Jefferson Lab

mini lecture series

"transverse thinking": an introduction to TMDs

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Preliminary plan

- Introduction
- Semi-inclusive DIS
- Theory of TMDs 1 (tree-level definition, interpretation, gauge link)
- Theory of TMDs 2 (high pT, resummation, evolution)
- Phenomenology of unpolarized SIDIS
- Phenomenology of polarized SIDIS

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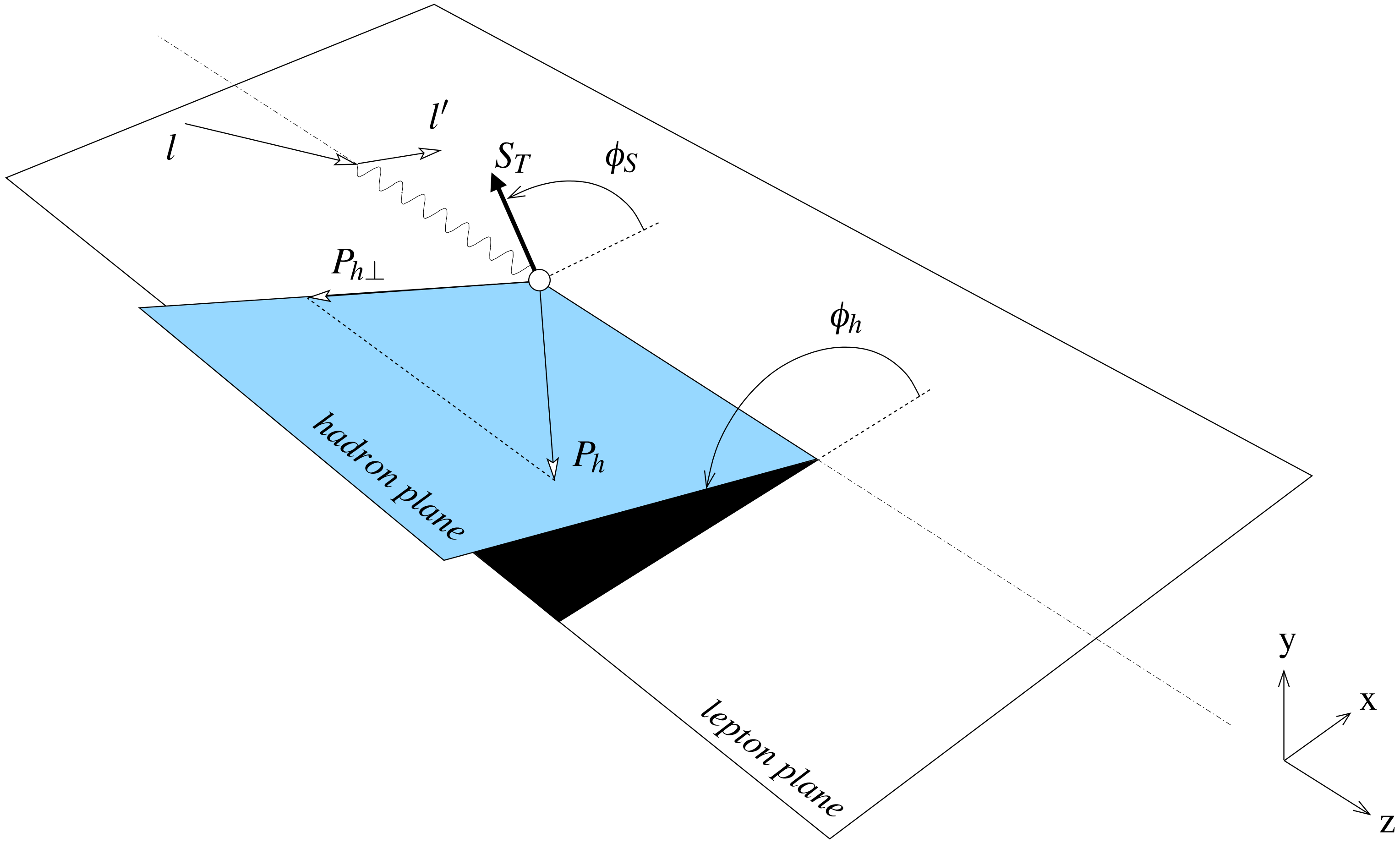
Part 2: Semi-inclusive DIS (SIDIS)

Deep inelastic scattering

$$\ell(l) + N(P) \rightarrow \ell(l') + h(P_h) + X,$$

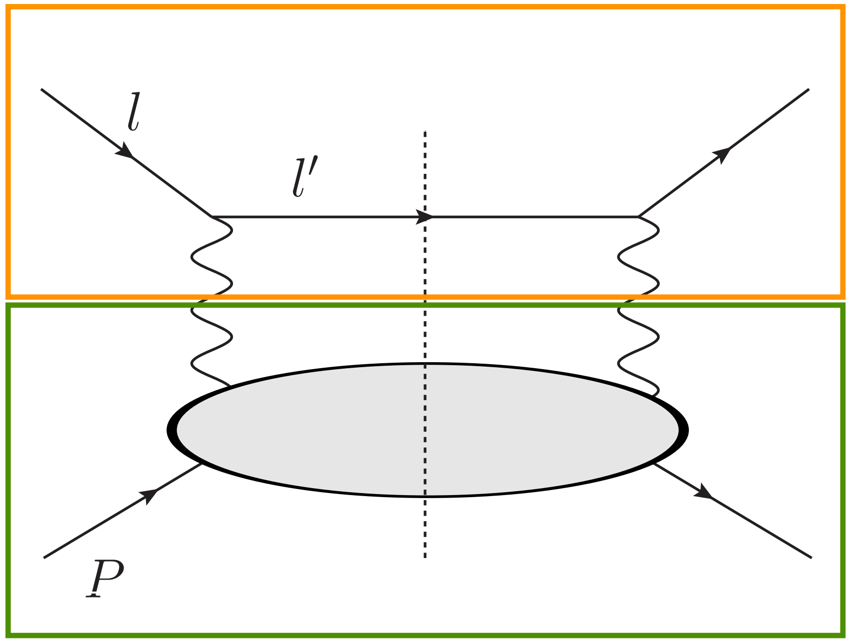
$$x_B = \frac{Q^2}{2\,P \cdot q}, \qquad y = \frac{P \cdot q}{P \cdot l}, \qquad z_h = \frac{P \cdot P_h}{P \cdot q}.$$



A.B., D'Alesio, Diehl, Miller, PRD70 (04)

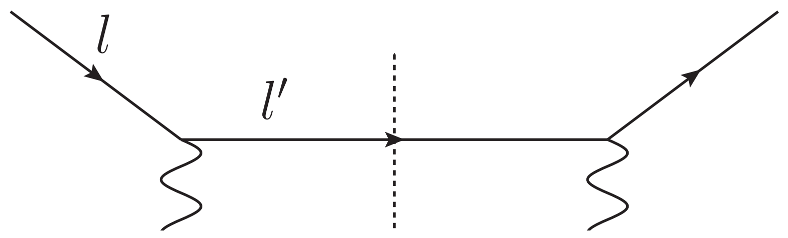
Inclusive DIS

$$\frac{d^3\sigma}{dx_B\, dy\, d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4} L_{\mu\nu}(l, l', \lambda_e)\, 2MW^{\mu\nu}(q, P, S)$$



Single-photon-exchange approximation

Inclusive DIS



$$L_{\mu\nu} = -Q^2 g_{\mu\nu} + 2\left(l_\mu l'_\nu + l'_\mu l_\nu\right)$$
$$= \frac{2Q^2}{y^2}\Big[-\Big(1 - y + \frac{y^2}{2}\Big) g_{\perp\mu\nu} + 2(1-y)\,\hat{t}_\mu \hat{t}_\nu$$
$$+ 2(1-y)\Big(\hat{l}_{\perp\mu}\hat{l}_{\perp\nu} + \frac{1}{2} g_{\perp\mu\nu}\Big) + \ldots\Big]$$

Lepton tensor

$$q^\mu = (0,\ 0,\ 0,\ Q)$$

$$l^\mu = \left(\frac{(2-y)Q}{2y},\ \frac{\sqrt{1-y}\,Q}{y},\ 0,\ \frac{Q}{2}\right)$$

$$l'^\mu = \left(\frac{(2-y)Q}{2y},\ \frac{\sqrt{1-y}\,Q}{y},\ 0,\ -\frac{Q}{2}\right)$$

$$L_{\mu\nu} = \frac{2Q^2}{y^2}\begin{pmatrix} 2(1-y) & \dots & 0 & 0 \\ \dots & \left(1-y+\frac{y^2}{2}\right)+(1-y) & 0 & 0 \\ 0 & 0 & \left(1-y+\frac{y^2}{2}\right)-(1-y) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

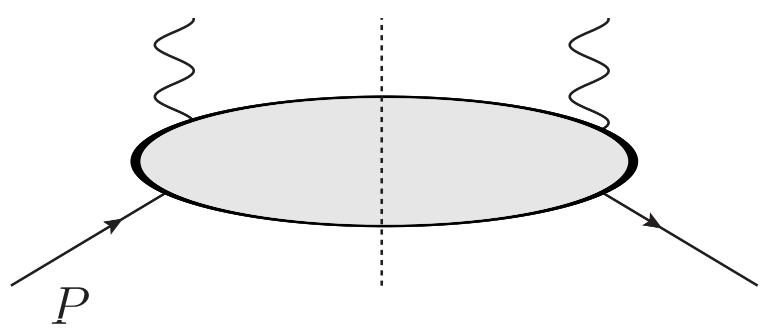
$$\epsilon_x^\mu = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad \epsilon_y^\mu = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad \epsilon_L^\mu = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Structure functions

$$\frac{d^3\sigma}{dx_B dy d\phi_S} = \frac{\alpha^2\, y}{2\,Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S)$$

$$F_{UU,T}(x, Q^2)$$

$$2MW^{\mu\nu} = \frac{1}{x}\left[-g_\perp^{\mu\nu}\, F_{UU,T} + \hat{t}^\mu \hat{t}^\nu\, F_{UU,L} + i S_L \epsilon_\perp^{\mu\nu}\, F_{LL} - i\hat{t}^{[\mu}\epsilon_\perp^{\nu]\rho} S_\rho F_{LT}^{\cos\phi_S}\right]$$



$$\frac{d\sigma}{dx_B\, dy\, d\phi_S} = \frac{2\alpha^2}{x_B y Q^2}\left\{\left(1 - y + \frac{y^2}{2}\right) F_{UU,T} + (1 - y)\, F_{UU,L} + S_L \lambda_e\, y\left(1 - \frac{y}{2}\right)\, F_{LL}\right.$$
$$\left. + \,|\boldsymbol{S}_T|\lambda_e\, y\sqrt{1 - y}\, \cos\phi_S\, F_{LT}^{\cos\phi_s}\right\}$$

see, e.g., A.B., Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

Connection with F_1, F_2, g_1, and g_2

$$F_{UU,T} = 2x_B F_1,$$

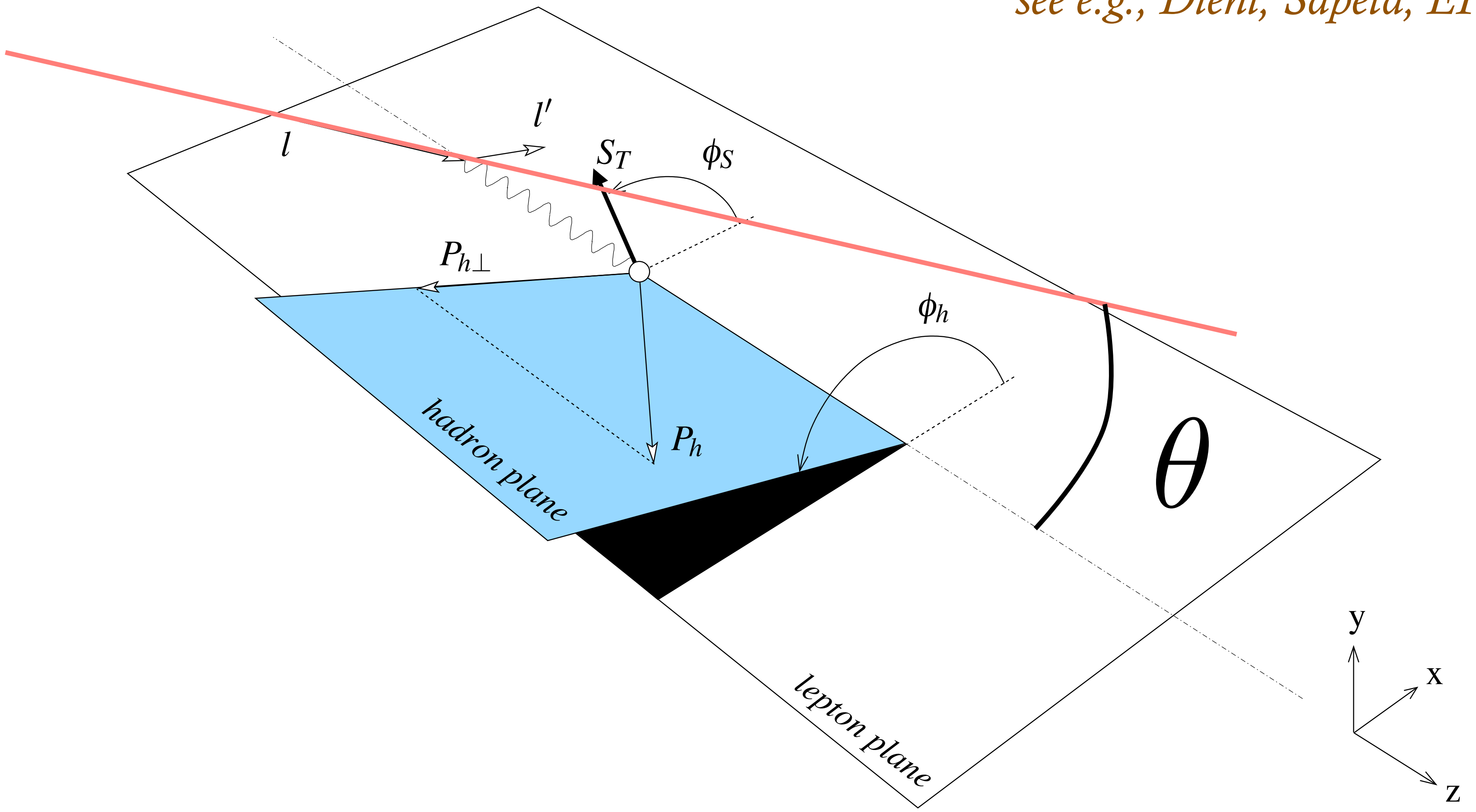
$$F_{UU,L} = (1+\gamma^2)F_2 - 2x_B F_1,$$

$$F_{LL} = 2x_B \left(g_1 - \gamma^2 g_2\right),$$

$$F_{LT}^{\cos\phi_S} = -2x_B \gamma \left(g_1 + g_2\right)$$

Beam direction vs virtual photon direction

see e.g., Diehl, Sapeta, EPJC41 (05)



$$S_L^\gamma = \cos\theta\, S_\parallel^e + \sin\theta\, |\boldsymbol{S}_\perp^e| \cos\psi,$$

$$|\boldsymbol{S}_T^\gamma| \cos\phi_S = \cos\theta\, |\boldsymbol{S}_\perp^e| \cos\psi - \sin\theta\, S_\parallel^e$$

$$|\boldsymbol{S}_T^\gamma| \sin\phi_S = |\boldsymbol{S}_\perp^e| \sin\psi$$

$$\cos\theta = \frac{1+\gamma^2 y/2}{\sqrt{1+\gamma^2}} = \frac{1-(1-y)\varepsilon}{\sqrt{1-\varepsilon^2}},$$

$$\sin\theta = \gamma\sqrt{\frac{1-y-\gamma^2 y^2/4}{1+\gamma^2}} = \frac{\varepsilon y}{\sqrt{2\,\varepsilon(1-\varepsilon)}}$$

Target mass corrections

$$\frac{d\sigma}{dx_B\,dy\,d\phi_S} = \frac{2\alpha^2}{x_B y Q^2}\left\{\left(1-y+\frac{y^2}{2}\right)F_{UU,T} + (1-y)\,F_{UU,L} + S_L\lambda_e\,y\left(1-\frac{y}{2}\right)\,F_{LL} + |\boldsymbol{S}_T|\lambda_e\,y\sqrt{1-y}\,\cos\phi_S\,F_{LT}^{\cos\phi_s}\right\}$$

$$\frac{d\sigma}{dx_B\,dy\,d\psi} = \frac{2\alpha^2}{x_B y Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left\{F_{UU,T}(x_B,Q^2) + \varepsilon F_{UU,L}(x_B,Q^2) + S_\parallel^\gamma\lambda_e\,\sqrt{1-\varepsilon^2}\,F_{LL}(x_B,Q^2) + |\boldsymbol{S}_\perp^\gamma|\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}(x_B,Q^2)\right\},$$

$$\varepsilon = \frac{1-y-\gamma^2y^2/4}{1-y+y^2/2+\gamma^2y^2/4} \qquad \gamma^2 = \frac{4x^2M^2}{Q^2}$$

Parallel and perpendicular asymmetry

$$\frac{d\sigma}{dx_B\,dy\,d\psi} = \frac{2\alpha^2}{x_B y Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\,F_{UU,T}\Big\{1+\varepsilon R + S_\parallel^e \lambda_e\,A_\parallel - |\boldsymbol{S}_\perp^e|\lambda_e \cos\psi\,A_\perp\Big\}$$

$$A_\parallel = \frac{1}{F_{UU,T}(1+\varepsilon R)}\Big[\cos\theta\,\sqrt{1-\varepsilon^2}\,F_{LL} - \sin\theta\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,F_{LT}^{\cos\phi_S}\Big]$$

$$A_\perp = -\frac{1}{F_{UU,T}(1+\varepsilon R)}\Big[\cos\theta\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,F_{LT}^{\cos\phi_S} + \sin\theta\,\sqrt{1-\varepsilon^2}\,F_{LL}\Big]$$

$$A_\parallel = \frac{2(1-\varepsilon)}{F_1\,y\,(1+\varepsilon R)}\bigg[\bigg(1-\frac{y}{2}-\frac{\gamma^2 y^2}{4}\bigg)g_1 - \frac{\gamma^2 y}{2}g_2\bigg],$$

$$A_\perp = \frac{2(1-\varepsilon)}{F_1\,y\,(1+\varepsilon R)}\,\gamma\sqrt{1-y-\frac{\gamma^2 y^2}{4}}\,\bigg[\frac{y}{2}\,g_1 + g_2\bigg]$$

Extracting the structure functions

$$A_{\parallel} - \frac{\sin\theta}{\cos\theta} A_{\perp} = \frac{(1-\varepsilon)(2-y)}{y(1+\varepsilon R)(1+\gamma^2 y/2)} \frac{F_{LL}}{F_{UU,T}}$$

$$A_{\parallel} + \frac{\gamma y}{2\sqrt{1-y-\gamma^2 y^2/4}} A_{\perp} = \frac{(1-\varepsilon)(2-y)}{y\,(1+\varepsilon R)} \frac{g_1}{F_1}$$

Semi-inclusive DIS

$$\frac{d^3\sigma}{dx_B dy d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S)$$

$$\frac{2E_h d^6\sigma}{d^3 P_h dx_B dy d\phi_S} = \frac{\alpha^2\, y}{2\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S, P_h)$$

$$\frac{d^6\sigma}{dx_B dy dz_h d\phi_S d\phi_h dP_{h\perp}^2} = \frac{\alpha^2\, y}{2\, z_h\, Q^4}\, L_{\mu\nu}(l, l', \lambda_e)\; 2MW^{\mu\nu}(q, P, S, P_h)$$

Structure functions

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz\,d\phi_h\,dP_{h\perp}^2}$$

$F_{UU,T}(x, z, P_{h\perp}^2, Q^2)$

$$= \frac{\alpha^2}{x\,y\,Q^2}\,\frac{y^2}{2\,(1-\varepsilon)}\Bigg\{ F_{UU,T} + \varepsilon\,F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

$$+ \lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} + S_L\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h} + \varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$

$$+ S_L\,\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$

$$+ S_T\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)} + \varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+ \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}$$

$$+ \sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] + S_T\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}$$

$$+ \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} + \sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\}$$

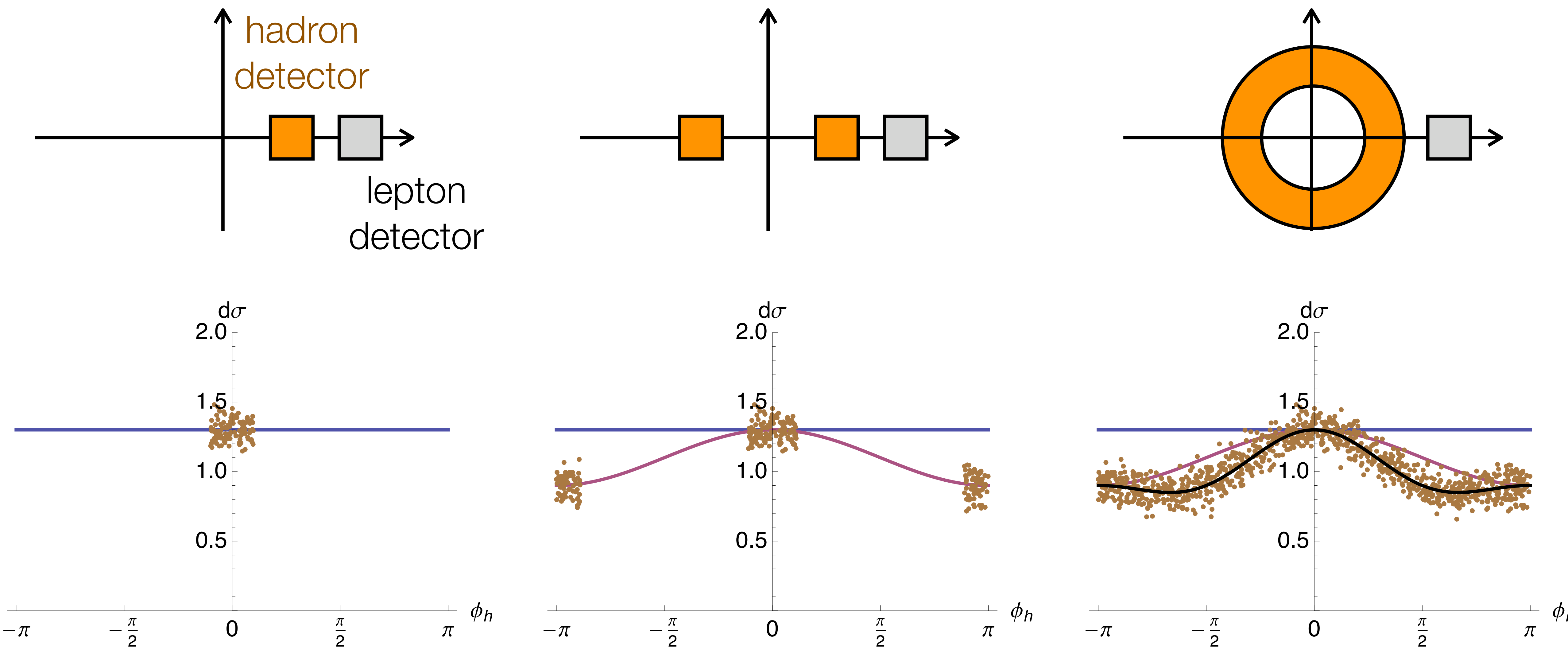
see e.g. AB, Diehl, Goeke, Metz, Mulders, Schlegel, JHEP093 (07)

Q: Most of the time we do SIDIS integrated over transverse momentum, so why should we care?

A: Because acceptance is never perfect

Beware: azimuthal coverage in experiments

$$d\sigma = A + B\cos\phi_h + C\cos 2\phi_h$$



$A = 1.3$ $\quad$ $A = 1.1, \quad B = 0.2$ $\quad$ $A = 1, \quad B = 0.2, \quad C = 0.1$

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Part 3: Theory of TMDs

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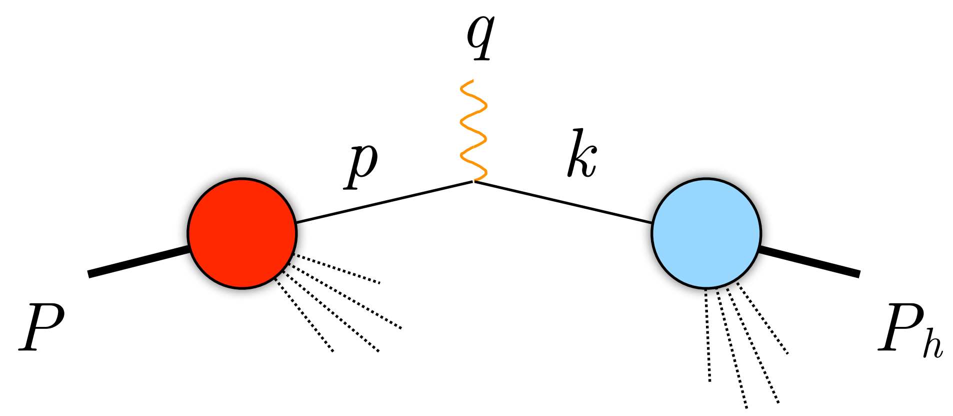
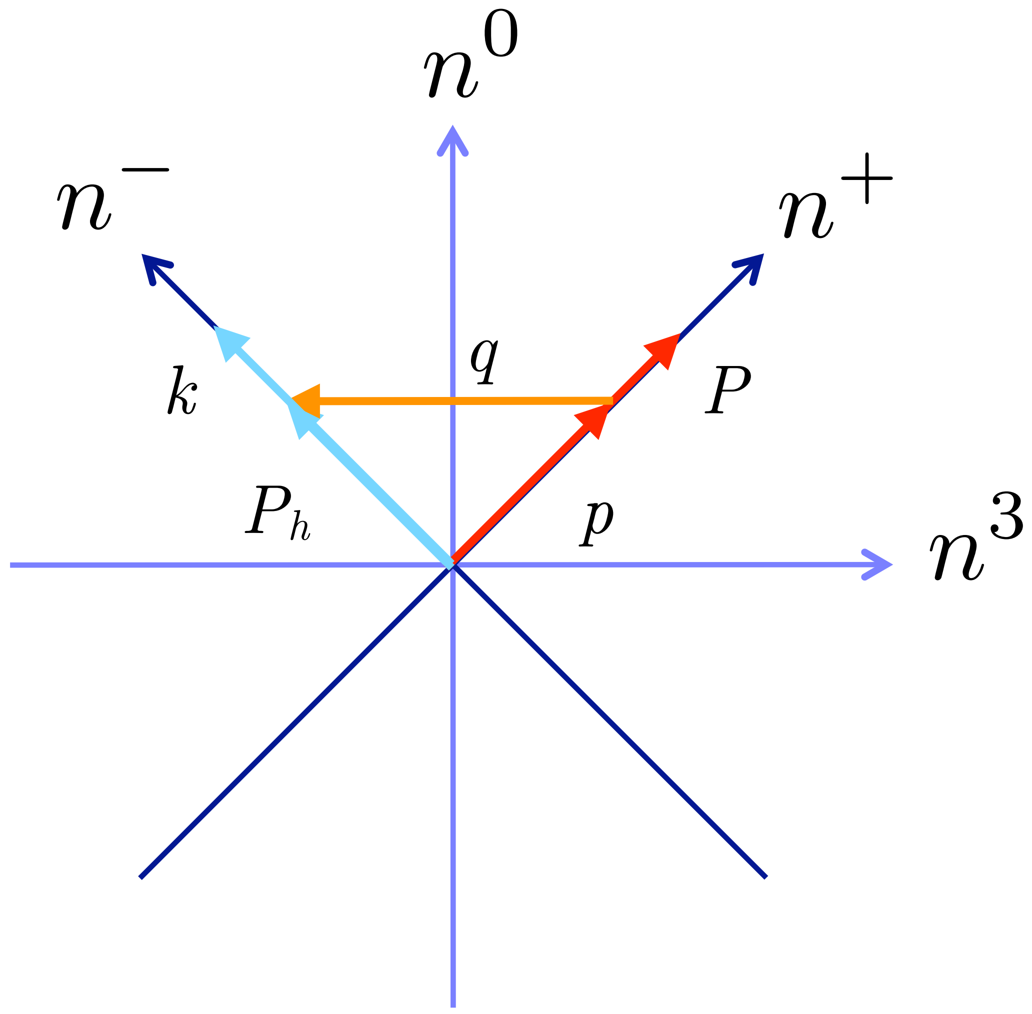
Part 3: Theory of TMDs (but first a review of collinear PDFs)

Inclusive DIS and collinear PDFs:
Tree-level analysis

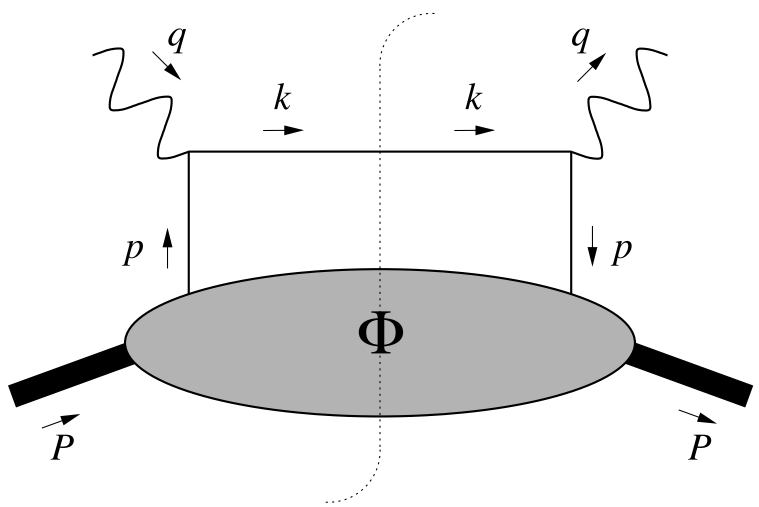
Light-cone vectors

Q^2 much higher than any other scalar product





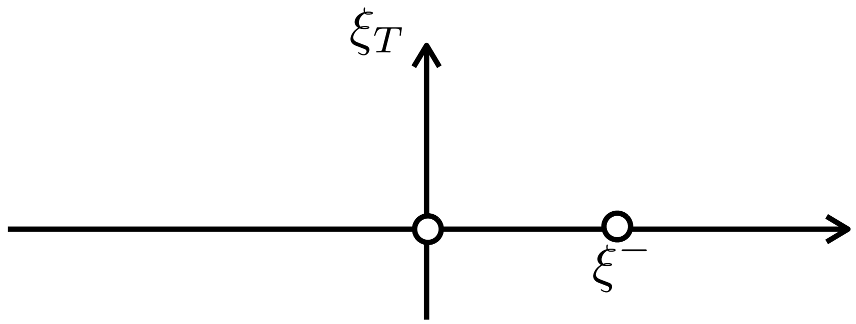
Correlation functions in DIS



$$2MW^{\mu\nu}(q,P,S) \approx \sum_q e_q^2\, \frac{1}{2}\mathrm{Tr}\left[\Phi(x_B,S)\,\gamma^\mu\gamma^+\gamma^\nu\right].$$

$$\Phi_{ij}(x,S) = \int d^2\boldsymbol{p}_T dp^-\ \Phi_{ij}(p,P,S)\Big|_{p^+=xP^+}$$

$$= \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi}\langle P,S|\,\overline{\psi}_j(0)\,\psi_i(\xi)\,|P,S\rangle\Big|_{\xi^+=\boldsymbol{\xi}_T=0}$$



$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\ e^{ip\cdot\xi}\langle P,S|\,\overline{\psi}_j(0)\,\psi_i(\xi)\,|P,S\rangle$$

The gauge link

Need of a gauge link

$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ip\cdot\xi}\langle P,S|\,\bar{\psi}_j(0)\,\psi_i(\xi)\,|P,S\rangle$$

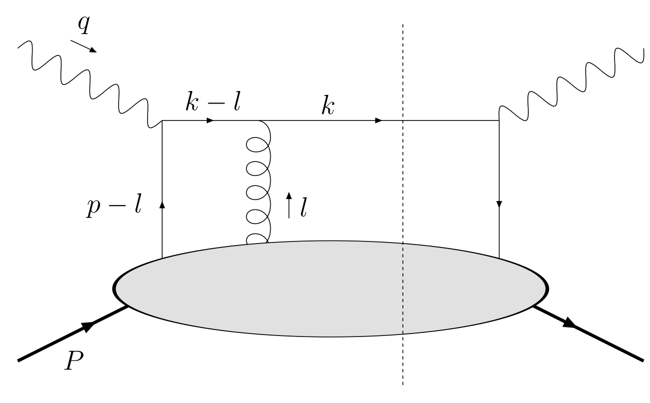
not invariant under $\psi(\xi) \to e^{i\alpha(\xi)}\,\psi(\xi)$

$$\Phi_{ij}(p,P,S) = \frac{1}{(2\pi)^4}\int d^4\xi\; e^{ip\cdot\xi}\langle P,S|\,\bar{\psi}_j(0)\,U_{[0,\xi]}\,\psi_i(\xi)\,|P,S\rangle$$

$$U(\xi_1,\xi_2) \to e^{i\alpha(\xi_1)}\,U(\xi_1,\xi_2)\,e^{-i\alpha(\xi_2)}.$$

$$U_{[a,b]} = \mathcal{P}\exp\left[-ig\int_a^b d\eta^\mu A_\mu(\eta)\right]$$

Birth of the gauge link (Feynman gauge)



(a)

$$2MW_{\mu\nu}^{(a)} \sim \int d^4l \int \frac{d^4\eta}{(2\pi)^4}\, e^{il\cdot(\eta-\xi)} \langle P,S|\overline{\psi}(0)\gamma_\mu\gamma^+ \gamma_\alpha \frac{\not{k}-\not{l}}{(k-l)^2+i\epsilon}\gamma_\nu g A^\alpha(\eta)\psi(\xi)|P,S\rangle$$

$$i\,\frac{\not{k}-\not{l}}{(k-l)^2+i\epsilon} \approx i\,\frac{k^-\gamma^+}{-2l^+k^-+i\epsilon} \approx \frac{i}{2}\,\frac{\gamma^+}{-l^++i\epsilon}$$

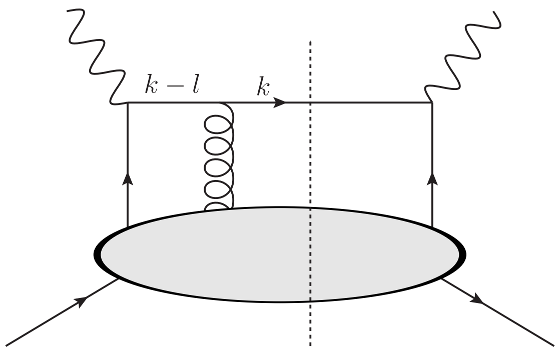
eikonal approximation

$$2MW_{\mu\nu}^{(a)} \sim \int \frac{d\eta^-}{2\pi} \int dl^+ e^{il^+(\eta^- - \xi^-)}\, \langle P,S|\overline{\psi}(0)\gamma_\mu\gamma^+ \frac{\gamma^-\gamma^+}{2}\gamma_\nu (ig)\frac{A^+(\eta)}{-l^++i\epsilon}\psi(\xi)|P,S\rangle\Big|_{\substack{\eta^+=\xi^+,\\ \boldsymbol{\eta}_T=\boldsymbol{\xi}_T}}$$

$$2MW_{\mu\nu}^{(a)} \sim \langle P,S|\overline{\psi}(0)\,\gamma_\mu\gamma^+\,\gamma_\nu\,(-ig)\int_{\infty^-}^{\xi^-} \mathrm{d}\eta^-\; A^+(\eta)\,\psi(\xi)|P,S\rangle\Big|_{\substack{\eta^+=\xi^+=0\\ \boldsymbol{\eta}_T=\boldsymbol{\xi}_T=0}}$$

Ji, Yuan, PLB 543 (02); Belitsky, Ji, Yuan, NPB656 (03)

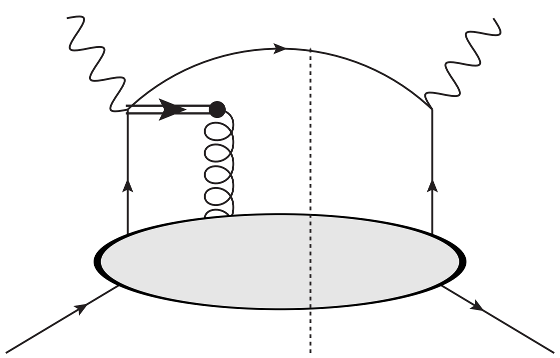
Birth of the gauge link



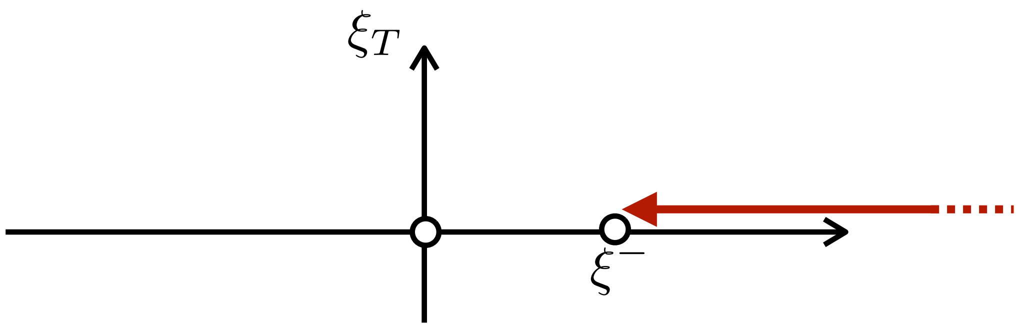
$$2MW_{\mu\nu}^{(a)} \sim \langle P,S|\overline{\psi}(0)\,\gamma_\mu\gamma^+\,\gamma_\nu\,(-ig)\int_{\infty^-}^{\xi^-} \mathrm{d}\eta^-\;\; A^+(\eta)\;\psi(\xi)|P,S\rangle$$

compare with:

$$2MW^{\mu\nu}(q,P,S) \approx \sum_q e_q^2\,\frac{1}{2}\mathrm{Tr}\left[\Phi(x_B,S)\,\gamma^\mu\gamma^+\gamma^\nu\right].$$

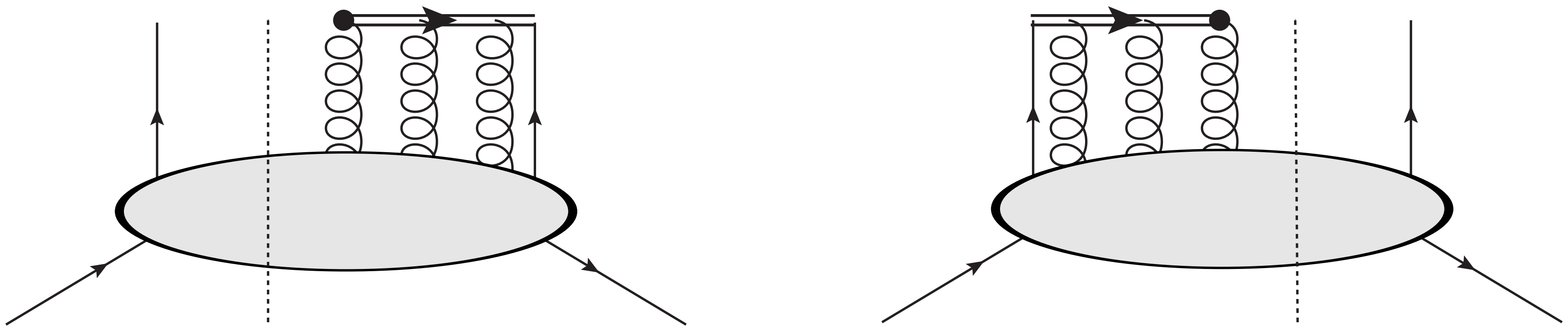


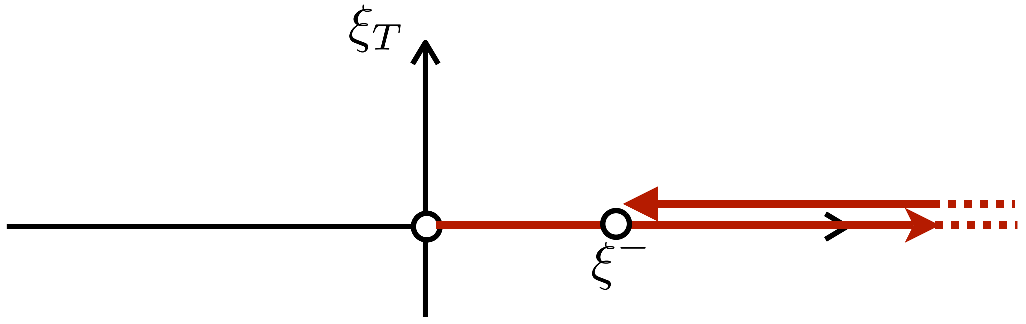
$$\Phi^{(a)}(x,S) \sim \langle P,S\big|\,\bar{\psi}(0)\,(-ig)\int_{\infty^-}^{\xi^-} \mathrm{d}\eta^-\;\; A^+(\eta)\;\psi(\xi)\,\big|P,S\rangle$$

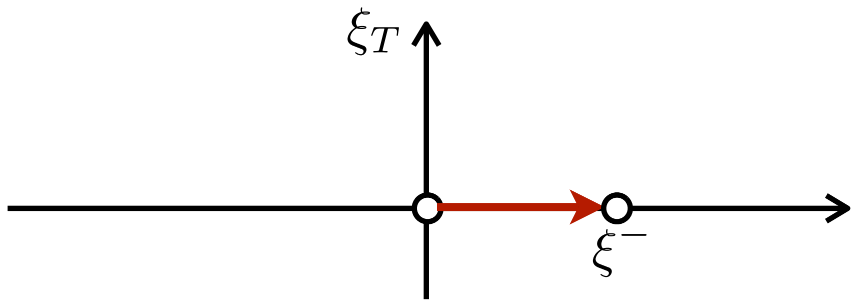


Shape of the gauge link

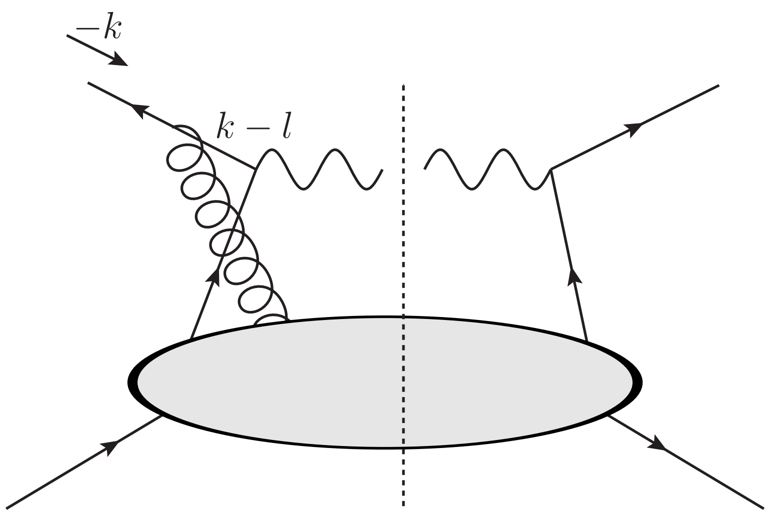
$$\Phi(x,S) \sim \langle P,S \big| \,\overline{\psi}(0)\, U_{[0,\infty^-]}\, U_{[\infty^-,\xi^-]} \psi(\xi) \,\big| P,S \rangle$$







Gauge link in Drell-Yan



$$2MW^{(a)}_{\mu\nu} \sim \int d^4l \int \frac{d^4\eta}{(2\pi)^4}\, e^{il\cdot(\eta-\xi)} \langle P,S|\overline{\psi}(0)\gamma_\mu\gamma^+\gamma_\alpha\, \frac{\not{k}-\not{l}}{(k-l)^2+i\epsilon}\,\gamma_\nu g A^\alpha(\eta)\psi(\xi)|P,S\rangle$$

$$i\,\frac{\not{k}-\not{l}+m}{(k-l)^2-m^2+i\epsilon} \approx i\,\frac{-(-k)^-\gamma^+}{2l^+(-k)^-+i\epsilon} \approx \frac{i}{2}\;\frac{\gamma^+}{-l^+-i\epsilon}$$

$$2MW^{(a)}_{\mu\nu} \sim \langle P,S|\overline{\psi}(0)\,\gamma_\mu\gamma^+\,\gamma_\nu\,(-ig)\int_{-\infty^-}^{\xi^-} \mathrm{d}\eta^-\; A^+(\eta)\;\psi(\xi)|P,S\rangle\bigg|_{\eta^+=0;\;\boldsymbol{\eta}_T=\boldsymbol{\xi}_T}$$

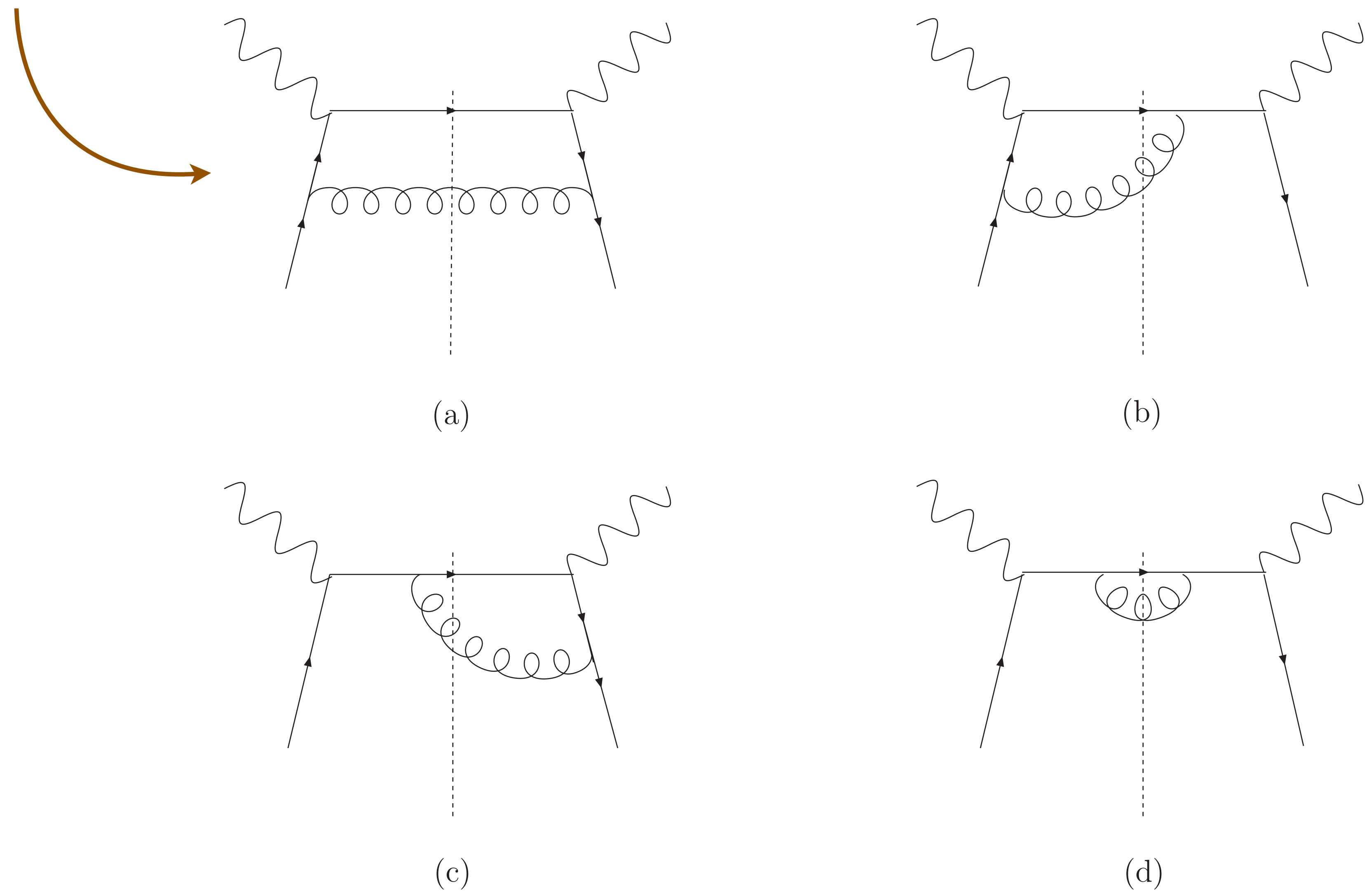
Collins, PLB 536 (02)

To summarize

- The gauge link is an essential part of the PDFs
- It comes from final-state (or initial-state) interactions, appears at leading twist, can be factorized
- Light-cone gauges where $A^{+}=0$ are appealing because the gauge link reduces to unity and we don’t have to come to terms with final-state interactions

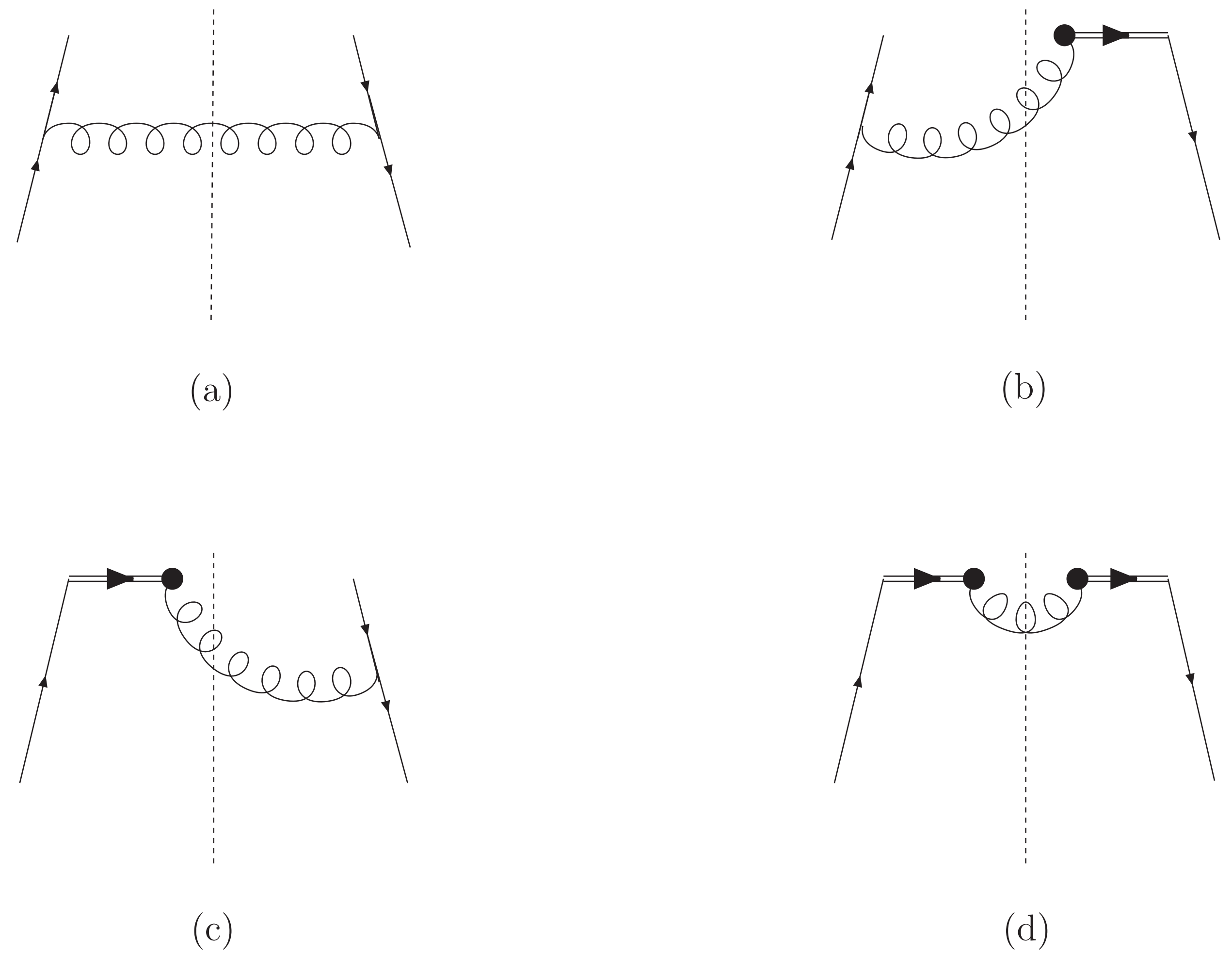
There's nothing special about the gauge link

Light-cone gauge



There’s nothing special about the gauge link

Light-cone gauge



Correlation functions and PDFs

Correlation function

$$\Phi_{ij}(x,S) = \int \frac{d\xi^-}{2\pi}\, e^{ip\cdot\xi} \langle P,S \big|\, \bar{\psi}_j(0)\, U_{[0,\xi]}\, \psi_i(\xi)\, \big| P,S\rangle \Big|_{\xi^+=\boldsymbol{\xi}_T=0}$$

Correlation function: decomposition

Available vectors $p,\ P,\ S,\ n_-$

Available Dirac matrices $\mathbf{1},\ \gamma_5,\ \gamma^\mu,\ \gamma^\mu\gamma_5,\ i\sigma^{\mu\nu}\gamma_5$ $\sigma^{\mu\nu} \equiv \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right].$

Constraints

Hermiticity: $\Phi(p,P,S) = \gamma^0\,\Phi^\dagger(p,P,S)\,\gamma^0,$ (1a)

parity: $\Phi(p,P,S) = \gamma^0\,\Phi(\tilde{p},\tilde{P},-\tilde{S})\,\gamma^0$ (1b)

$\tilde{p}^\nu = \delta^{\nu\mu}p_\mu$

see, e.g., Mulders, Tangerman, NPB 461 (96)
Goeke, Metz, Schlegel, PLB 618 (05)

Unpolarized target

$$\Phi(p, P, S|n_-) = M A_1 + \not{P} A_2 + \not{p} A_3 + \frac{i}{2M} [\not{P}, \not{p}] A_4$$
$$+ \frac{M^2}{P \cdot n_-} \not{n}_- B_1 + \frac{iM}{2P \cdot n_-} [\not{P}, \not{n}_-] B_2 + \frac{iM}{2P \cdot n_-} [\not{p}, \not{n}_-] B_3$$
$$+ \frac{1}{P \cdot n_-} \varepsilon^{\mu\nu\rho\sigma} \gamma_\mu \gamma_5 P_\nu p_\rho n_{-\sigma} B_4$$

$A_i, \ B_i$ are real scalar functions with dimension $1/[m]^4$.

If we keep only the leading terms in $1/P^+$ (leading twist)

$$\Phi(p, P) \approx P^+ (A_2 + x A_3) \not{n}_+ + P^+ \frac{i}{2M} [\not{n}_+, \not{p}_T] A_4,$$

Enter the Parton Distribution Functions

$$\Phi(p, P) \approx P^+ \left(A_2 + x A_3\right) \not{n}_+ + P^+ \frac{i}{2M} \left[\not{n}_+, \not{p}_T\right] A_4,$$

$$\Phi(x) \equiv \int d^2\boldsymbol{p}_T dp^- \, \Phi(p, P) = f_1(x) \frac{\not{n}_+}{2}$$

$$f_1(x) = 2P^+ \int d^2\boldsymbol{p}_T dp^- \left(A_2 + x A_3\right)$$

A twist on twist

OPE: for local operators
twist = dimension - spin

PDFs: “working redefinition”
twist = 2 + power of M/P^+

PDFs are nonlocal, but can be expanded in local operators, all of the same twist

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A twist on twist

- There are at least three ways to give a working definition of twist (1/P+ powers, M/Q powers with which they appear in the cross section, good/bad fields decomposition -- see later--, twist of the first nonzero moment)
- All of them correspond to the OPE definitions in the local limit, but are more flexible