Preliminary plan

1. Introduction
2. Inclusive and semi-inclusive DIS (structure functions)

   Basics of collinear PDFs at tree level (definition, gauge link)

- Basics of collinear PDFs (interpretation)
- Basics of TMDs at tree level (definition, gauge link, interpretation)
- Basics of factorization

- Theory of TMDs: advanced topics
- Phenomenology of unpolarized SIDIS
- Phenomenology of polarized SIDIS

Before starting: on light-cone vectors

$$n_+ \cdot n_- = 1 \qquad n_+^2 = n_-^2 = 0$$

Convenient choice for semi-inclusive DIS theory

$$P^\mu = P^+ n_+^\mu + \frac{M^2}{2P^+}\, n_-^\mu$$

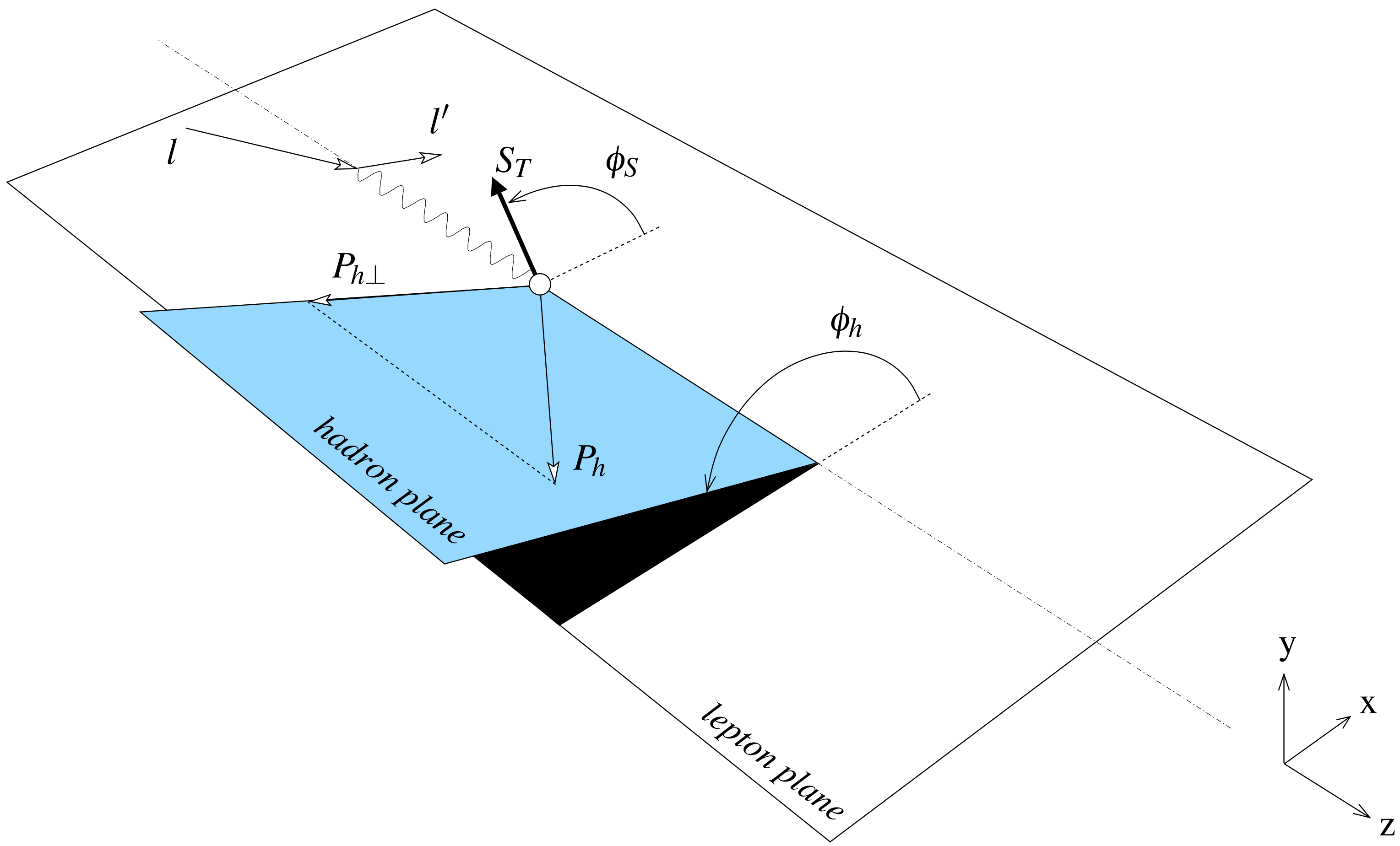
$$P_h^\mu = P_h^- n_-^\mu + \frac{M_h^2}{2P_h^-}\, n_+^\mu$$

$$q^\mu = P_h^\mu / z_h - (1-r) x_B P^\mu + q_T^\mu$$

$$q_T \cdot n_- = 0 \qquad q_T \cdot n_+ = 0 \qquad ~~r = \frac{q_T^2}{Q^2}~~ \qquad (q_T^2 \leq 0)$$

Most of the time

Before starting: on light-cone vectors



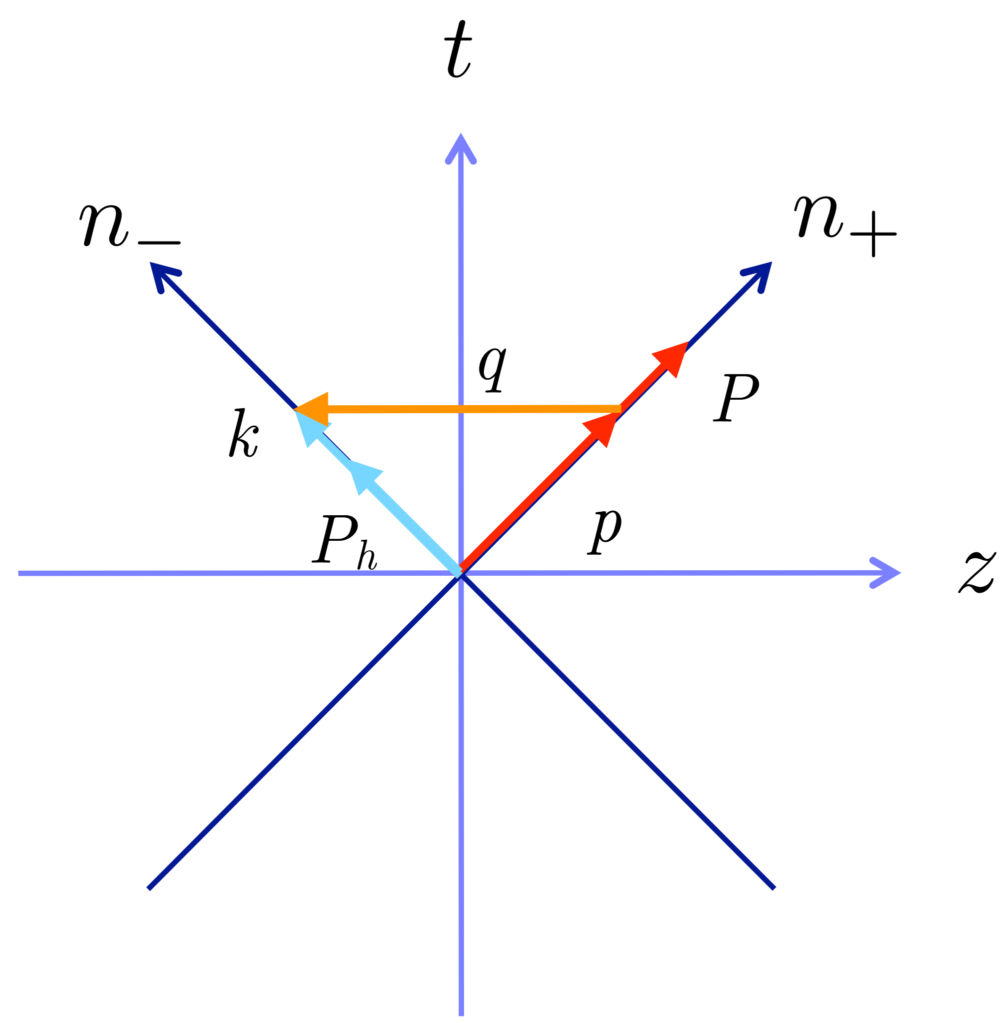
$$P^{\mu}_{h\perp} = -z_h q^{\mu}_T - 2r z_h x_B P^{\mu}$$

Convenient frame

$$n_+ = \frac{1}{\sqrt{2}}(1,0,0,1) \qquad n_- = \frac{1}{\sqrt{2}}(1,0,0,-1)$$

$$P^+ = \frac{Q}{x_B\sqrt{2}} \qquad P_h^- = z_h\frac{Q}{\sqrt{2}}$$

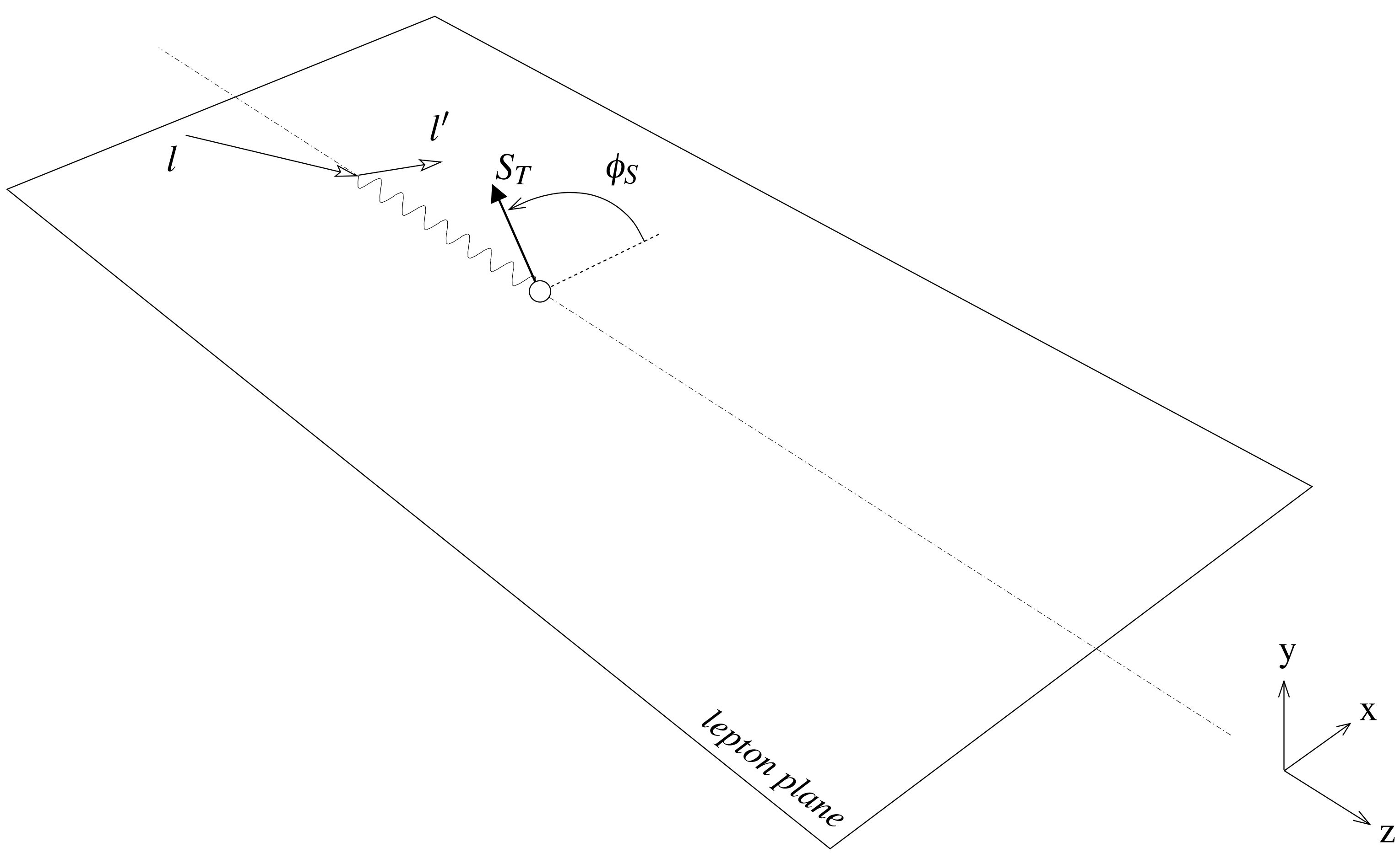
$$Q \gg M,\ \ M_h,\ \ q_T$$



Summary of last lecture

Inclusive DIS

$$\ell(l) + N(P) \rightarrow \ell(l') + X$$

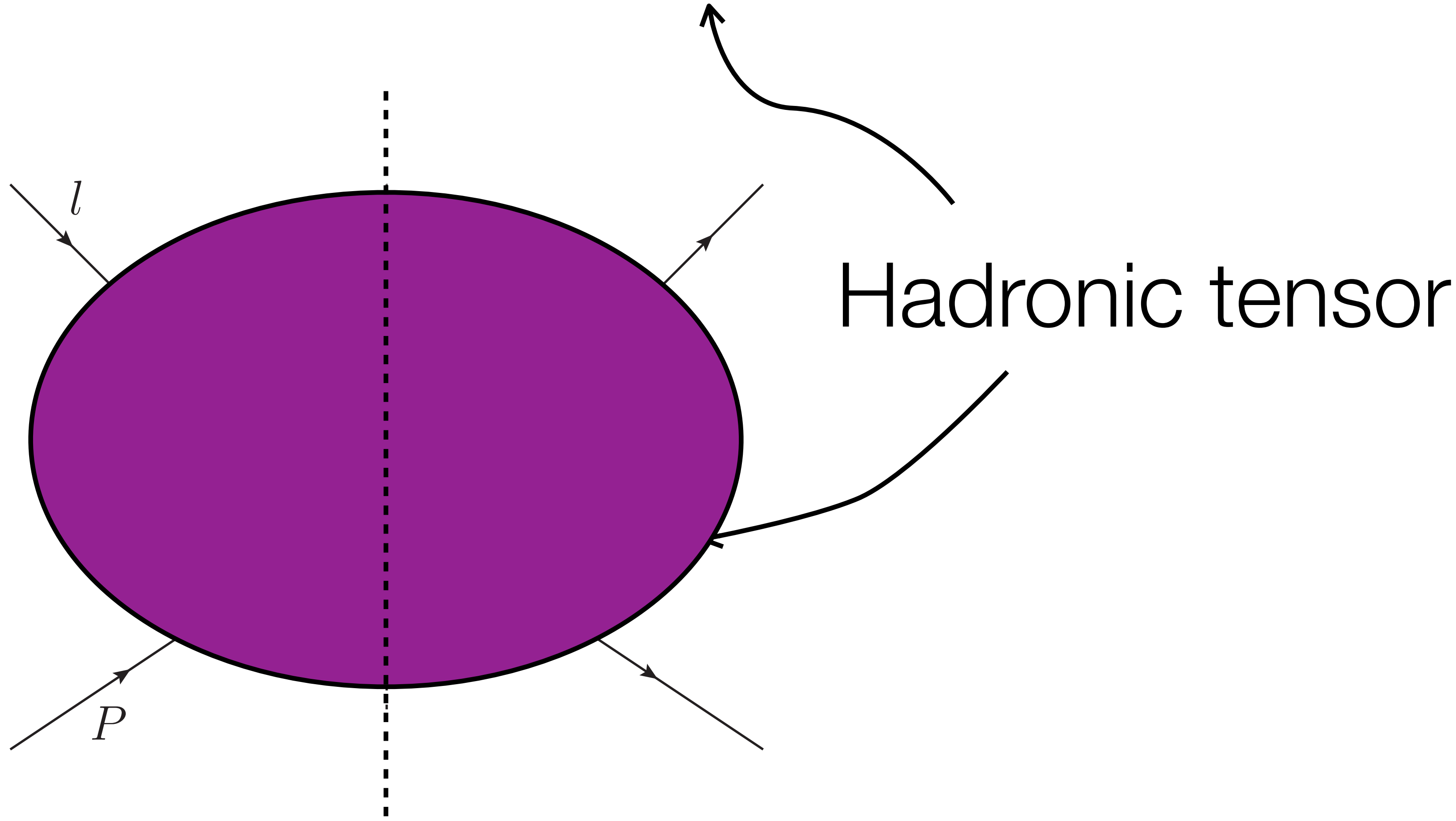


4 structure functions

$$\frac{d\sigma}{dx_B\,dy\,d\phi_S} = \frac{2\alpha^2}{x_B y Q^2}\left\{\left(1-y+\frac{y^2}{2}\right)F_{UU,T} + (1-y)\,F_{UU,L} + S_L\lambda_e\,y\left(1-\frac{y}{2}\right)F_{LL} + |\boldsymbol{S}_T|\lambda_e\,y\sqrt{1-y}\,\cos\phi_S\,F_{LT}^{\cos\phi_s}\right\}$$

Cross section

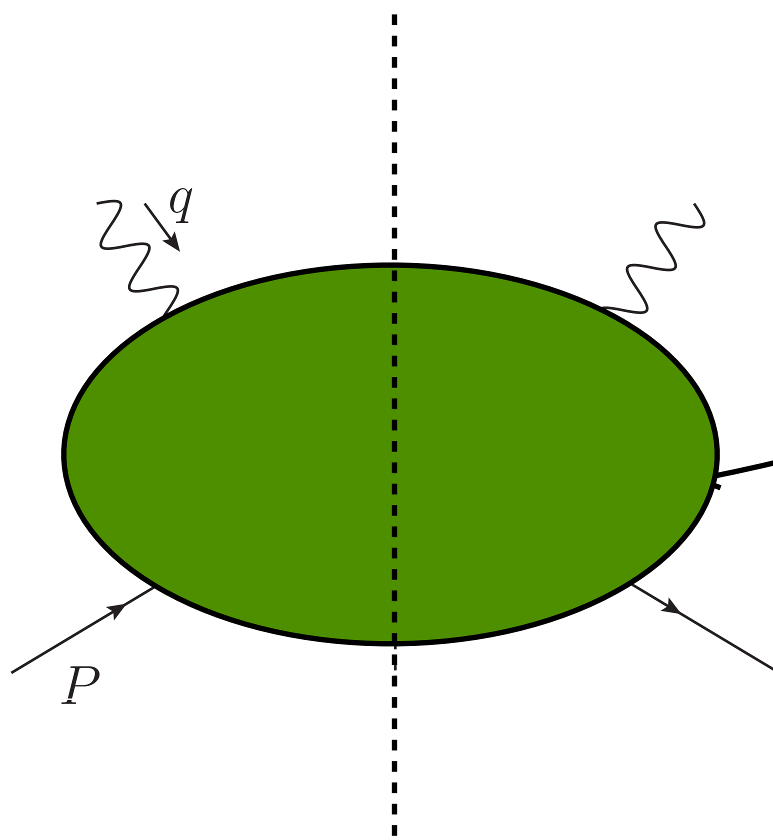
$$\frac{d^3\sigma}{dx_B dy d\phi_S} = \frac{\alpha^2 y}{2Q^4} L_{\mu\nu}(l, l', \lambda_e)\, 2MW^{\mu\nu}(q, P, S)$$



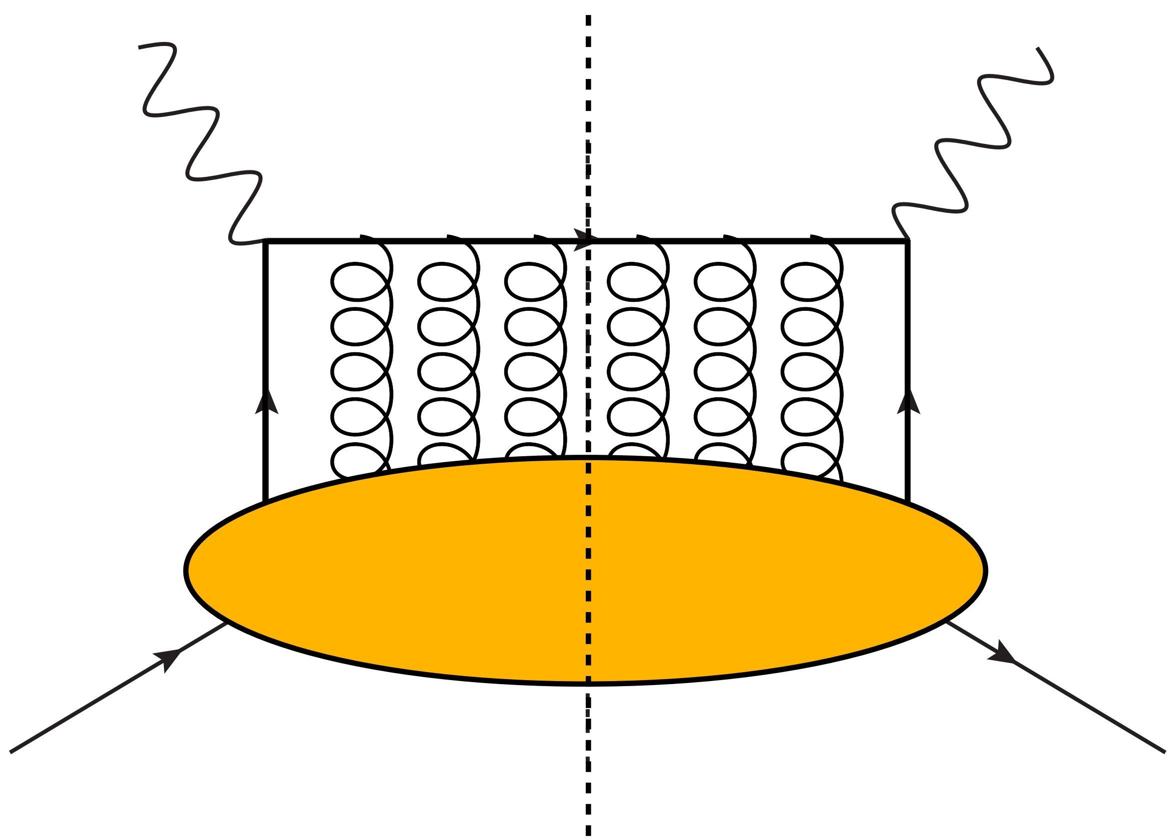
Hadronic tensor

$$2MW^{\mu\nu}(q,P,S) \approx \sum_q e_q^2 \, \frac{1}{2} \mathrm{Tr}\left[\Phi(x_B,S)\,\gamma^{\mu}\gamma^{+}\gamma^{\nu}\right].$$

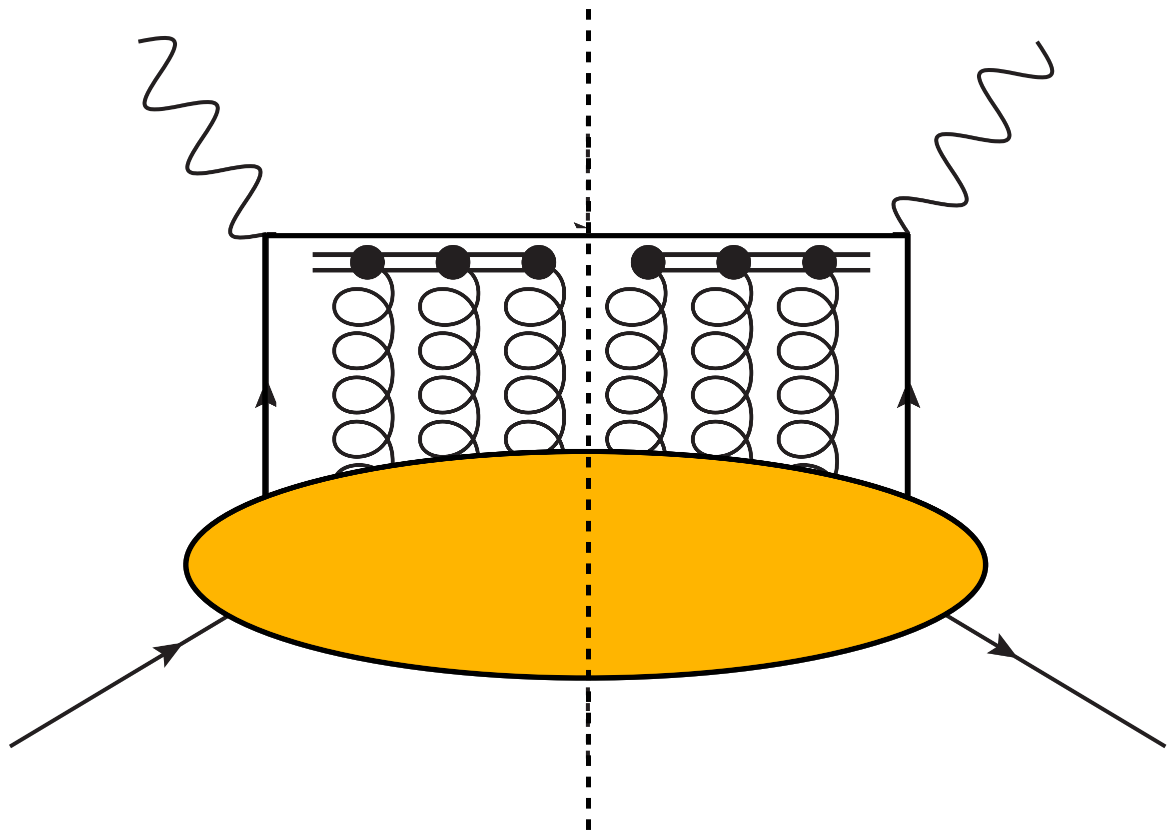
Quark correlator



Gauge link



Gauge link



Decomposition of correlator

$$\Phi(x,S) = \frac{1}{2}\left\{ f_1 \gamma^- + S_L\, g_1 \gamma_5 \gamma^- + h_1 \gamma_5\, \not{S}_T \gamma^- \right\}$$

Twist 2

$$+ \frac{M}{2P^+}\left\{ e + g_T \gamma_5\, \not{S}_T + h_L\, S_L\, \frac{[\gamma^-, \gamma^+]\gamma_5}{2} \right\}$$

Twist 3

$$\not{n}_\pm = \gamma^\mp = \frac{1}{\sqrt{2}}(\gamma^0 \mp \gamma^3)$$

Twist

- OPE: for local operators

  twist = dimension - spin

  for local operators, more directly related to the dynamics, less flexible

- Working redefinition:

  twist = 2 + power of M/P^+

  for nonlocal operators, more directly related to phenomenology, more flexible

  Jaffe hep-ph/9602236

mini lecture series

“transverse thinking”: an introduction to TMDs

Part 3

Interpretation of PDFs

Interpretation of PDFs

$$\Phi(x,S) = \frac{1}{2}\left\{ f_1\, \not{n}_+ + S_L\, g_1 \gamma_5\, \not{n}_+ + h_1\, \frac{[\, \not{S}_T,\, \not{n}_+] \gamma_5}{2} \right\}$$

$$\Phi(x) = f_1\, \frac{\not{n}_+}{2}$$

Only unpolarized first

Dirac matrices: an unusual representation

$$\gamma^0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \qquad \gamma^3 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

$$\gamma^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \qquad \gamma^2 = \begin{pmatrix} 0 & i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & -i & 0 \end{pmatrix},$$

$$\gamma_5 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Good/Bad and Right/Left projectors

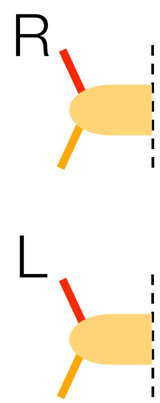
$$\mathcal{P}^+ = \gamma^-\gamma^+/2, \qquad \mathcal{P}_R = (1+\gamma_5)/2,$$
$$\mathcal{P}^- = \gamma^+\gamma^-/2, \qquad \mathcal{P}_L = (1-\gamma_5)/2$$

Good

$$\mathcal{P}_R\mathcal{P}^+ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad \mathcal{P}_L\mathcal{P}^+ = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\mathcal{P}_R\mathcal{P}^- = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad \mathcal{P}_L\mathcal{P}^- = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The correlator as probability density matrix



$$\Phi\gamma^0 = \begin{pmatrix} f_1 & 0 & \frac{M}{P^+}e & 0 \\ 0 & f_1 & 0 & \frac{M}{P^+}e \\ \frac{M}{P^+}e & 0 & 0 & 0 \\ 0 & \frac{M}{P^+}e & 0 & 0 \end{pmatrix} \sim \psi_i \big|P\big\rangle\big\langle P\big| \psi_j^\dagger$$

Twist 2

Twist 3

Pictures of the “good” stuff only

Twist 2 PDFs with polarization

$$\begin{array}{c|cc} & R & L \\ \hline R & f_1 & h_1 \\ L & h_1 & f_1 \end{array} \qquad \begin{pmatrix} f_1 & h_1 \\ h_1 & f_1 \end{pmatrix}$$

$$S_x = +1$$

$$\begin{array}{c|cc} & R & L \\ \hline R & f_1 + g_1 & 0 \\ L & 0 & f_1 - g_1 \end{array} \qquad \begin{pmatrix} f_1 + g_1 & 0 \\ 0 & f_1 - g_1 \end{pmatrix}$$

$$S_L = +1$$

$$\begin{array}{c|cc} & R & L \\ \hline R & f_1 & 0 \\ L & 0 & f_1 \end{array} \qquad \begin{pmatrix} f_1 & 0 \\ 0 & f_1 \end{pmatrix}$$

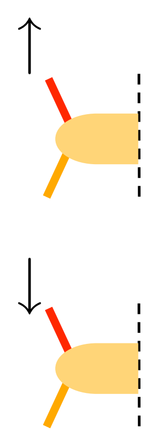
$$S = 0$$

In transversity basis

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\chi_\uparrow = \frac{1}{\sqrt{2}}(\chi_+ + e^{i\phi_s}\chi_-) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\phi_s} \end{pmatrix} \qquad \chi_\downarrow = \frac{1}{\sqrt{2}}(\chi_+ + e^{i(\phi_s+\pi)}\chi_-) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -e^{i\phi_s} \end{pmatrix}$$

In *x*-transversity basis



$$\begin{pmatrix} f_1 + h_1 & 0 \\ 0 & f_1 - h_1 \end{pmatrix}$$

$$S_x = +1$$

A short note on g_T

$$\Phi(x, S_x)\gamma^0 = \frac{1}{2}\left\{f_1\gamma^- + S_L\, g_1\gamma_5\gamma^- - h_1\, S_T\gamma_5\gamma^1\gamma^-\right\}\gamma^0$$
$$+ \frac{M}{2P^+}\left\{e - g_T\, S_T\gamma_5\gamma^1 + h_L\, S_L\, \frac{[\gamma^-, \gamma^+]\gamma_5}{2}\right\}\gamma^0$$

$$\gamma_5\gamma^-\gamma^0 = \sqrt{2}\begin{pmatrix} \sigma^3 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\gamma_5\gamma^1\gamma^-\gamma^0 = \sqrt{2}\begin{pmatrix} \sigma^1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\gamma_5\gamma^1\gamma^0 = \sqrt{2}\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}$$

This might be relevant for the discussion of the Bakker-Trueman-Leader transverse angular momentum sum-rule (PRD 70 (04))

http://www.ts.infn.it/eventi/transversitySR/

Back to the structure functions

Results for inclusive DIS

$$2MW^{\mu\nu}(q,P,S) \approx \sum_q e_q^2 \, \frac{1}{2} \mathrm{Tr}\left[\Phi(x_B,S)\,\gamma^\mu \gamma^+ \gamma^\nu\right] .$$

$$F_{UU,T} = x_B \sum_a e_a^2 \; f_1^a(x_B)$$

$$F_{UU,L} = 0$$

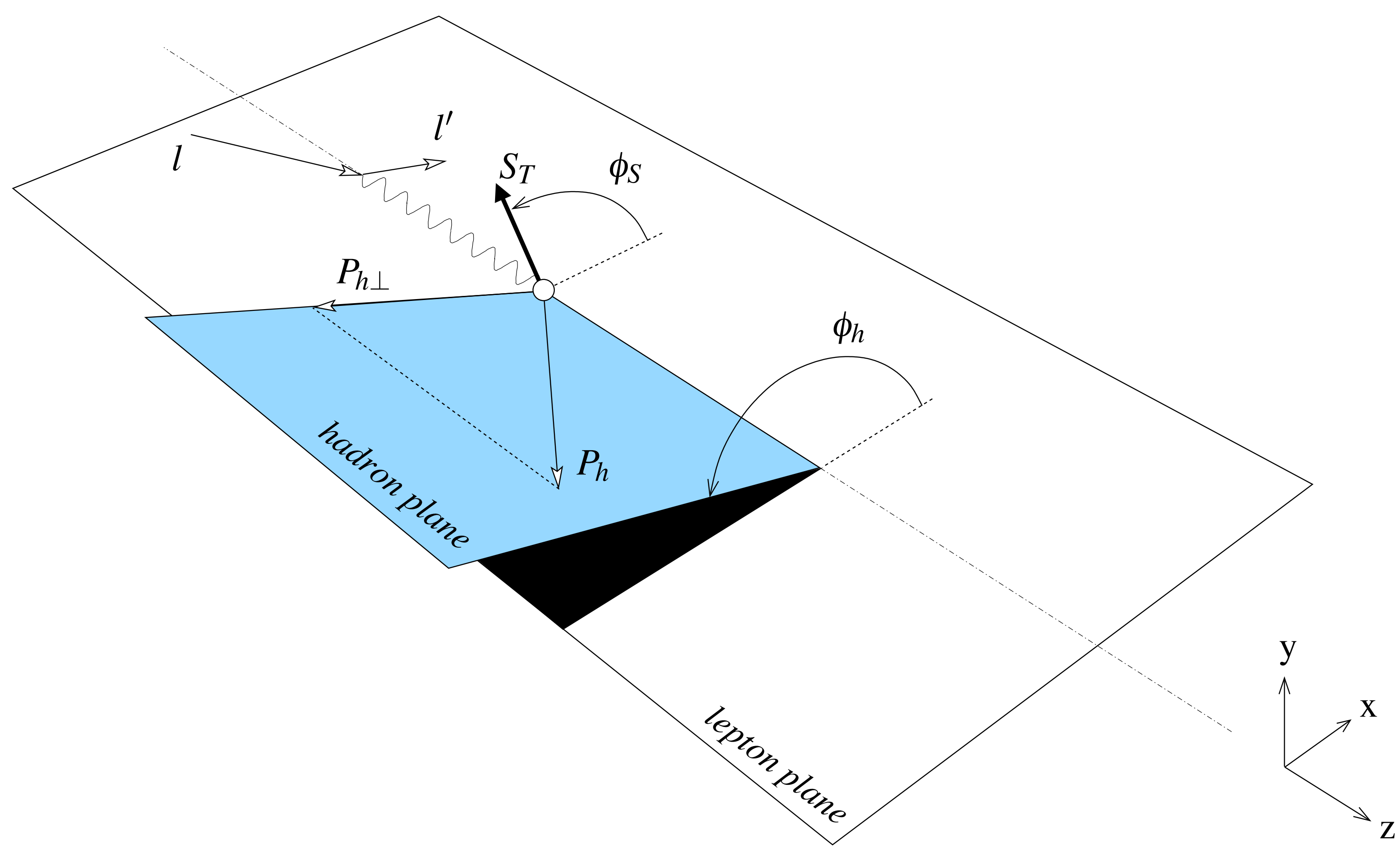
$$F_{LL} = x_B \sum_a e_a^2 \; g_1^a(x_B)$$

$$F_{LT}^{\cos\phi_S} = -\gamma x_B \sum_a e_a^2 \; g_T^a(x_B)$$

Semi-inclusive DIS and TMDs: what is different?

Semi-inclusive DIS

$$\ell(l) + N(P) \rightarrow \ell(l') + h(P_h) + X,$$



A.B., D'Alesio, Diehl, Miller, PRD70 (04)

18 structure functions

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}$$

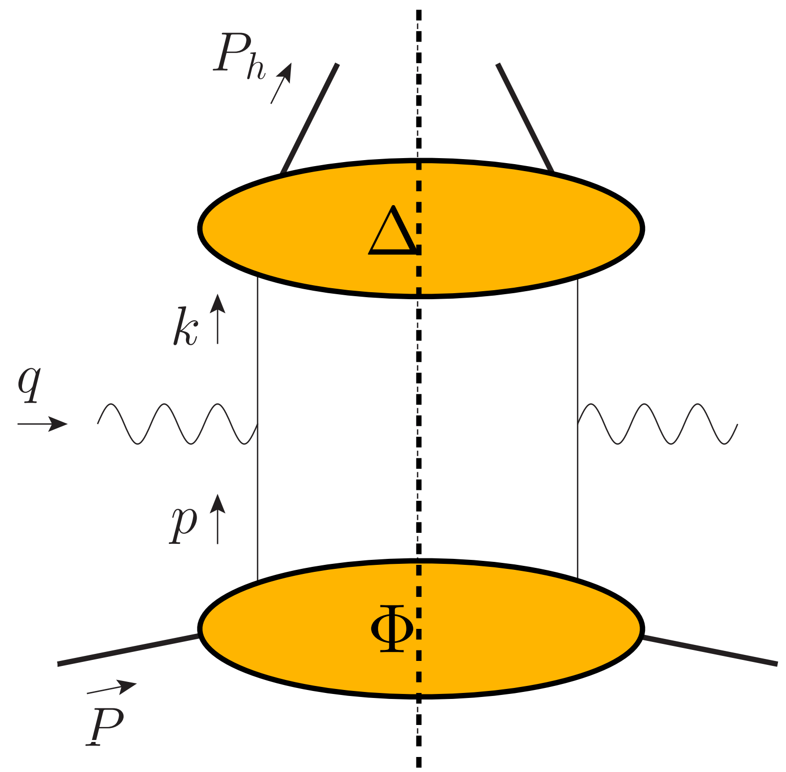
$F_{UU,T}(x, z, P_{h\perp}^2, Q^2)$

$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$
$$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$
$$+ S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$
$$+ S_T\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+ \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$$
$$+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] + S_T\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$
$$+ \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}$$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

Correlation functions in SIDIS

$$2MW^{\mu\nu}(q,P,S,P_h) = 2z_h\,\mathcal{I}\Big[\mathrm{Tr}\big(\Phi(x_B,\boldsymbol{p}_T,S)\,\gamma^\mu\,\Delta(z_h,\boldsymbol{k}_T)\,\gamma^\nu\big)\Big]$$



$$\mathcal{I}\Big[\cdots\Big] \equiv \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\,\delta^{(2)}\Big(\boldsymbol{p}_T+\boldsymbol{q}_T-\boldsymbol{k}_T\Big)\Big[\cdots\Big] = \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\,\delta^{(2)}\Big(\boldsymbol{p}_T-\frac{\boldsymbol{P}_{h\perp}}{z}-\boldsymbol{k}_T\Big)\Big[\cdots\Big]$$

Only at low transverse momentum $\boldsymbol{P}_{h\perp}^2 \ll Q^2$

Integrated vs unintegrated correlators

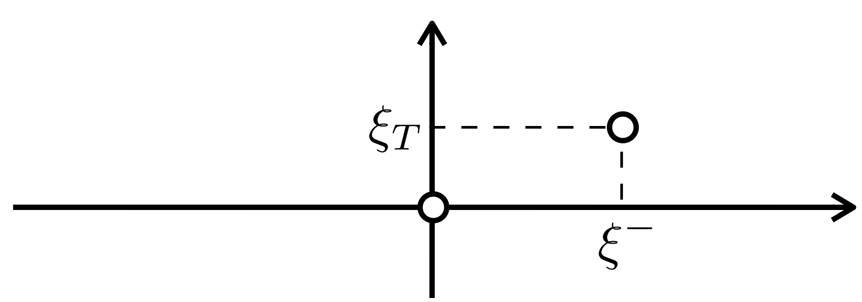
$$\Phi_{ij}(x,S) = \int d^2\boldsymbol{p}_T\;\Phi_{ij}(x,\boldsymbol{p}_T)$$

$$= \int \frac{d\xi^-}{2\pi}\;e^{ip\cdot\xi}\langle P,S\big|\,\overline{\psi}_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\,\big|P,S\rangle\Big|_{\xi^+=\boldsymbol{\xi}_T=0}$$

$$A^+(\eta)\Big|_{\substack{\eta^+ = \xi^+ = 0 \\ \boldsymbol{\eta}_T = \boldsymbol{\xi}_T = 0}}$$

$$\Phi_{ij}(x,\boldsymbol{p}_T,S) = \int dp^-\;\Phi(p,P,S)\Big|_{p^+=xP^+}$$

$$= \int \frac{d\xi^-\,d^2\boldsymbol{\xi}_T}{(2\pi)^3}\;e^{ip\cdot\xi}\langle P,S\big|\,\overline{\psi}_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\,\big|P,S\rangle\Big|_{\xi^+=0}$$

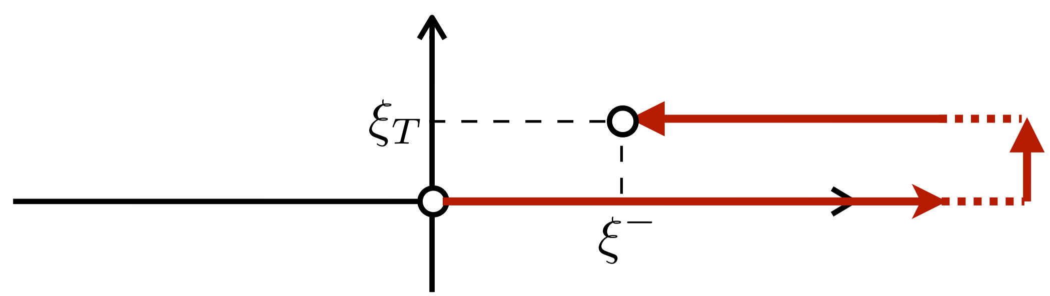


$$A^+(\eta)\Big|_{\substack{\eta^+ = \xi^+ = 0 \\ \boldsymbol{\eta}_T = \boldsymbol{\xi}_T}}$$

Gauge link for TMDs

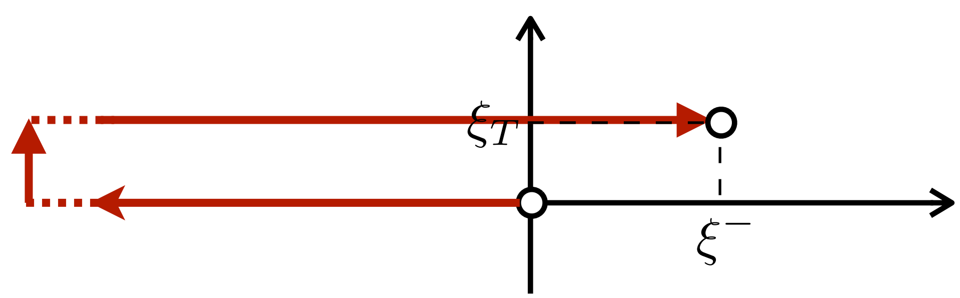
$$\Phi_{ij}(x, p_T) = \int \frac{d\xi^- d^2\xi_T}{8\pi^3}\, e^{ip\cdot\xi} \langle P|\bar{\psi}_j(0)U_{[0,\xi]}\psi_i(\xi)|P\rangle\Big|_{\xi^+=0}$$

SIDIS



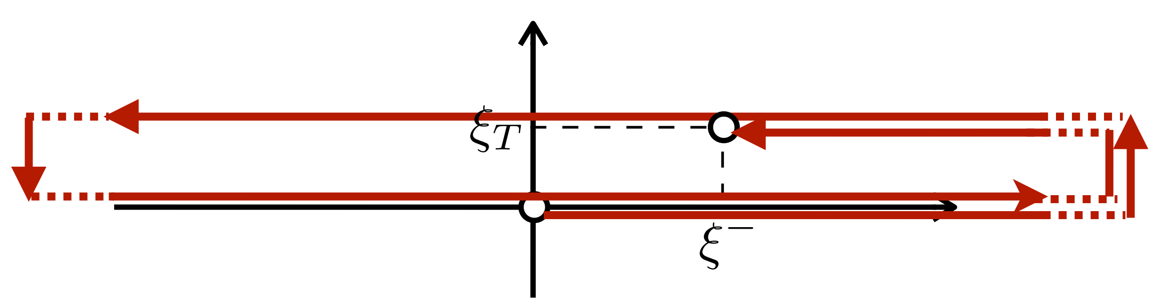
$U_{[+]}$

Drell-Yan



$U_{[-]}$

pp to hadrons



$U_{[\Box]}U_{[+]}$

+ several others

Bomhof, Mulders, Pijlman, PLB 596 (04)

Unintegrated correlation functions and TMDs

Parton distribution functions

If we keep only the leading terms in $1/P^+$ (leading twist)

$$\Phi(p,P) \approx P^+ (A_2 + xA_3)\, \not{n}_+ + P^+ \frac{i}{2M} \left[\not{n}_+,\, \not{p}_T\right] A_4,$$

$$\Phi(x,p_T) \equiv \int dp^- \Phi(p,P) = \frac{1}{2}\left\{ f_1\, \not{n}_+ + i h_1^\perp \frac{\left[\not{p}_T,\, \not{n}_+\right]}{2M} \right\}.$$

Here we introduced the parton distribution functions

$$f_1(x,p_T^2) = 2P^+ \int dp^- \,(A_2 + xA_3)\,, \qquad h_1^\perp(x,p_T^2) = 2P^+ \int dp^- \,(-A_4)\,.$$

A twist on twist

OPE: for local operators
twist = dimension - spin

TMDs: “working redefinition”
twist = 2 + power of M/P^+

When we expand TMDs in local operators, every time we take a transverse moment, we increase the twist

An example

$$\frac{1}{2}\mathrm{Tr}[\Phi\gamma^+\gamma_5] = \frac{1}{(2\pi)^4}\int d^4\xi\, e^{ip\cdot\xi}\langle P,S\big|\,\overline{\psi}_j(0)\gamma^+\gamma_5\,\psi_i(\xi)\,\big|P,S\rangle$$

$$\frac{1}{2}\int dp^-\mathrm{Tr}[\Phi\gamma^+\gamma_5] = \ldots\langle P,S\big|\,\overline{\psi}_j(0)\gamma^+\gamma_5\,\psi_i(\xi)\,\big|P,S\rangle = S_L\,g_1(x,p_T^2) - \frac{p_T\cdot S_T}{M}\,g_{1T}(x,p_T^2)$$

$$\frac{1}{2}\int dp^- d^2p_T\,\mathrm{Tr}[\Phi\gamma^+\gamma_5] = \ldots\langle P,S\big|\,\overline{\psi}_j(0)\gamma^+\gamma_5\,\psi_i(\xi)\,\big|P,S\rangle = S_L\,g_1(x)$$

twist 2

$$\frac{1}{2}\int dp^- d^2p_T\,\frac{p_T^\alpha}{M}\mathrm{Tr}[\Phi\gamma^+\gamma_5] = \ldots\langle P,S\big|\,\overline{\psi}_j(0)i\delta_{T\alpha}\gamma^+\gamma_5\,\psi_i(\xi)\,\big|P,S\rangle = S_T^\alpha g_{1T}^{(1)}(x)$$

twist 3

$$\frac{1}{2}\int dp^- d^2p_T\,\frac{p_T^\alpha}{M}\frac{p_T^\beta}{M}\mathrm{Tr}[\Phi\gamma^+\gamma_5] = \ldots\langle P,S\big|\,\overline{\psi}_j(0)\delta_{T\alpha}\delta_{T\beta}\gamma^+\gamma_5\,\psi_i(\xi)\,\big|P,S\rangle = S_L g^{\alpha\beta}g_1^{(1)}(x)$$

twist 4

To summarize

When I try to connect a TMD with a local operator, I need to take p_T moments and I increase the twist.

This is not due to the nature of the TMD, but to the fact that I have to take p_T moments of it.

Therefore, I'm still happy with the working definition of twist

Collinear PDFs and TMDs

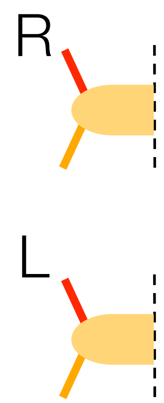
$f_1(x)$

$$\Phi(x) = \frac{1}{2}\left\{ f_1\, \not{n}_+ + S_L\, g_1 \gamma_5\, \not{n}_+ + h_1 \frac{[\not{S}_T, \not{n}_+]\gamma_5}{2} \right\}$$

$f_1(x, p_T^2)$

$$\Phi(x, p_T) = \frac{1}{2}\left\{ f_1\, \not{n}_+ - f_{1T}^{\perp} \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M}\, \not{n}_+ + S_L\, g_{1L} \gamma_5\, \not{n}_+ - g_{1T} \frac{p_T \cdot S_T}{M} \gamma_5\, \not{n}_+ \right.$$
$$+ h_{1T} \frac{[\not{S}_T, \not{n}_+]\gamma_5}{2} + S_L\, h_{1L}^{\perp} \frac{[\not{p}_T, \not{n}_+]\gamma_5}{2M}$$
$$\left. - h_{1T}^{\perp} \frac{p_T \cdot S_T}{M} \frac{[\not{p}_T, \not{n}_+]\gamma_5}{2M} + i\, h_1^{\perp} \frac{[\not{p}_T, \not{n}_+]}{2M} \right\}$$

The correlator as probability density matrix

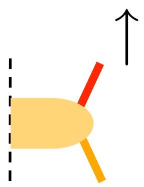


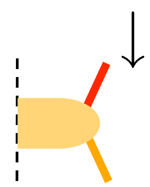
$$\Phi\gamma^0 = \begin{pmatrix} f_1 & i\frac{(p_x+ip_y)}{M}h_1^\perp & 0 & 0 \\ -i\frac{(p_x-ip_y)}{M}h_1^\perp & f_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \psi_i \left|P\right\rangle\left\langle P\right| \psi_j^\dagger$$

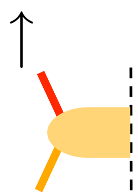
- Probabilistic interpretation
- Positivity bounds
- Need of orbital angular momentum

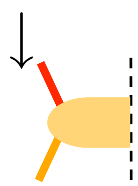
AB, M. Boglione, A. Henneman, P.J. Mulders, PRL 85 (00)

In *x*-transversity basis









$$\Phi\gamma^0 = \begin{pmatrix} f_1 - \frac{p_y}{M} h_1^\perp & i\frac{p_x}{M} h_1^\perp & 0 & 0 \\ -i\frac{p_x}{M} h_1^\perp & f_1 + \frac{p_y}{M} h_1^\perp & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \psi_i \left|P\right\rangle\left\langle P\right| \psi_j^\dagger$$

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\chi_\uparrow = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \chi_\downarrow = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

TMDs and their probabilistic interpretation

quark pol.

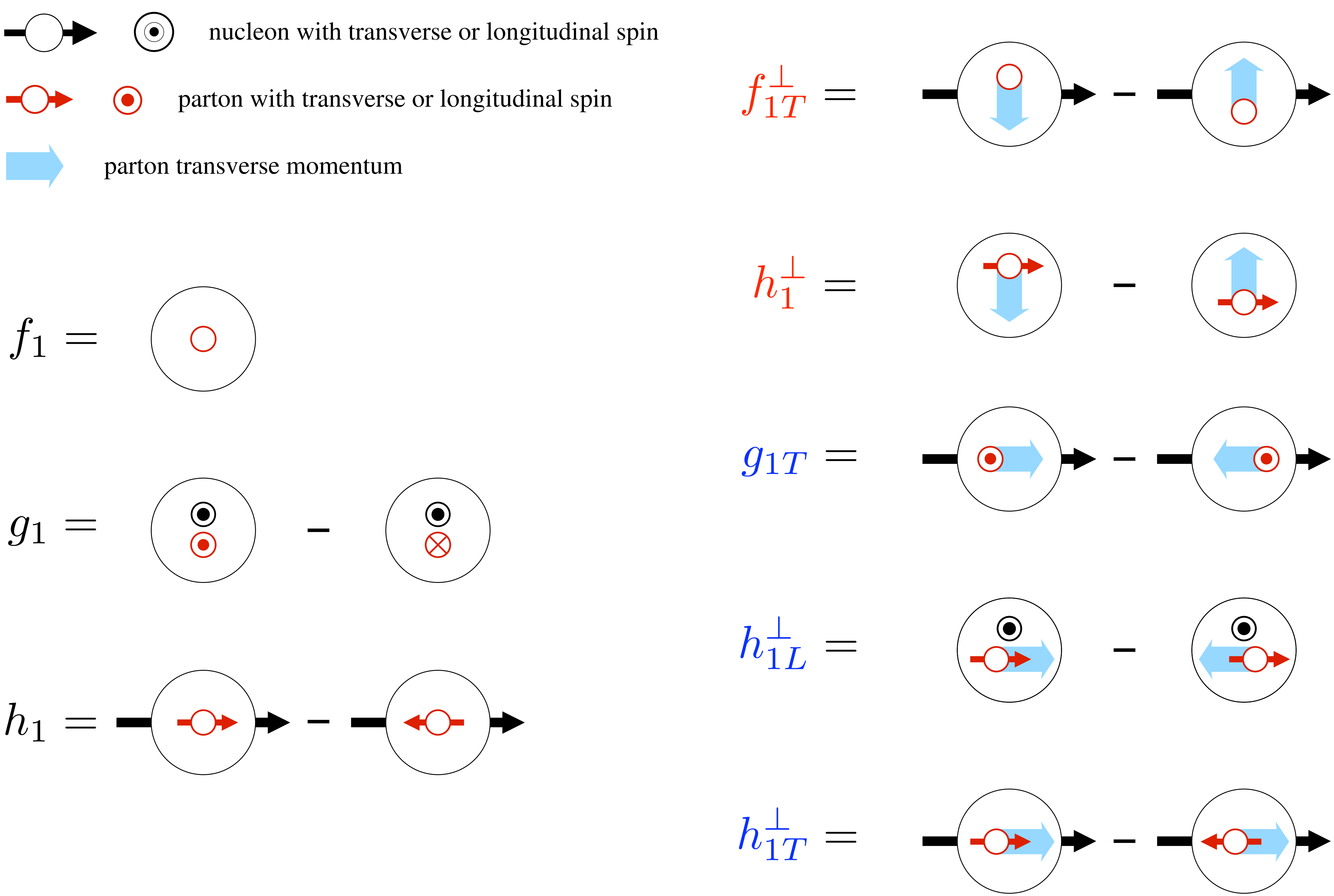
nucleon pol.	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1, $h_{1T}^\perp$

Twist-2 TMDs

TMDs in black survive transverse-momentum integration

TMDs in red are T-odd

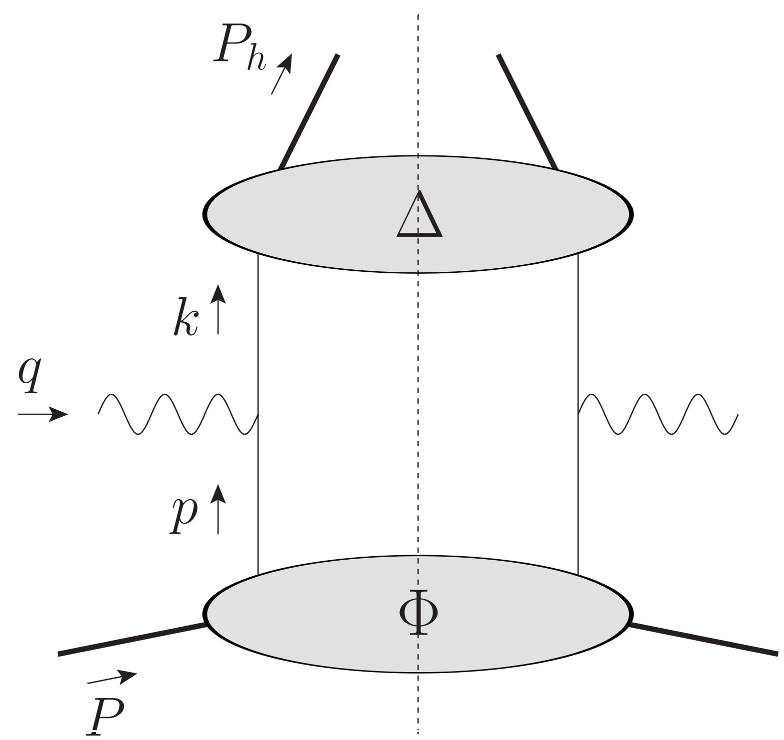
TMDs and their probabilistic interpretation



Back to the structure functions

Starting formula

$$2MW^{\mu\nu}(q,P,S,P_h) = 2z_h\,\mathcal{I}\Big[\mathrm{Tr}(\Phi(x_B,\boldsymbol{p}_T,S)\,\gamma^\mu\,\Delta(z_h,\boldsymbol{k}_T)\,\gamma^\nu)\Big]$$



$$\mathcal{I}\Big[\cdots\Big] \equiv \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\,\delta^{(2)}\Big(\boldsymbol{p}_T+\boldsymbol{q}_T-\boldsymbol{k}_T\Big)\,\Big[\cdots\Big] = \int d^2\boldsymbol{p}_T d^2\boldsymbol{k}_T\,\delta^{(2)}\Big(\boldsymbol{p}_T-\frac{\boldsymbol{P}_{h\perp}}{z}-\boldsymbol{k}_T\Big)\,\Big[\cdots\Big]$$

18 structure functions

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2} \\
&= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} \\
&\quad + \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad + S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad + S_T\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} \\
&\qquad + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} \\
&\qquad + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] + S_T\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} \\
&\qquad + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}
\end{aligned}
$$

see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

Unpolarized sector

$$F_{UU,T} = \mathcal{C}\big[f_1 D_1\big],$$

$$F_{UU,L} = \mathcal{O}\left(\frac{M^2}{Q^2}, \frac{q_T^2}{Q^2}\right),$$

$$F_{UU}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\left(xh\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{D}^{\perp}}{z}\right) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\left(xf^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{H}}{z}\right)\right],$$

$$F_{UU}^{\cos 2\phi_h} = \mathcal{C}\left[-\frac{2\,\big(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\big)\,\big(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\big) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_1^{\perp} H_1^{\perp}\right],$$

$$\mathcal{C}\big[wfD\big] = \sum_a x\,e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2),$$

Longitudinally polarized beam or/and target

$$F_{LU}^{\sin\phi_h} = \mathcal{O}\left(\frac{M}{Q}\right),$$

$$F_{UL}^{\sin\phi_h} = \mathcal{O}\left(\frac{M}{Q}\right),$$

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\left[-\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h} h_{1L}^{\perp} H_1^{\perp}\right],$$

$$F_{LL} = \mathcal{C}\left[g_{1L} D_1\right],$$

$$F_{LL}^{\cos\phi_h} = \mathcal{O}\left(\frac{M}{Q}\right)$$

Transversely polarized beam

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

$$F_{UT,L}^{\sin(\phi_h-\phi_S)} = \mathcal{O}\left(\frac{M^2}{Q^2},\frac{q_T^2}{Q^2}\right),$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right],$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)\left(\boldsymbol{p}_T\cdot\boldsymbol{k}_T\right)+\boldsymbol{p}_T^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)-4\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)}{2M^2M_h}\, h_{1T}^{\perp} H_1^{\perp}\right],$$

$$F_{UT}^{\sin\phi_S} = \mathcal{O}\left(\frac{M}{Q}\right),$$

$$F_{UT}^{\sin(2\phi_h-\phi_S)} = \mathcal{O}\left(\frac{M}{Q}\right)$$

Trasversely pol. target and long. pol. beam

$$F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} g_{1T} D_1\right],$$

$$F_{LT}^{\cos\phi_S} = \mathcal{O}\left(\frac{M}{Q}\right),$$

$$F_{LT}^{\cos(2\phi_h-\phi_S)} = \mathcal{O}\left(\frac{M}{Q}\right)$$

Appendix

Longitudinally polarized beam or/and target

$$F_{LU}^{\sin\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xe\,H_1^{\perp} + \frac{M_h}{M}\,f_1\frac{\tilde{G}^{\perp}}{z}\Big) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xg^{\perp}D_1 + \frac{M_h}{M}\,h_1^{\perp}\frac{\tilde{E}}{z}\Big)\Big],$$

$$F_{UL}^{\sin\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xh_L H_1^{\perp} + \frac{M_h}{M}\,g_{1L}\frac{\tilde{G}^{\perp}}{z}\Big) + \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xf_L^{\perp}D_1 - \frac{M_h}{M}\,h_{1L}^{\perp}\frac{\tilde{H}}{z}\Big)\Big],$$

$$F_{UL}^{\sin 2\phi_h} = \mathcal{C}\Big[-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T) - \boldsymbol{k}_T\cdot\boldsymbol{p}_T}{M M_h}h_{1L}^{\perp}H_1^{\perp}\Big],$$

$$F_{LL} = \mathcal{C}\big[g_{1L}D_1\big],$$

$$F_{LL}^{\cos\phi_h} = \frac{2M}{Q}\,\mathcal{C}\Big[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h}\Big(xe_L H_1^{\perp} - \frac{M_h}{M}\,g_{1L}\frac{\tilde{D}^{\perp}}{z}\Big) - \frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M}\Big(xg_L^{\perp}D_1 + \frac{M_h}{M}\,h_{1L}^{\perp}\frac{\tilde{E}}{z}\Big)\Big],$$

Transversely polarized beam

$$F_{UT,T}^{\sin(\phi_h-\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} f_{1T}^{\perp} D_1\right],$$

$$F_{UT,L}^{\sin(\phi_h-\phi_S)} = \mathcal{O}\left(\frac{M^2}{Q^2}, \frac{q_T^2}{Q^2}\right),$$

$$F_{UT}^{\sin(\phi_h+\phi_S)} = \mathcal{C}\left[-\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T}{M_h} h_1 H_1^{\perp}\right],$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)} = \mathcal{C}\left[\frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)\left(\boldsymbol{p}_T\cdot\boldsymbol{k}_T\right)+\boldsymbol{p}_T^2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)-4\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T)}{2M^2M_h}\, h_{1T}^{\perp} H_1^{\perp}\right],$$

$$F_{UT}^{\sin\phi_S} = \frac{2M}{Q}\,\mathcal{C}\left\{\left(x f_T D_1 - \frac{M_h}{M}\, h_1 \frac{\tilde{H}}{z}\right) - \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(x h_T H_1^{\perp} + \frac{M_h}{M} g_{1T}\,\frac{\tilde{G}^{\perp}}{z}\right) - \left(x h_T^{\perp} H_1^{\perp} - \frac{M_h}{M} f_{1T}^{\perp}\,\frac{\tilde{D}^{\perp}}{z}\right)\right]\right\},$$

$$F_{UT}^{\sin(2\phi_h-\phi_S)} = \frac{2M}{Q}\,\mathcal{C}\left\{\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-\boldsymbol{p}_T^2}{2M^2}\left(x f_T^{\perp} D_1 - \frac{M_h}{M}\, h_{1T}^{\perp} \frac{\tilde{H}}{z}\right) - \frac{2\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\right)\left(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\right)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\left[\left(x h_T H_1^{\perp} + \frac{M_h}{M} g_{1T}\,\frac{\tilde{G}^{\perp}}{z}\right) + \left(x h_T^{\perp} H_1^{\perp} - \frac{M_h}{M} f_{1T}^{\perp}\,\frac{\tilde{D}^{\perp}}{z}\right)\right]\right\},$$

Trasversely pol. target and long. pol. beam

$$F_{LT}^{\cos(\phi_h-\phi_S)} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T}{M} g_{1T} D_1\right],$$

$$F_{LT}^{\cos\phi_S} = \frac{2M}{Q}\,\mathcal{C}\biggl\{-\biggl(x g_T D_1 + \frac{M_h}{M}\,h_1\frac{\tilde{E}}{z}\biggr) + \frac{\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\biggl[\biggl(x e_T H_1^{\perp} - \frac{M_h}{M} g_{1T}\,\frac{\tilde{D}^{\perp}}{z}\biggr) + \biggl(x e_T^{\perp} H_1^{\perp} + \frac{M_h}{M} f_{1T}^{\perp}\,\frac{\tilde{G}^{\perp}}{z}\biggr)\biggr]\biggr\},$$

$$F_{LT}^{\cos(2\phi_h-\phi_S)} = \frac{2M}{Q}\,\mathcal{C}\biggl\{-\frac{2\,(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T)^2-\boldsymbol{p}_T^2}{2M^2}\biggl(x g_T^{\perp} D_1 + \frac{M_h}{M}\,h_{1T}^{\perp}\frac{\tilde{E}}{z}\biggr) + \frac{2\,\bigl(\hat{\boldsymbol{h}}\cdot\boldsymbol{k}_T\bigr)\,\bigl(\hat{\boldsymbol{h}}\cdot\boldsymbol{p}_T\bigr)-\boldsymbol{k}_T\cdot\boldsymbol{p}_T}{2MM_h}\biggl[\biggl(x e_T H_1^{\perp} - \frac{M_h}{M} g_{1T}\,\frac{\tilde{D}^{\perp}}{z}\biggr) - \biggl(x e_T^{\perp} H_1^{\perp} + \frac{M_h}{M} f_{1T}^{\perp}\,\frac{\tilde{G}^{\perp}}{z}\biggr)\biggr]\biggr\}$$