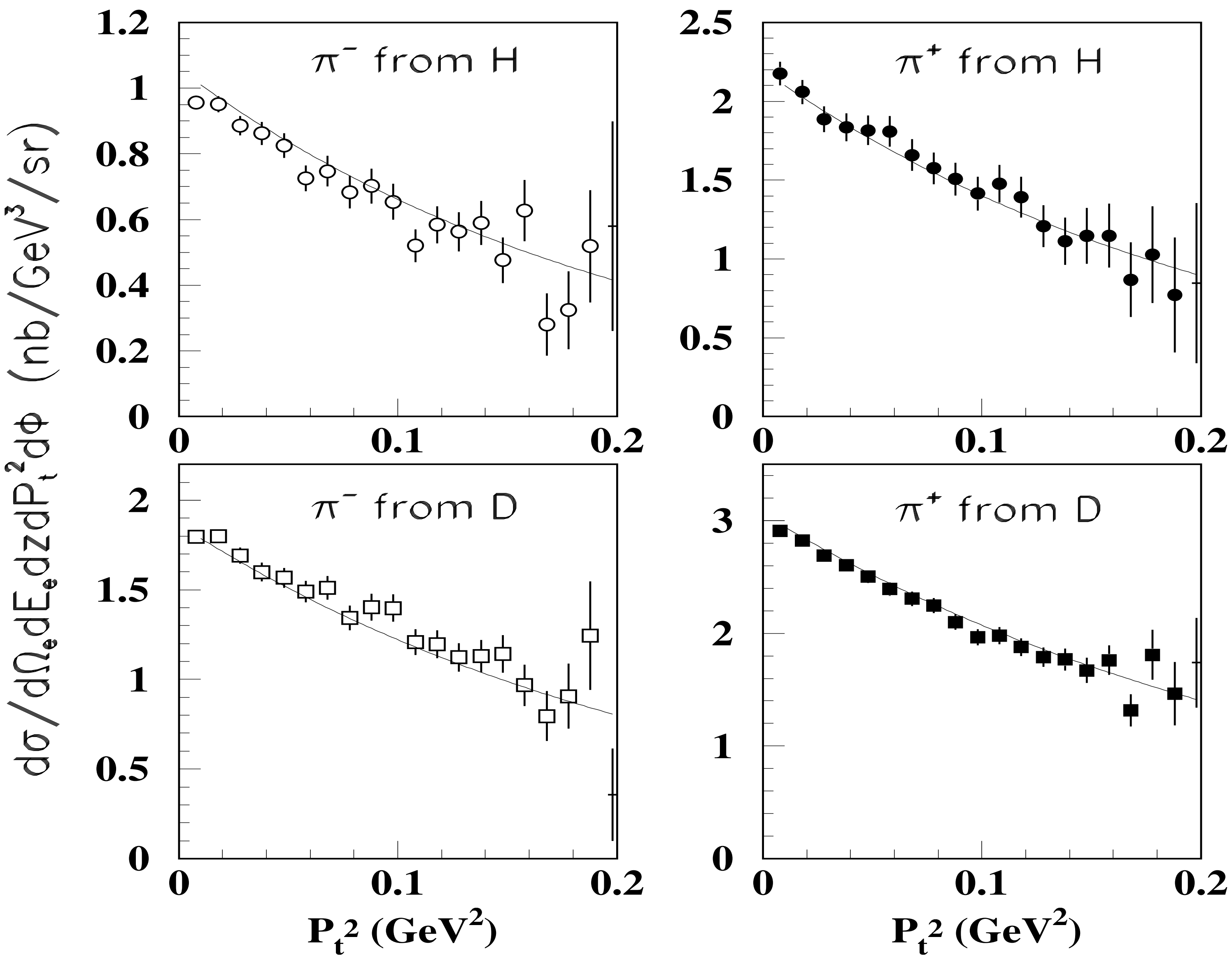
$$f_1^a \otimes D_1^a = \frac{1}{\pi(z^2\rho_a^2+\sigma_a^2+\tau^2)}\, e^{-\boldsymbol{P}_{h\perp}^2/(z^2\rho_a^2+\sigma_a^2+\tau^2)}$$

Interesting ratio

$$\sigma_{\rm f}^2 = \sigma_{\rm d}^2 = 0.3\ {\rm GeV}^2$$
$$f_1^u / f_1^d \approx 0.25$$
$$D_1^{\rm d} / D_1^{\rm f} \approx 0.40$$



Hall-C results



JLab Hall C, Mkrtchyan et al., PLB665 (08)