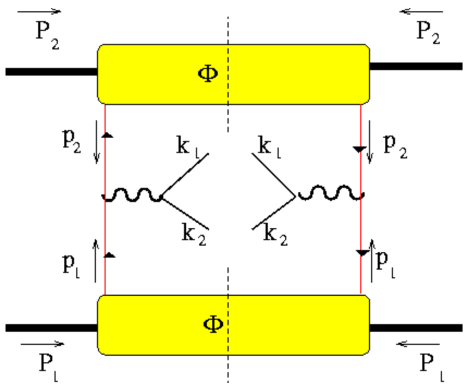
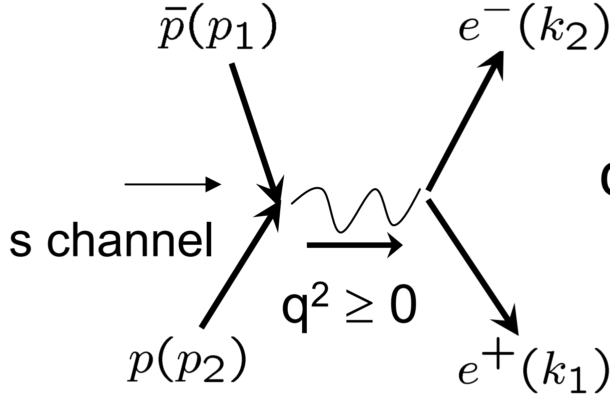
Riassunto della lezione precedente

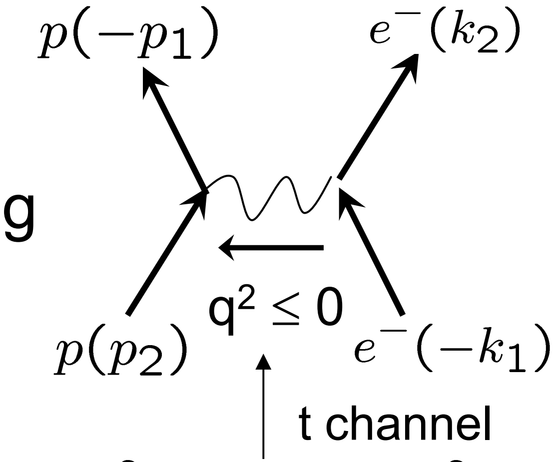
- GPD a twist=2: proprietà, interpretazione intuitiva, trasformata di Fourier nel parametro di impatto $\mathbf{b}_\perp$ e localizzazione dei partoni nel piano $\perp$
- generalizzazione della momentum sum rule per operatori "tower" di tipo vettoriale (γ^μ), assiale-vettoriale ($\gamma^\mu\gamma_5$), etc..
- polinomialità delle GPD non polarizzate H,E
- def. gauge-invariante di operatori momento (da tensore energia-impulso $T^{\mu\nu}$) e momento angolare (da tensore momento angolare $M^{\lambda\mu\nu}$)
- polinomialità : fattori di forma generalizzati $\leftrightarrow$ regola di somma di momento e di spin
- scomposizione gauge-invariante di operatore momento angolare: definizione di momento angolare orbitale
- regola di somma di spin e sua evoluzione con scala μ^2

"exclusive" Drell-Yan





crossing



$$\tau = \frac{M^2}{s} = 1$$

$$\begin{aligned} q^2 &= (k_1 + k_2)^2 \\ &= (p_1 + p_2)^2 = s \\ t &= (k_1 - p_2)^2 \to \theta \end{aligned}$$

$$\begin{aligned} q^2 &= (k_1 - k_2)^2 \\ &= (p_1 - p_2)^2 = t \\ s &= (k_1 + p_2)^2 \end{aligned}$$

4 vectors ($k_1+k_2 \to p_1+p_2$) $\to$ 8 elements in J^μ : $(\gamma^\mu, k_1^\mu, k_2^\mu, p^\mu) \otimes (1, \not{q})$

$p=(p_1+p_2)/2$
$q=(p_1-p_2)$

$q \cdot J = 0 \to$ 6 elements $\to$ 3 complex form factors
a possible parametrization:

N.B. eliminate p^μ and $\not{q}\, p^\mu$

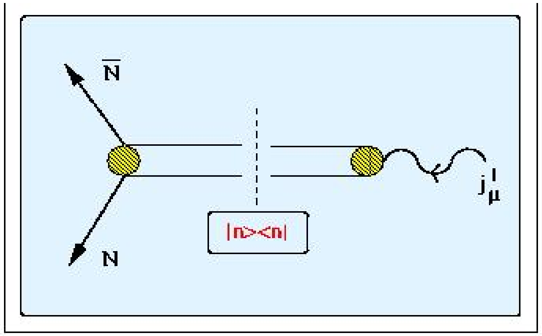
$$\begin{aligned} J_\mu^v &= \bar{v}(p_1)\, \Gamma^\mu \left[G_E(q^2),\, G_M(q^2) \right] u(p_2) \\ J_\mu^a &= \bar{v}(p_1)\, \gamma^\mu \gamma_5\, A(q^2)\, u(p_2) \end{aligned}$$

$G_E(q^2)$, $G_M(q^2)$ $\xrightarrow[\text{continuation}]{\text{analytic}}$ $G_E(Q^2=q^2e^{-i\pi})$, $G_M(Q^2=q^2e^{-i\pi})$

$A(q^2)$ 0

$q^2 \geq 0$ complex $q^2 \leq 0$ real

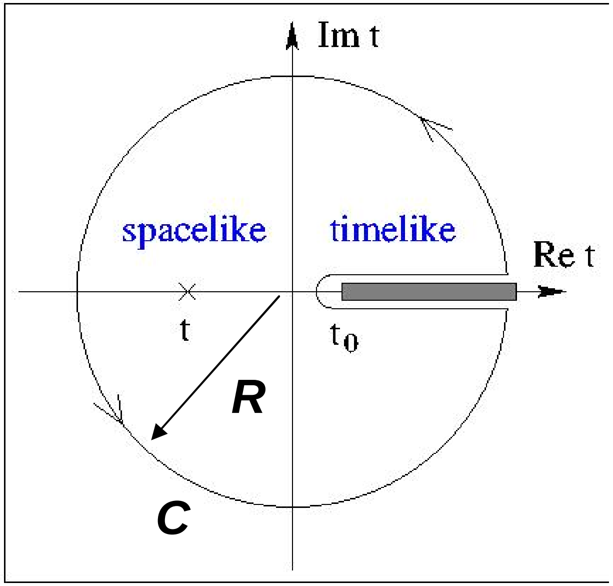
$$Im\,\langle N(p)\bar{N}(p')|J^\mu|0\rangle$$
$$\sim \sum_n \langle N(p)\bar{N}(p')|n\rangle\langle n|J^\mu|0\rangle$$
$$\Rightarrow Im\,F(q^2)$$



hermitean Hamiltonian

vector-meson poles and
multi-hadron continuum

Dispersion Relation



F(t) analytic function in $t \in \mathbb{C}$ with
cut $[t_0 = 4m_\pi^2, \infty)$

$$F(t) = \lim_{R\to\infty} \frac{1}{2\pi i} \oint_C dz \frac{F(z)}{z-t} = \int_{t_0}^{\infty} dt' \frac{Im F(t')}{t'-t}$$

Im F(t') $\neq 0$ only in $[t_0=4m_\pi^2, \infty)$

question in space-like domain

elastic scattering cross section $N\,(e,e')$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\,E'\,\cos^2\frac{\theta}{2}}{4E^3\,\sin^4\frac{\theta}{2}}\left[\frac{G_E^2+\tau G_M^2}{1+\tau} + 2\tau\,G_M^2\,\tan^2\frac{\theta}{2}\right]$$

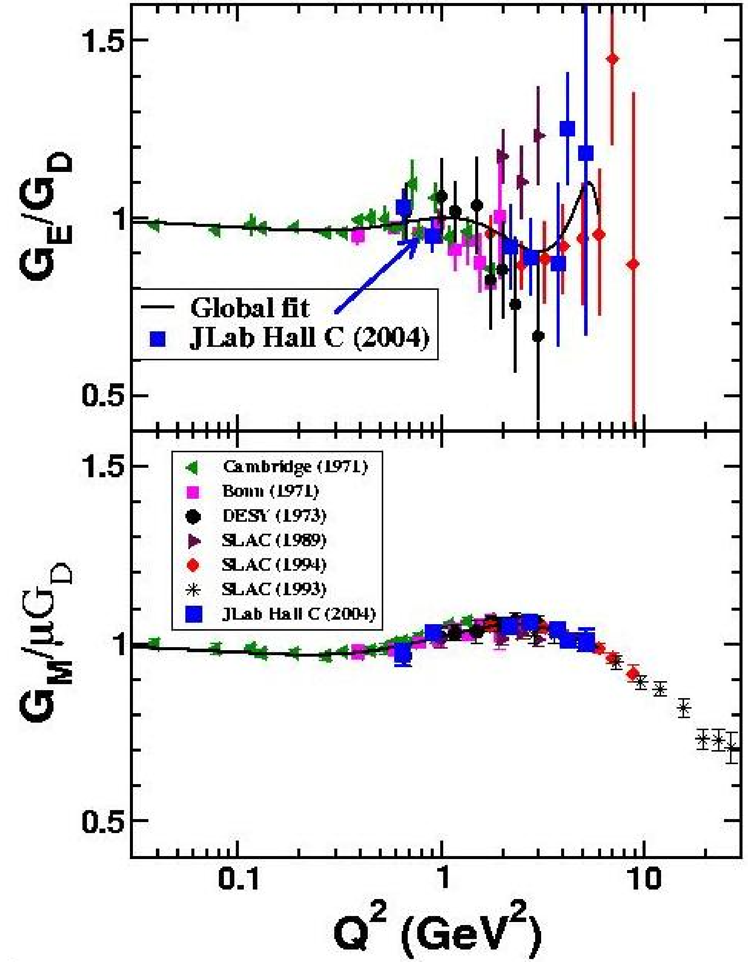
Rosenbluth separation

$$\sigma_R = \frac{\epsilon\,(1+\tau)}{\tau\,\sigma_{\text{Mott}}}\,\frac{d\sigma}{d\Omega} = \frac{\epsilon}{\tau}G_E^2 + G_M^2$$

fixed Q^2, vary $\epsilon = \left[1+2(1+\tau)\tan^2\frac{\theta_e}{2}\right]^{-1}$

G_E slope , G_M intercept

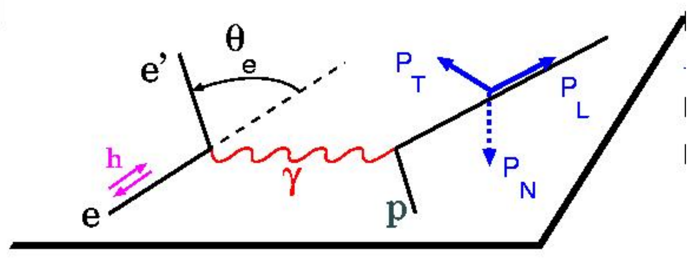
$$\tau = \frac{Q^2}{4M^2} \;\Rightarrow\; \frac{\epsilon}{\tau} \sim \frac{1}{Q^2} + \dots$$ large errors in G_E



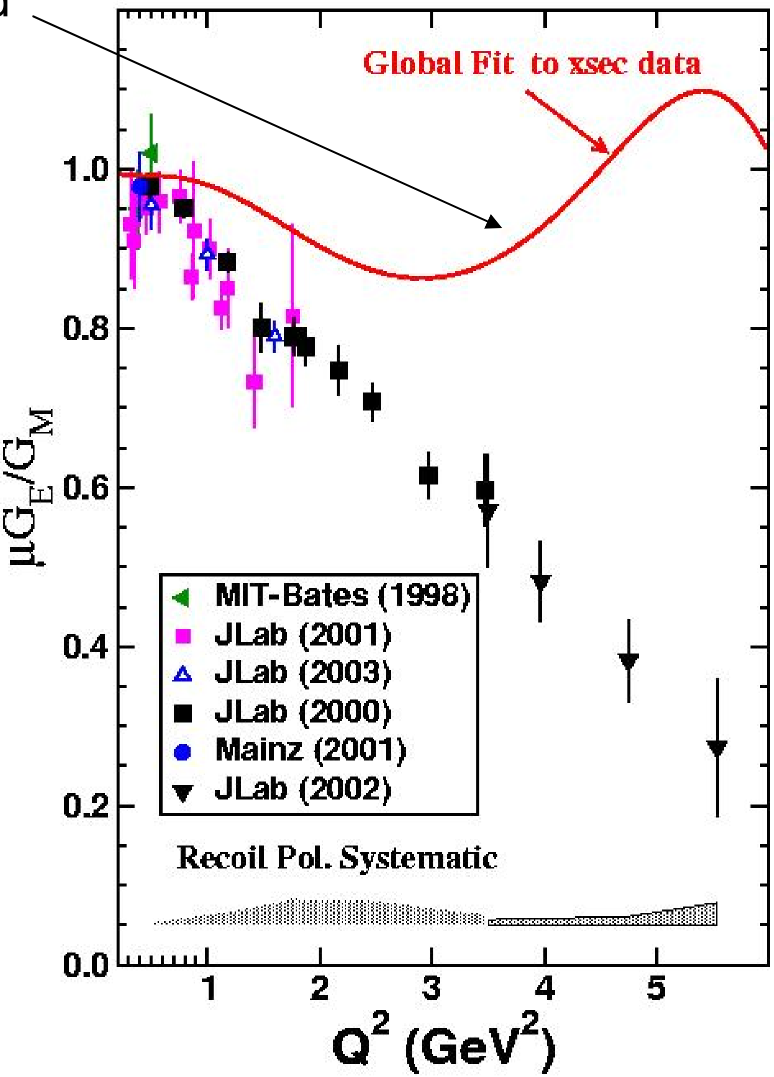
polarization transfer method

Rosenbluth method

$$^1H\,(\vec{e},e'\vec{p})$$

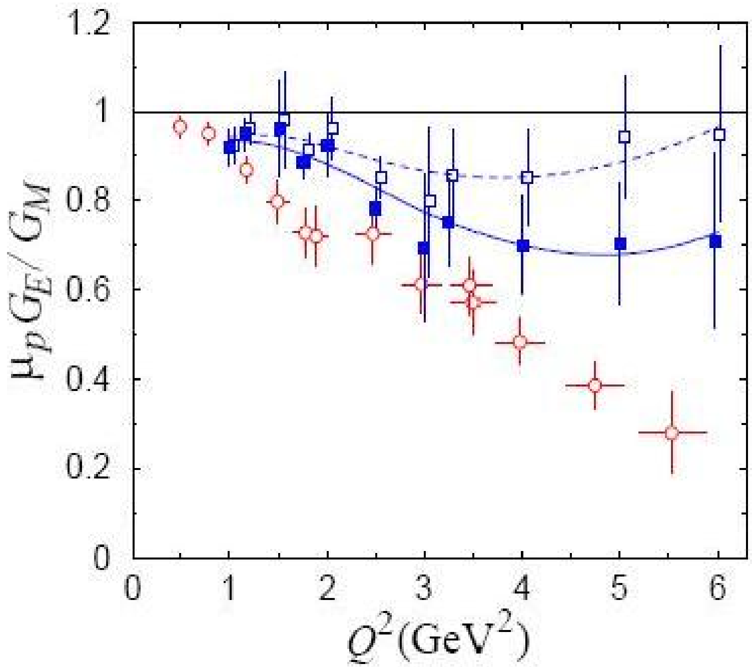


$$\frac{G_E}{G_M} = -\frac{P_T}{P_L}\sqrt{\frac{\tau(1+\epsilon)}{2\epsilon}}$$



space-like form factor puzzle

Blunden *et al.*
P.R. C**72** (05) 034612



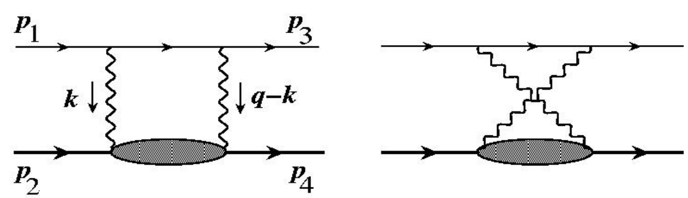
1. Rosenbluth method

2. after correction for nonBorn (2γ)

?

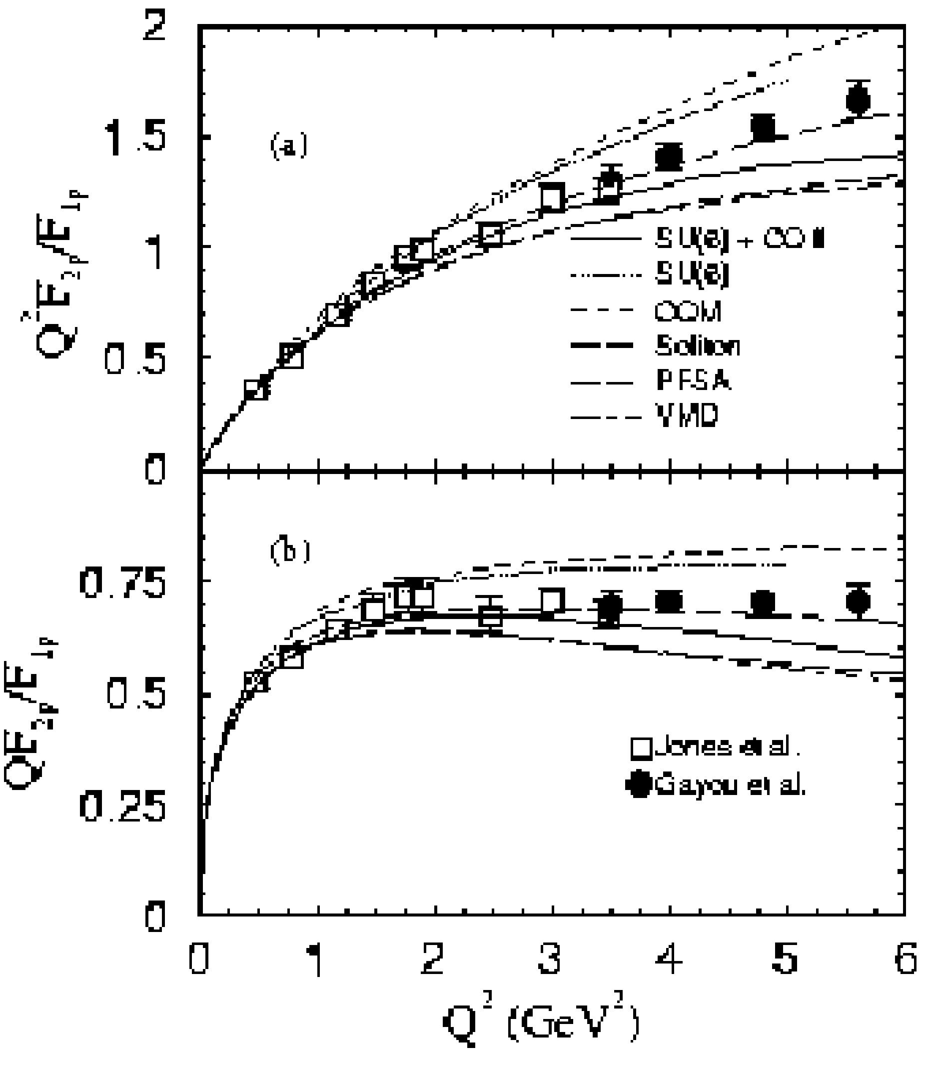
3. polarization transfer method

exp's repeated and checked for consistency; see also
JLab – Hall A   Qattan *et al.*
P.R.L. **94** (05) 142301



$$\sigma_R = \frac{\epsilon}{\tau} G_E^2 + G_M^2 + \epsilon\, \sigma_{2\gamma}(\epsilon, Q^2)$$

space-like form factor puzzle: where is onset of QCD scaling?



pQCD $$\frac{F_2}{F_1} \to \frac{1}{Q^2}$$

JLab polar. transfer data

$$\frac{F_2}{F_1} \sim \frac{1}{Q}$$

$Q^2 \leq 10$ no scaling!

exclusive Drell-Yan cross section

parametrizzazione dati sperimentali (analisi di Fourier)

twist 2

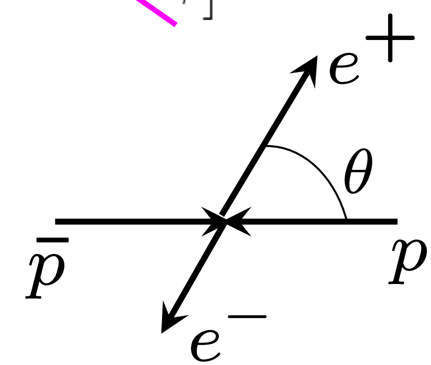
$$\int d\phi \quad \frac{1}{\sigma}\frac{d\sigma}{d\Omega} = \frac{3}{4\pi}\frac{1}{\lambda+3}\Big[\underbrace{1+\lambda\cos^2\theta}_{\text{annichilazione collineare} \sim \text{QPM} \quad R_T} + \cancel{\mu\sin^2\theta\cos\phi}_{\text{1 spin-flip} \quad R_{LT}} + \cancel{\frac{\nu}{2}\sin^2\theta\cos 2\phi}_{\text{2 spin-flip} \quad h_1^\perp \otimes \bar{h}_1^\perp \quad R_{TT}}$$

$$+\,\alpha\cos\theta + \cancel{\beta\sin\theta\cos\phi + \gamma\sin\theta\sin\phi}\Big]$$

$\lambda \sim 1 >> \mu$; $\nu \sim 0.3$

$1-\lambda \neq 2\nu$ violazione regola di Lam-Tung

$$\frac{d\sigma}{d\cos\theta} = a(q^2)\left(1+R\cos^2\theta\right) - b(q^2)\,\mathrm{Re}[G_M\,A^*]\,\cos\theta$$



$$a(q^2) = \frac{\alpha^2\pi}{2q^2}\frac{1}{\tau}\sqrt{1-\frac{1}{\tau}}\left(\tau|G_M|^2+|G_E|^2\right)$$

$$b(q^2) = \frac{2\pi\alpha^2}{q^2}\frac{\tau-1}{\tau}, \qquad \tau = \frac{q^2}{4M^2}, \qquad R = \frac{\tau|G_M|^2-|G_E|^2}{\tau|G_M|^2+|G_E|^2} = \frac{\tau-r}{\tau+r}$$

$$\sigma_{tot} = a(q^2)\left(2+\frac{2}{3}R\right)$$

R angular asymmetry $\quad r = \dfrac{|G_E|^2}{|G_M|^2}$

(continua)

$$R=\frac{\tau|G_M|^2-|G_E|^2}{\tau|G_M|^2+|G_E|^2}=\frac{\tau-r}{\tau+r}\qquad r=\frac{|G_E|^2}{|G_M|^2}$$

$$\sigma_{tot}=\frac{4\pi\alpha^2}{3q^2}\sqrt{1-\frac{1}{\tau}}\left(|G_M|^2+\frac{1}{2\tau}|G_E|^2\right)$$

$$\Big\} \quad |G_M|\;, \qquad |G_E|$$

$$\frac{d\sigma}{d\cos\theta} \;=\; a(q^2)\left(1+R\cos^2\theta\right)-b(q^2)\,\mathrm{Re}[G_M\,A^*]\,\cos\theta$$

$q^2 \leq 0$ Rosenbluth plot
change E, θ_e at fixed q^2
$\rightarrow$ linear plot in ε

$q^2 \geq 0$ measure θ at fixed q^2
$\rightarrow$ get $R \rightarrow r$
$\cos^2\theta$ typical of Born diagram

non $\cos^2\theta$
$\rightarrow$ explore 2γ mechanisms?

need good coverage over whole range in θ

$$G_E(Q^2) \;=\; F_1(Q^2)+\tau F_2(Q^2) \qquad \text{with } \tau=\frac{q^2}{4M^2}$$
$$G_M(Q^2) \;=\; F_1(Q^2)+F_2(Q^2)$$

threshold τ =1 $\Rightarrow$ $|G_E|=|G_M|$

poor exp. angular coverage; data mainly from σ_{tot} with $|G_E|=|G_M|$
true only at threshold $\rightarrow$ $|G_E|$ is unknown!

polarized exclusive Drell-Yan cross section

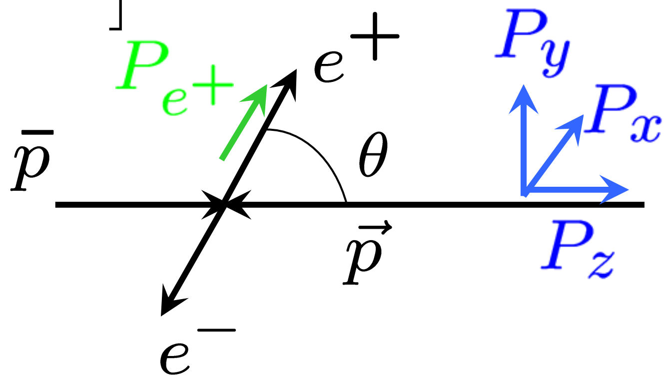
$$\bar{p}\,\vec{p} \longrightarrow e^+ e^-$$

$$A_y = \frac{1}{P_y}\frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{b(q^2)}{2\sqrt{\tau-1}\,d\sigma^o} \times \sin\theta \left[\cos\theta\,\mathrm{Im}[G_M\,G_E^*] - \sqrt{\frac{\tau-1}{\tau}}\,\mathrm{Im}[G_E\,A^*]\right]$$

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^o}{d\Omega}\left(1 + \vec{A}\cdot\vec{P}\right)$$

$$A_x = -P_{e+}\frac{b(q^2)}{2\sqrt{\tau-1}\,d\sigma^o}\,2\sin\theta\,\mathrm{Re}[G_M\,G_E^*]$$

$$A_z = P_{e+}\frac{b(q^2)\sqrt{\tau}}{2\sqrt{\tau-1}\,d\sigma^o}\,2\cos\theta\,|G_M|^2$$



A_x, A_z require polarization of the electron: $P_{e+} \neq 0$

FSI $\rightarrow$ T-odd mechanisms are allowed : $\vec{p}_{e+} \times \vec{p} \cdot \vec{P}$ generates $A_y \neq 0$

phases of FF from residual interactions (FSI) of baryon system :
interference of channels with different phases (*Im* $[G_E^* G_M]$)

$$A_y \propto Im(G_E^* G_M) \propto \sin(\delta_M - \delta_E)$$

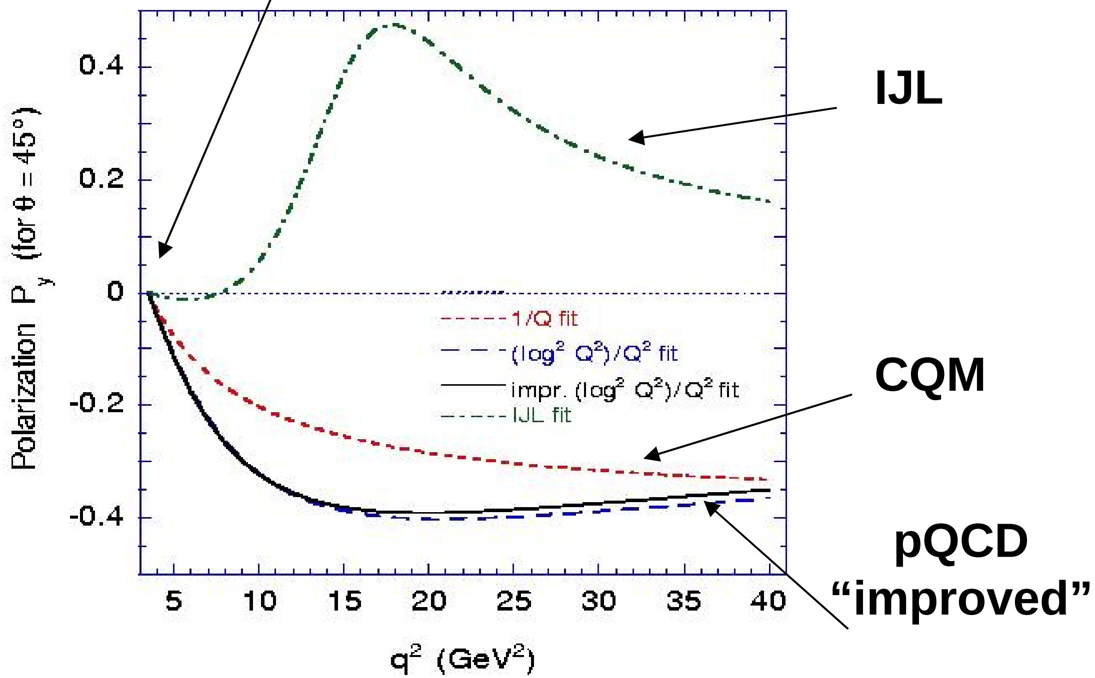
ambiguity $\delta \leftrightarrow \pi - \delta$ solved by

$$\frac{A_y}{A_x} \propto -\frac{Im(G_E^* G_M)}{Re(G_E^* G_M)} \propto \tan(\delta_E - \delta_M)$$

Interesting model selectivity

threshold $q^{o2}=4M^2$ $|G_E|=|G_M| \Rightarrow \mathrm{Im}\,[G_E^* G_M] = 0$

A_y



Brodsky *et al.*
P.R. D**69** (04) 054022

polarized data would discriminate among models giving the same space-like results

7

but no polarization data available → phases of $G_{E/M}$ unknown !

puzzles in the time-like domain

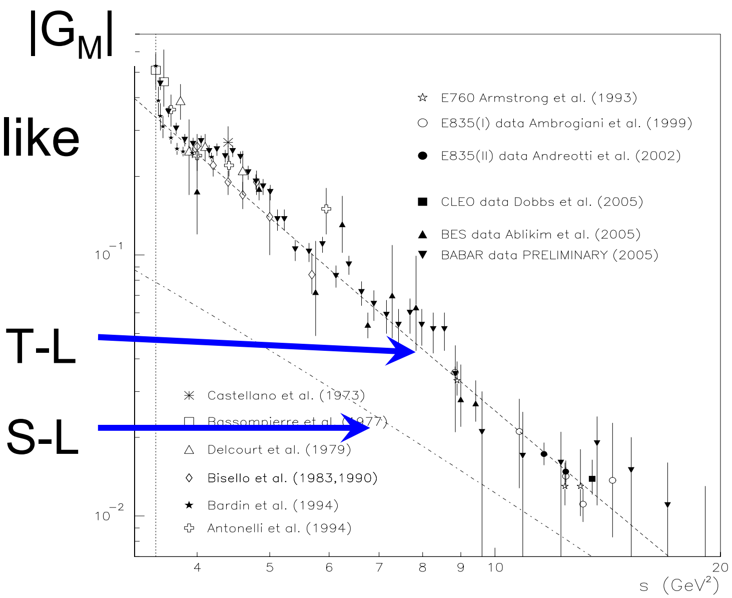
- space like $\leftarrow$ analytic continuation $\rightarrow$ time-like

$$\lim_{q^2 \to \infty} |G_M(q^2 \geq 0)| = G_M(-q^2 \geq 0)$$

but $|G_M^p(q^2)| \approx 2\, G_M^p(-q^2)$

fit to pQCD T-L

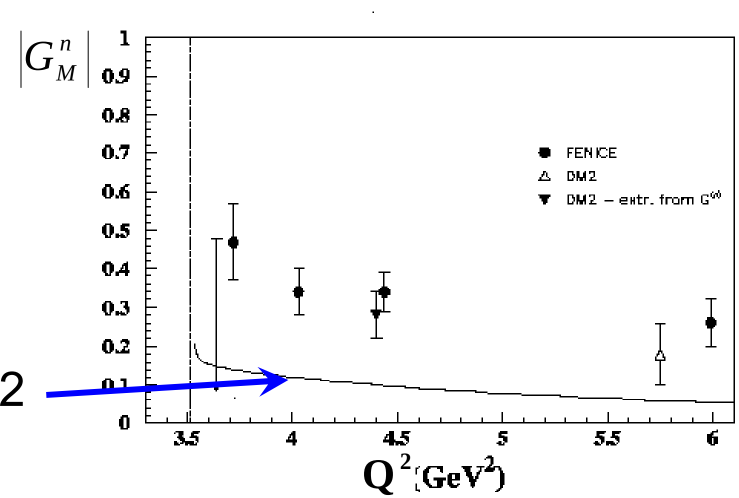
fit to pQCD S-L



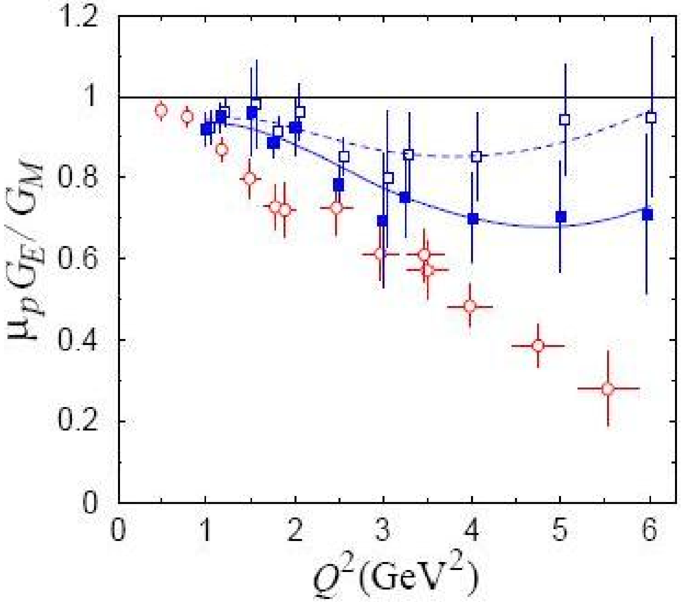
only 1 measurement for neutron (ADONE-1998)
again with $|G_E^n| = |G_M^n|$

- pQCD + analyticity $\rightarrow \left| \dfrac{G_M^n}{G_M^p} \right|^2 \approx \left(\dfrac{e_d}{e_u} \right)^2 = \dfrac{1}{4}$

but $|G_M^p| \leq |G_M^n|$ fit to $|G_M^p| / 2$

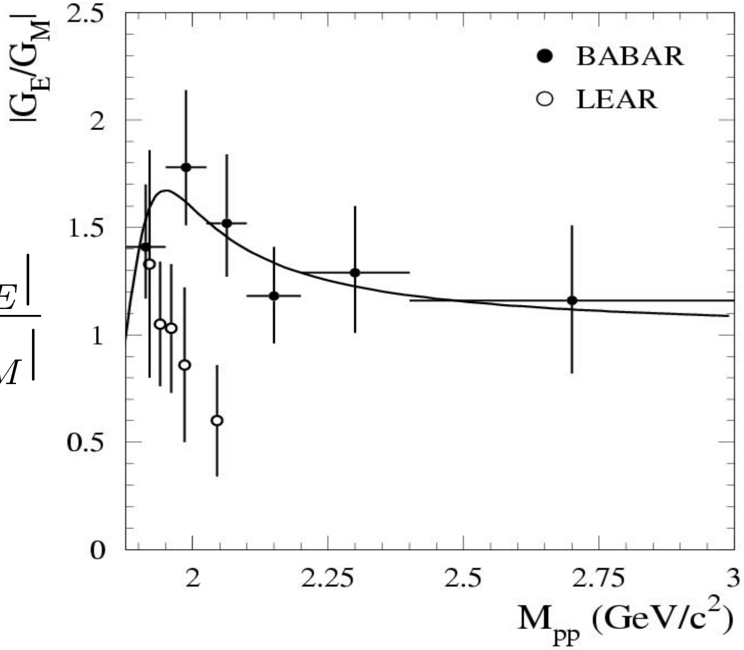


continued



space-like

$$\frac{G_E}{G_M} < 1 < \frac{|G_E|}{|G_M|}$$

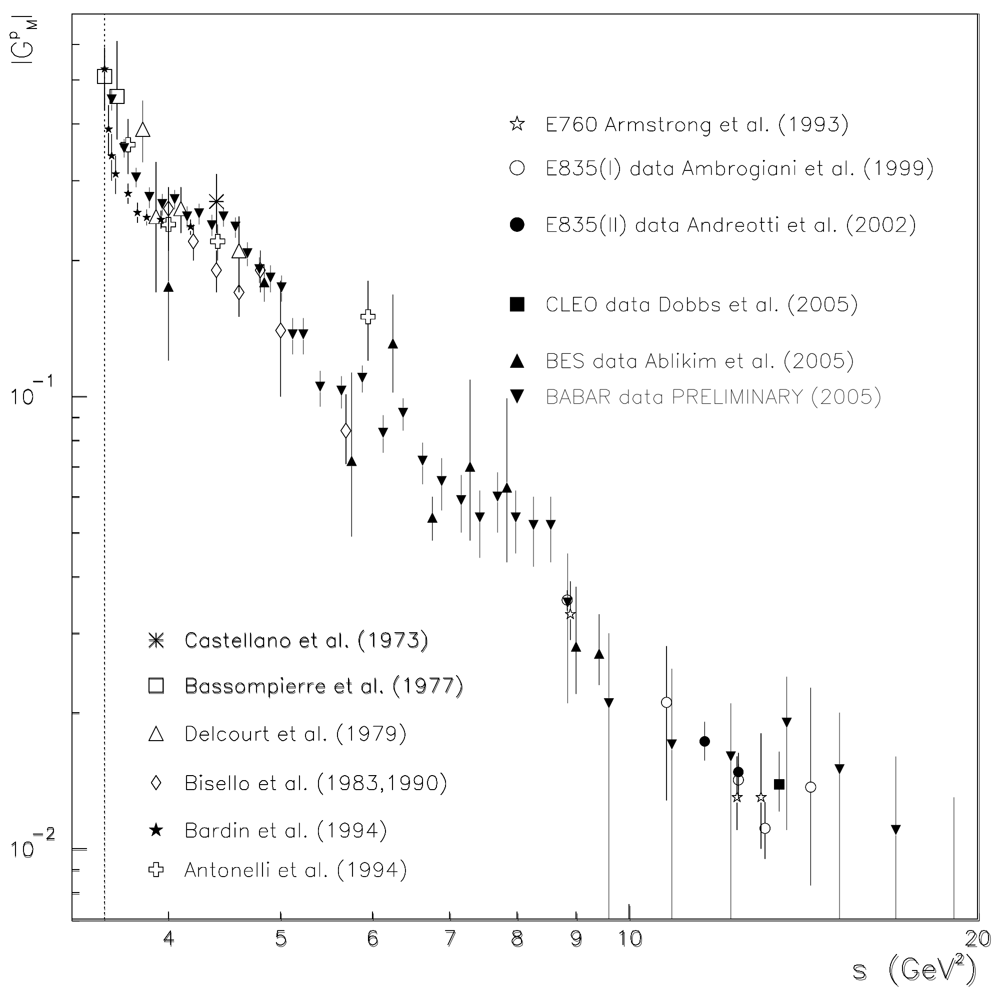


time-like

World data

$$e^+e^- \longrightarrow p\bar{p}$$
$$p\bar{p} \longrightarrow e^+e^-$$

$|G_M^p|$



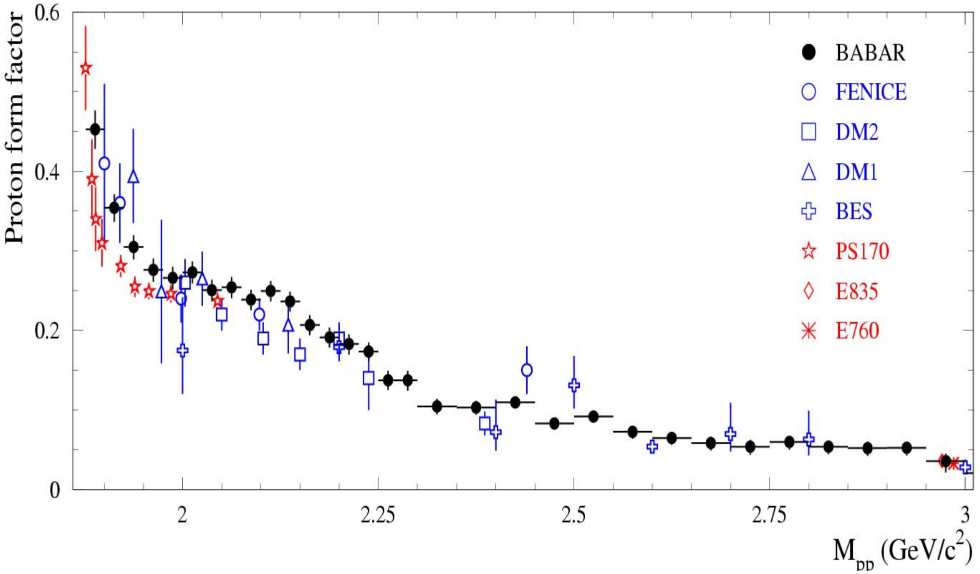
ADONE	Q^2 = 4.4 GeV	(1973)
CERN	Q^2 ~ 3.6	(1977)
Orsay-DM1	Q^2 ~ 3.75-4.56	(1979)
Orsay-DM2	Q^2 =4-5	(1983)
LEAR	Q^2 ~3.5-4.2	(1994)
E760	Q^2 ~8.9-13	(1993)
FENICE	Q^2 ~3.7-6	(1994)
E835	Q^2 ~8.8-18.4	(1999)
	11.6-18.2	(2003)
CLEO	Q^2 ~11-12	(2005)
BES	Q^2 ~4-9	(2005)
BaBar	Q^2 ~2-20	(2005)

below threshold ?

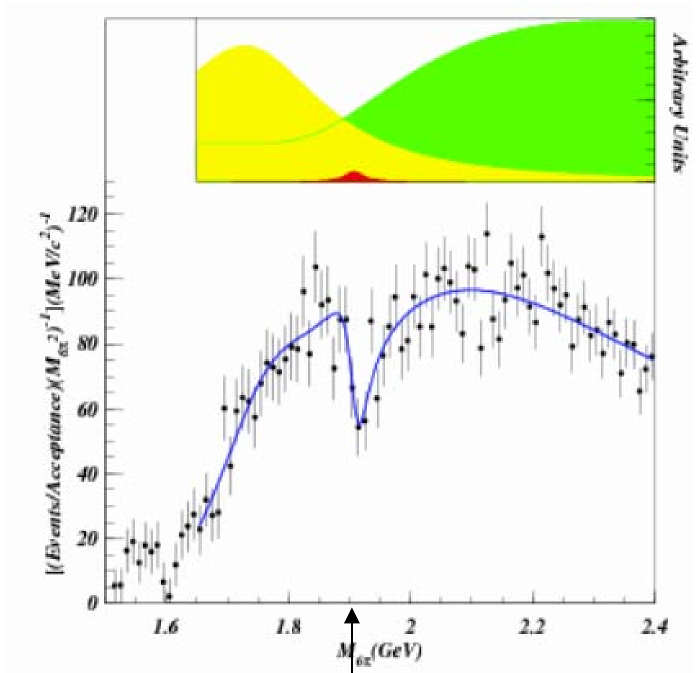
world data for $|G_M|$ show steep rise at threshold

↓

tail of resonance at $q^2 \leq 4M^2$:
$\rho(1900)$ or baryonium or..?



should show up as a dip in some hadronic cross section



E687 diffractive photoproduction of 6π

W $\sim$ 1.9 GeV