Lez. 16 19-12-2013

Slide 2

① $$2 M W^{\mu\nu} = \frac{1}{2\pi} \int d^4\xi \sum_f e_f^2 \, e^{iq\cdot\xi} \langle P | [J_f^\mu(\xi), J_f^\nu(0)] | P \rangle$$

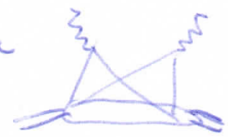
$J_f^\mu(\xi) = \bar{\psi}_f(\xi)\gamma^\mu \psi_f(\xi)$

corrente di quark libero con flavor f

OPE

$$[J^\mu(\xi), J^\nu(0)] = \mathrm{Tr}[\gamma^\mu, \gamma^\nu] \, \psi(\xi)\bar{\psi}(0) \, \psi(0)\bar{\psi}(\xi)$$
$$+ \; :\bar{\psi}(\xi)\gamma^\mu \psi(\xi)\bar{\psi}(0)\gamma^\nu\psi(0):$$
$$+ \; :\bar{\psi}(0)\gamma^\nu \psi(0)\bar{\psi}(\xi)\gamma^\mu\psi(\xi):$$
$$+ \; :\bar{\psi}(\xi)\gamma^\mu \psi(\xi) \, \bar{\psi}(0)\gamma^\nu\psi(0):$$

1° termine escluso perché corrisponde a diagramma  sconnesso

4° termine escluso perché corrisponde a 

Rimangono 2° e 3° termine dove

$\psi_f(\xi)\bar{\psi}(0) = S_F(\xi)$ propagatore di particella (con $\xi^0 > 0$)

$\psi(0)\bar{\psi}(\xi) = S_F(-\xi)$ propagatore di particella a $\xi^0 < 0$

$\Leftrightarrow$ propagatore di antiparticella a $\xi^0 > 0$

cioè

$$:\bar{\psi}(0)\gamma^\nu \psi(0)\bar{\psi}(\xi)\gamma^\mu\psi(\xi): \;\Leftrightarrow\; :\psi(\xi)\gamma^\nu\bar{\psi}(\xi)\psi(0)\gamma^\mu\bar{\psi}(0):$$

(1) continua slide 2

$$2MW^{\mu\nu} = \frac{1}{2\pi}\sum_f e_f^2 \int d^4\xi\, e^{iq\cdot\xi} \Big\{ \langle P|\bar\psi_f(\xi)\gamma^\mu\psi_f(\xi)\bar\psi_f(0)\gamma^\nu\psi_f(0)|P\rangle +$$
$$\langle P|\psi_f(\xi)\gamma^\nu\bar\psi_f(\xi)\psi_f(0)\gamma^\mu\bar\psi_f(0)|P\rangle \Big\}$$

inseriamo completezze $|0\rangle\langle 0|$

$$\int\frac{d\vec P_x}{(2\pi)^3 2P_x^0}|P_x\rangle\langle P_x| \quad \int\frac{d\vec k}{(2\pi)^3 2k^0}|k\rangle\langle k|$$

cioè stato finale visto come prodotto tensoriale di stato di 1 quark libero con vettore d'onda k e del resto ($\equiv X$)

E' giustificato da operatore di corrente J^μ per quark libero

$$2MW^{\mu\nu} = \frac{1}{2\pi}\sum_f e_f^2 \int d^4\xi \int d^4P_x \int d^4k\; e^{iq\cdot\xi}$$
$$\times\Big\{\langle P|\bar\psi_f(\xi)|0\rangle\langle 0|\gamma^\mu\psi_f(\xi)|P_x\rangle|k\rangle\langle k|\langle P_x|\bar\psi_f(0)\gamma^\nu\psi_f(0)|P\rangle +$$
$$\langle P|\psi_f(\xi)|0\rangle\langle 0|\gamma^\nu\bar\psi_f(\xi)|P_x\rangle|k\rangle\langle k|\langle P_x|\psi_f(0)\gamma^\mu\bar\psi_f(0)|P\rangle\Big\}$$

traslliamo i campi in ξ al punto 0 in modo da avere solo elementi di matrice nello stesso punto spazio-temporale

N.B. Non c'è più bisogno della contrazione, perché così i campi sono già ordinati temporalmente

$$2MW^{\mu\nu} = \frac{1}{2\pi}\sum_f e_f^2 \int d^4\xi \int d^4P_x \int d^4k\; e^{iq\cdot\xi}$$
$$\times\Big\{\langle P|e^{i\hat P\cdot\xi}\bar\psi_f(0)e^{-i\hat P\cdot\xi}|0\rangle\langle 0|\gamma^\mu e^{i\hat P\cdot\xi}\psi_f(0)e^{-i\hat P\cdot\xi}|P_x\rangle|k\rangle\langle k|\langle P_x|\bar\psi_f(0)\gamma^\nu\psi_f(0)|P\rangle$$
$$+\langle P|e^{i\hat P\cdot\xi}\psi_f(0)e^{-i\hat P\cdot\xi}|0\rangle\langle 0|\gamma^\nu e^{i\hat P\cdot\xi}\bar\psi_f(0)e^{-i\hat P\cdot\xi}|P_x\rangle|k\rangle\langle k|\langle P_x|\psi_f(0)\gamma^\mu\bar\psi_f(0)|P\rangle\Big\}$$

$$= \frac{1}{2\pi}\sum_f e_f^2 \int d^4\xi \int d^4P_x \int d^4k\; e^{i(P+q-P_x-k)\cdot\xi}$$
$$\times\Big\{\langle P|\bar\psi_f(0)|0\rangle\langle 0|\gamma^\mu\psi_f(0)|P_x\rangle|k\rangle\langle k|\langle P_x|\bar\psi_f(0)\gamma^\nu\psi_f(0)|P\rangle +$$
$$\langle P|\psi_f(0)|0\rangle\langle 0|\gamma^\nu\bar\psi_f(0)|P_x\rangle|k\rangle\langle k|\langle P_x|\psi_f(0)\gamma^\mu\bar\psi_f(0)|P\rangle\Big\}$$

① continua

slide 2

$$2MW^{\mu\nu} = \frac{1}{2\pi}\sum_f e_f^2 \int d^4\xi \int d^4P_x \int d^4k \; e^{i(q+P-P_x-k)\cdot\xi}$$

$$\times\Big\{ \langle P|\bar\psi_f(0)\gamma^\mu \psi_f(0)|k\rangle\, |P_x\rangle\langle P_x|\, \langle k|\bar\psi_f(0)\gamma^\nu\psi_f(0)|P\rangle \;+$$

$$\langle P|\psi_f(0)\gamma^\nu \bar\psi_f(0)|k\rangle\, |P_x\rangle\langle P_x|\, \langle k|\psi_f(0)\gamma^\mu\bar\psi_f(0)|P\rangle \Big\}$$

$\psi_f(0)|k\rangle \equiv u_k$ campo di particella libera con flavor f e vettore d'onda k

quindi $\psi_f(0)|k\rangle\langle k|\bar\psi_f(0) \equiv u_k\bar u_k = (\not k + m)\,\delta(k^2-m^2)\,\theta(k^0-m)$

cioè propagatore di particella libera con vettore d'onda k (se ha massa m)

particella $\to \theta(k^0-m)$ cioè propagazione ad energie positive

libera $\to$ on shell $\to \delta(k^2-m^2)$

idem per $\bar\psi_f(0)|k\rangle\langle k|\psi_f(0) \equiv v_k\bar v_k$ antiparticella

$$2MW^{\mu\nu} = \frac{1}{2\pi}\sum_f e_f^2 \int d^4\xi \int d^4P_x \int d^4k \; e^{i(q+P-P_x-k)\cdot\xi}\,\delta(k^2-m^2)\,\theta(k^0-m)$$

$$\times\Big\{ \langle P|\bar\psi_f(0)\gamma^\mu(\not k+m)|P_x\rangle\langle P_x|\gamma^\nu\psi_f(0)|P\rangle \;+$$

$$\langle P|\psi_f(0)\gamma^\nu(\not k-m)|P_x\rangle\langle P_x|\gamma^\mu\bar\psi_f(0)|P\rangle \Big\}$$

$$= \sum_f e_f^2 \int d^4P_x \int d^4k\; \delta(k^2-m^2)\,\theta(k^0-m)\;\delta(q+P-P_x-k) \times\Big\{ \;.....\; \Big\}$$

$$= \sum_f e_f^2 \int d^4P_x \int d^4k\; \delta(k^2-m^2)\,\theta(k^0-m) \int d^4p \int \frac{d^4x}{(2\pi)^4}\, e^{i(P-P_x-p)\cdot x}\,\delta(p+q-k)\times\{\ldots\}$$

$$= \sum_f e_f^2 \int d^4P_x \int d^4p\; \delta((p+q)^2-m^2)\,\theta(p^0+\nu-m) \int \frac{d^4x}{(2\pi)^4}\, e^{-ip\cdot x}\, e^{i(P-P_x)\cdot x}$$

$$\times\Big\{ \langle P|\bar\psi_f(0)\gamma^\mu(\cancel{\not k}\not p+\not q+m)|P_x\rangle\langle P_x|\gamma^\nu\psi_f(0)|P\rangle \;+$$

$$\langle P|\psi_f(0)\gamma^\nu(\cancel{\not k}\not p+\not q-m)|P_x\rangle\langle P_x|\gamma^\mu\bar\psi_f(0)|P\rangle \Big\}$$

$$= \sum_f e_f^2 \int d^4P_x \int d^4p\; \delta((p+q)^2-m^2)\,\theta(p^0+\nu-m) \int \frac{d^4x}{(2\pi)^4}\, e^{-ip\cdot x}\Big\{ \langle P|\bar\psi_f(x)\gamma^\mu(\not p+\not q+m)|P_x\rangle\langle P_x|\gamma^\nu\psi_f(0)|P\rangle$$

$$+\langle P|\psi_f(x)\gamma^\nu(\not p+\not q-m)|P_x\rangle\langle P_x|\gamma^\mu\bar\psi_f(0)|P\rangle\Big\}$$

$$= \sum_f e_f^2 \int d^4p\; \delta((p+q)^2-m^2)\,\theta(p^0+\nu-m) \int \frac{d^4x}{(2\pi)^4}\, e^{\pm ip\cdot x}\Big\{ \langle P|\bar\psi_f(x)\gamma^\mu(\not p+\not q+m)\gamma^\nu\psi_f(0)|P\rangle \;+$$

$$\langle P|\psi_f(x)\gamma^\nu(\not p+\not q-m)\gamma^\mu\bar\psi_f(0)|P\rangle \Big\}$$

① continua

$$2\pi W^{\mu\nu} = \sum_f e_f^2 \int d^4p \; \delta\left((p+q)^2 - m^2\right) \theta(p^0+\nu-m) \int \frac{d^4x}{(2\pi)^4} e^{-ip\cdot x}$$

$$\times \left\{ \langle P| \bar{\psi}_f(x) \gamma^\mu (\not{p}+\not{q}+m) \gamma^\nu \psi_f(0) |P\rangle + \langle P| \psi_f(x) \gamma^\nu (\not{p}+\not{q}-m) \gamma^\mu \bar{\psi}_f(0) |P\rangle \right\}$$

Si mettano in evidenza gli indici di Dirac per ciascun operatore nell'elemento di matrice adronico

$$2\pi W^{\mu\nu} = \sum_f e_f^2 \int d^4p \; \delta\left((p+q)^2 - m^2\right) \theta(p^0+\nu-m) \int \frac{d^4x}{(2\pi)^4} e^{-ip\cdot x}$$

$$\times \left\{ \langle P| \bar{\psi}_{f_j}(x) (\gamma^\mu)_{il} (\not{p}+\not{q}+m)_{lm} (\gamma^\nu)_{mj} \psi_{f_i}(0) |P\rangle + \langle P| \psi_{f_j}(x) (\gamma^\nu)_{il} (\not{p}+\not{q}-m)_{lm} (\gamma^\mu)_{mj} \bar{\psi}_{f_i}(0) |P\rangle \right\}$$

$$= \sum_f e_f^2 \int d^4p \; \delta\left((p+q)^2 - m^2\right) \theta(p^0+\nu-m) \, \mathrm{Tr}\Big[(\Phi_f)_{ij}(p,P) (\gamma^\mu)_{il} (\not{p}+\not{q}+m)_{lm} (\gamma^\nu)_{mj} + (\bar{\Phi}_f)_{ij}(p,P) (\gamma^\nu)_{il} (\not{p}+\not{q}-m)_{lm} (\gamma^\mu)_{mj} \Big]$$

con
$$(\Phi_f)_{ij}(p,P) = \int \frac{d^4x}{(2\pi)^4} e^{-ip\cdot x} \langle P| \bar{\psi}_{f_j}(x) \, \psi_{f_i}(0) |P\rangle$$

$$(\bar{\Phi}_f)_{ij}(p,P) = \int \frac{d^4x}{(2\pi)^4} e^{-ip\cdot x} \langle P| \psi_{f_j}(x) \, \bar{\psi}_{f_i}(0) |P\rangle$$

in termini diagrammatici