light-cone (LC) variables

4-vector a^μ

$$a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3) \quad , \quad \mathbf{a}_\perp = (a^1, a^2)$$

$$a^\mu = (a^0, a^1, a^2, a^3) = (a^+, a^-, \mathbf{a}_\perp)$$

scalar product

$$a \cdot b = a^+ b^- + a^- b^+ - \mathbf{a}_\perp \cdot \mathbf{b}_\perp$$

$$a^2 = 2a^+ a^- - \mathbf{a}_\perp^2$$

metric

$$g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}_{(0,1,2,3)} \rightarrow \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}_{(+,-,1,2)}$$

LC "basis":

$$n_+^\mu = (1, 0, \mathbf{0}_\perp) \quad , \quad n_-^\mu = (0, 1, \mathbf{0}_\perp) \quad ; \quad n_\pm^2 = 0\ , \ n_+ \cdot n_- = 1$$

$$a^\pm = a \cdot n_\mp \longrightarrow a^\mu = (a \cdot n_-)\, n_+^\mu + (a \cdot n_+)\, n_-^\mu + \mathbf{a}_\perp$$

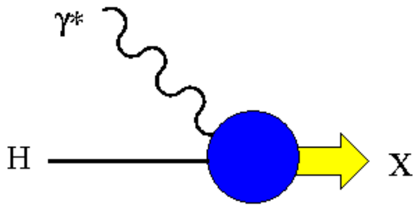
"transverse" metric

$$g_\perp^{\mu\nu} = g^{\mu\nu} - n_+^\mu n_-^\nu - n_+^\nu n_-^\mu = g^{\mu\nu} - n_+^{\{\mu} n_-^{\nu\}}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}_{(+,-,1,2)}$$

hadron target at rest $P^\mu \overset{restframe}{=} (M,0,0,0) = \frac{1}{\sqrt{2}}(M,M,\mathbf{0}_\perp)$

inclusive DIS



target absorbes momentum from γ^* ; for example,

2 if **q** || z $P_z=0 \rightarrow P'_z = q \gg M$ in DIS regime

$$P'^{\,\mu} = (\sqrt{M^2 + P'^{\,2}_z}, 0, 0, P'_z) \overset{Q^2\to\infty}{\longrightarrow} (P'_z, 0, 0, P'_z) = (\sqrt{2}\,P'_z, 0, \mathbf{0}_\perp)$$

DIS regime ⇒ direction "+" dominant
direction "-" suppressed

boost of 4-vector $a^\mu \rightarrow a'^\mu$ along z axis

$$a'^0 = \frac{a^0 + \beta\, a^3}{\sqrt{1-\beta^2}} \qquad a'^3 = \frac{\beta\, a^0 + a^3}{\sqrt{1-\beta^2}} \qquad \mathbf{a}'_\perp = \mathbf{a}_\perp$$

$$a'^+ = \frac{1}{\sqrt{2}} \frac{(1+\beta)\,(a^0 + a^3)}{\sqrt{1-\beta^2}} = a^+ \sqrt{\frac{1+\beta}{1-\beta}} = a^+ e^{\psi}$$

$$a'^- = a^- \sqrt{\frac{1-\beta}{1+\beta}} = a^- e^{-\psi}$$

N.B. rapidity

$$\psi = \frac{1}{2} \log\left(\frac{1+\beta}{1-\beta}\right) \Rightarrow \beta = \tanh\psi$$

$$P^\mu = \frac{1}{\sqrt{2}}(M, M, \mathbf{0}_\perp)$$

boost along z axis

$$P'^{\,\mu} = \frac{1}{\sqrt{2}}\left(A, \frac{M^2}{A}, \mathbf{0}_\perp\right)$$

$A = M \rightarrow$ hadron rest frame

$A = Q \rightarrow$ **I**nfinite **M**omentum **F**rame (IFM)

$$P^\mu = \frac{1}{\sqrt{2}}\left(A,\, \frac{M^2}{A},\, \mathbf{0}_\perp\right)$$

A = *M* → hadron rest frame
A = *Q* → **I**nfinite **M**omentum **F**rame (IFM)

LC kinematics ⇔ boost to IFM

$$p^\mu = \frac{1}{\sqrt{2}}\left(xA,\, \frac{p^2+\mathbf{p}_\perp^2}{xA},\, \mathbf{p}_\perp\right)$$

definition : $x = \frac{p^+}{P^+}$

fraction of LC ("longitudinal") momentum

$$p^2 = 2p^+p^- - \mathbf{p}_\perp^2$$

in QPM $x \sim x_B$

$$q^\mu = \frac{1}{\sqrt{2}}\left(-x_N A,\, \frac{Q^2}{x_N A},\, \mathbf{0}_\perp\right)$$

it turns out $x_N \sim x_B + o(M/Q)$
LC components not suppressed

$$q^2 = 2q^+q^- - \mathbf{q}_\perp^2 = -Q^2$$

Quantum Field Theory on the light-cone

rules at time x^0=t=0 evolution in x^0	→	Rules at "light-cone" time x^+=0 evolution in x^+
variables **x**	→	x^- , $\mathbf{x}_\perp$
conjugated momenta **k**	→	k^+ , $\mathbf{k}_\perp$
Hamiltonian k^0	→	k^-
field quantum	→	

$$\psi(x) = \int \frac{d\mathbf{k}}{(2\pi)^3 2k^0}\, b_\alpha(\mathbf{k})\, u(\mathbf{k})\, e^{-ik\cdot x} + d^\dagger_\alpha(\mathbf{k})\, v(\mathbf{k})\, e^{ik\cdot x}$$

$$\psi_+(x) = \int \frac{dk^+ d\mathbf{k}_\perp}{(2\pi)^3 k^+}\, b_\alpha(\mathbf{k})\, u_+(\mathbf{k})\, e^{-ik\cdot x} + d^\dagger_\alpha(\mathbf{k})\, v_+(\mathbf{k})\, e^{ik\cdot x}$$

$$[b_\alpha(\mathbf{k}),\, b^\dagger_{\alpha'}(\mathbf{k}')] = (2\pi)^3\, 2k^0\, \delta(\mathbf{k}-\mathbf{k}')\, \delta_{\alpha\alpha'}$$

$$[b_\alpha(\mathbf{k}),\, b^\dagger_{\alpha'}(\mathbf{k}')] = (2\pi)^3\, k^-\, \delta(k^+ - k'^+)\, \delta(\mathbf{k}_\perp - \mathbf{k}'_\perp)\, \delta_{\alpha\alpha'}$$

.....

Fock space

.....

$$b^\dagger|0\rangle \to q \quad d^\dagger|0\rangle \to \bar{q}$$

Dirac algebra on the light-cone

usual representation of Dirac matrices

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad \gamma_5 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$$

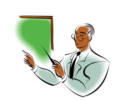
so (anti-)particles have only upper (lower) components in Dirac spinor

new representation in light-cone field theory

$$\gamma^0 = \begin{pmatrix} 0 & \sigma_3 \\ \sigma_3 & 0 \end{pmatrix} \quad \gamma^3 = \begin{pmatrix} 0 & -\sigma_3 \\ \sigma_3 & 0 \end{pmatrix} \quad \boldsymbol{\gamma}_\perp = \begin{pmatrix} i\boldsymbol{\sigma}_\perp & 0 \\ 0 & i\boldsymbol{\sigma}_\perp \end{pmatrix} \quad \gamma_5 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$$

$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \quad \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ ok

definitions : $\gamma^\pm = \frac{1}{\sqrt{2}}(\gamma^0 \pm \gamma^3)$ $\qquad P_\pm = \frac{1}{2}\gamma^\mp\gamma^\pm$



projectors $(P_\pm)^2 = P_\pm \quad ; \quad P_+ P_- = P_- P_+ \quad , \quad P_+ + P_- = \mathbb{1}$

$$P_+ = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & 0 \end{pmatrix}, \; P_- = \begin{pmatrix} 0 & 0 \\ 0 & \mathbb{1} \end{pmatrix} \quad |\psi\rangle = \begin{vmatrix} \phi \\ \chi \end{vmatrix} \quad , \quad P_+|\psi\rangle = \phi \, , \; P_-|\psi\rangle = \chi$$

project Dirac eq.

$$P_\pm \quad (i\gamma \cdot D + m) \begin{vmatrix} \phi \\ \chi \end{vmatrix} = 0 \qquad D_\pm = \partial_\pm - igA^\mp$$

$$\begin{cases} i\gamma^- D_- \chi &= -i\boldsymbol{\gamma}_\perp \mathbf{D}_\perp \phi - m\phi \\ i\gamma^+ D_+ \phi &= -i\boldsymbol{\gamma}_\perp \mathbf{D}_\perp \chi - m\chi \end{cases}$$

⟵ does not contain "time" x^+ :
χ depends from ϕ and $\mathbf{A}_\perp$ at fixed x^+
ϕ, $\mathbf{A}_\perp$ independent degrees of freedom

$$|\psi\rangle = \begin{vmatrix} \phi \\ \chi \end{vmatrix} \begin{matrix} \longleftarrow \text{“good”} \\ \longleftarrow \text{“bad”} \end{matrix}$$

light-cone components $P_+|\psi\rangle$, $P_-|\psi\rangle$

component "good" → independent and leading
component "bad" → dependent from interaction (quark-gluon) and therefore at higher order

quark polarization

generator of spin rotations around z $\Sigma^3 = \frac{i}{2}[\gamma^1, \gamma^2] = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$

if momentum k || z, it gives helicity

γ^1, γ^2, γ_5 commute with $P_\pm$ → 2 possible choices :

- diagonalize γ_5 and Σ^3 → helicity basis
- diagonalize γ^1 (or γ^2) → "transversity" basis

N.B. in helicity basis $\gamma_5 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}$ $\Sigma^3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & \sigma_3 \end{pmatrix}$

helicity = chirality for component "good" ϕ

helicity = - chirality for component "bad" χ

N.B. projector for transverse polarization $P_{\uparrow/\downarrow} = \frac{1}{2}\left(1 \mp \gamma_5\,\gamma^1\right)$ $\left[P_{\uparrow/\downarrow}\,,\,P_\pm\right] = 0$

we define $P_{\uparrow/\downarrow}\,\phi = \phi_{\perp/\top} = f(\phi_\pm)$

go back to OPE for inclusive DIS

$$W_{\mu\nu} \propto \int d^4x\, e^{iq\cdot x} \langle P|[J_\mu(x), J_\nu(0)]|P\rangle$$

quark current $J^\mu \sim \sum_f e_f^2\, \bar\psi_f\, \gamma^\mu\, \psi_f$

$$2MW^{\mu\nu} \sim \sum_f e_f^2 \int d^4p\, \delta((p+q)^2 - m^2)\, \theta(p^0+q^0-m)$$

$$\mathrm{Tr}\left[\Phi(p,P)\, \gamma^\mu\, (\not p + \not q + m)\, \gamma^\nu + \overline{\Phi}(p,P)\, \gamma^\nu\, (\not p + \not q - m)\, \gamma^\mu\right]$$

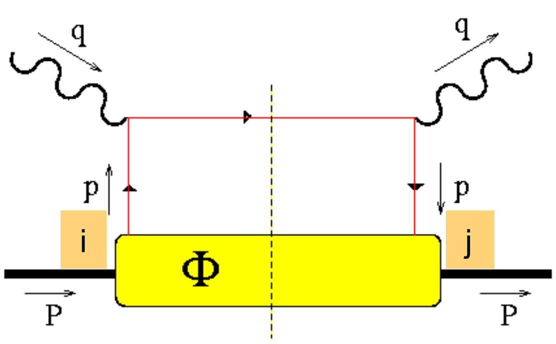
ij im mr nj

$$\Phi(p,P) = \int \frac{d^4\xi}{(2\pi)^4}\, e^{-ip\cdot\xi} \langle P|\bar\psi_f(\xi)\, \psi_f(0)|P\rangle$$

ij j i

$$= \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} \langle P|\bar\psi_f(0)\,|P_X\rangle\langle P_X|\psi_f(0)|P\rangle\, \delta(P-p-P_X)$$

bilocal operator, contains twist ≥ 2



IFM ($Q^2 \to \infty$) ⇒ isolate leading contribution in 1/Q
equivalently calculate **Φ** on the Light-Cone (LC)

IFM: leading contribution

$$\begin{cases} P^\mu = \frac{1}{\sqrt{2}}\left(A, \frac{M^2}{A}, \mathbf{0}_\perp\right) \xrightarrow{A=Q} P^\mu \sim \left(Q, \frac{1}{Q}, \mathbf{0}_\perp\right) \xrightarrow{Q^2\to\infty} (Q, 0, \mathbf{0}_\perp) \\ p^\mu = \frac{1}{\sqrt{2}}\left(xA, \frac{p^2+\mathbf{p}_\perp^2}{xA}, \mathbf{p}_\perp\right) \xrightarrow{A=Q} \frac{1}{\sqrt{2}}\left(xQ, \frac{p^2+\mathbf{p}_\perp^2}{xQ}, \mathbf{p}_\perp\right) \sim (Q,\ 0,\ \mathbf{0}_\perp) \\ q^\mu = \frac{1}{\sqrt{2}}\left(-x_N A, \frac{Q^2}{x_N A}, \mathbf{0}_\perp\right) \xrightarrow{A=Q} \frac{1}{\sqrt{2}}\left(-x_N Q, \frac{Q}{x_N}, \mathbf{0}_\perp\right) \end{cases}$$

N.B. $p^+ \sim Q \rightarrow (p+q)^- \sim Q$

$$\begin{aligned} 2MW^{\mu\nu} &= \sum_f e_f^2 \int d^4p\, \delta((p+q)^2 - m^2)\, \theta(p^0+q^0-m) \\ &\qquad \times \mathrm{Tr}\left[\Phi(p,P)\, \gamma^\mu\, (\gamma\cdot p + \gamma\cdot q + m)\, \gamma^\nu\right] \\ &\sim \frac{1}{2}\sum_f e_f^2 \int dp^- d\mathbf{p}_\perp\, \mathrm{Tr}\left[\Phi(p,P)\, \gamma^\mu\, \gamma^+\, \gamma^\nu\right]\Big|_{p^+=xP^+} \end{aligned}$$

$$\delta(p^+ + q^+) = \delta(xP^+ - x_N P^+) \rightarrow x \sim x_N \sim x_B$$

(analogously for antiquark)

(cont'ed)

- decomposition of Dirac matrix Φ(p,P,S) on basis of Dirac structures with 4-(pseudo)vectors p,P,S compatible with Hermiticity and parity invariance

$$\Phi(p,P,S) \; = \; \gamma^0\,\Phi^\dagger(p,P,S)\,\gamma^0$$
$$\Phi(p,P,S) \; = \; \gamma^0\,\Phi(\tilde{p},\tilde{P},-\tilde{S})\,\gamma^0$$

Dirac basis $\quad \mathbb{1}\,,\; i\gamma_5\,,\; \gamma^\mu\,,\; \gamma^\mu\gamma_5\,,\; \sigma^{\mu\nu}\,,\; i\sigma^{\mu\nu}\gamma_5$ $\qquad \tilde{a}^\mu = (a_0, -\mathbf{a})$

$$\begin{aligned}\Phi(p,P,S) \; = \; & A_1 M + A_2 \not{P} + A_3 \not{p} + A_4 \sigma_{\mu\nu} P^\mu p^\nu + i A_5 p\cdot S \gamma_5 + A_6 M \not{S}\gamma_5 \\ + \; & A_7 \frac{p\cdot S}{M} \not{P}\gamma_5 + A_8 \frac{p\cdot S}{M} \not{p}\gamma_5 + i A_9 \sigma_{\mu\nu}\gamma_5 S^\mu P^\nu + i A_{10} \sigma_{\mu\nu}\gamma_5 S^\mu p^\nu \\ + \; & i A_{11} \frac{p\cdot S}{M^2} \sigma_{\mu\nu}\gamma_5 P^\mu p^\nu + A_{12} \epsilon_{\mu\nu\rho\sigma} \frac{\gamma^\mu P^\nu p^\rho S^\sigma}{M}\end{aligned}$$

time-reversal $\rightarrow$ 0

$$\Phi^*(p,P,S) \; = \; -\mathrm{i}\gamma^1\gamma^3\,\Phi(\tilde{p},\tilde{P},\tilde{S})\,\mathrm{i}\,\gamma^1\gamma^3$$

$$\mathrm{Tr}\left[\Phi(p,P)\,\gamma^\mu\,\gamma^+\,\gamma^\nu\right]\Big|_{p^+=xP^+} = -4 g_\perp^{\mu\nu} \underbrace{(A_2 + x A_3)\; P^+}$$

$\int dp^- d\mathbf{p}_\perp \ldots|_{p^+=xP^+} \;\rightarrow$ $q_f(x)$

similar for antiquark

$$\Downarrow$$

$$2MW^{\mu\nu} \sim -g_\perp^{\mu\nu}\,\frac{1}{2}\sum_f e_f^2\,[q_f(x) + \bar{q}_f(x)] + o\left(\frac{1}{Q}\right)$$

(cont'ed)

$$2MW^{\mu\nu} \sim -g_\perp^{\mu\nu} \underbrace{\frac{1}{2} \sum_f e_f^2 \left[q_f(x) + \bar{q}_f(x)\right]}_{F_1(x_B)} + o\left(\frac{1}{Q}\right)$$

$x \approx x_B$ $F_1(x_B)$ → QPM result

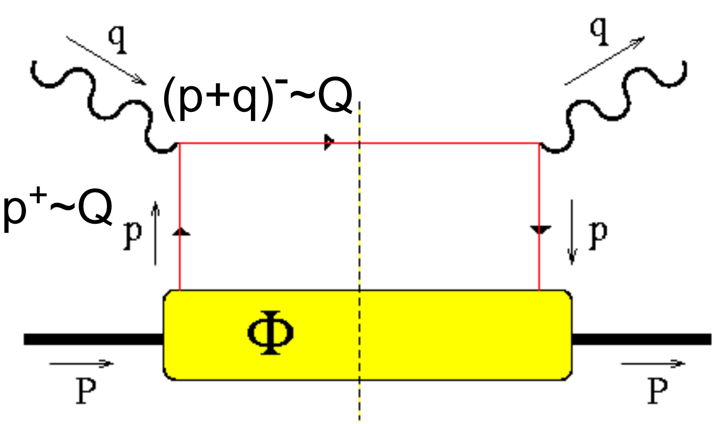
$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1 + \frac{\tilde{P}^\mu \tilde{P}^\nu}{M^2} W_2$$

W_1 response to transverse polarization of γ^* $\longrightarrow -g_\perp^{\mu\nu} F_1$

Summary :

bilocal operator Φ has twist ≥ 2 ; leading-twist contribution extracted in IFM selecting the dominant term in 1/Q ($Q^2 \to \infty$) ; equivalently, calculating Φ on the LC

at leading twist (t=2) recover QPM result for unpolarized $W^{\mu\nu}$; but what is the general result for t=2 ?



Decomposition of Φ at leading twist

Dirac basis $\{\mathbb{1}\,,\ \gamma^\mu\,,\ \gamma^\mu\gamma_5\,,\ i\gamma_5\,,\ i\sigma^{\mu\nu}\gamma_5\}$

$$\Phi(p,P,S) = \frac{1}{2}\left[S\,\mathbb{1} + V_\mu\,\gamma^\mu + A_\mu\,\gamma^\mu\gamma_5 + iP\,\gamma_5 + iT_{\mu\nu}\,\sigma^{\mu\nu}\ \gamma_5\right]$$

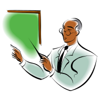
$$S = \frac{1}{2}\,\mathrm{Tr}(\Phi) = C_1(p^2, p\cdot P)$$

$$V^\mu = \frac{1}{2}\,\mathrm{Tr}(\gamma^\mu\,\Phi) = C_2\,P^\mu + C_3\,p^\mu$$

$$A^\mu = \frac{1}{2}\,\mathrm{Tr}(\gamma^\mu\gamma_5\,\Phi) = C_4\,S^\mu + C_5\,p\cdot S\,P^\mu + C_6\,P\cdot S\,p^\mu$$

$$P_5 = \frac{1}{2i}\,\mathrm{Tr}(\gamma_5\,\Phi) = 0$$

$$T^{\mu\nu} = \frac{1}{2i}\,\mathrm{Tr}(\sigma^{\mu\nu}\,\Phi) = C_7\,P^{[\mu}S^{\nu]} + C_8\,p^{[\mu}S^{\nu]} + C_9\,p\cdot S\,P^{[\mu}p^{\nu]}$$



$\mathrm{Tr}\,[\gamma^+\ldots] \rightarrow$ $$q_f(x) = \Phi^{[\gamma^+]} = \int \frac{d\xi^-}{2\pi}\,e^{-ixP^+\xi^-}\,\langle P|\bar{\psi}_f(\xi^-)\,\gamma^+\,\psi_f(0)|P\rangle$$

$\mathrm{Tr}\,[\gamma^+\gamma_5\ldots] \rightarrow$ $$\Delta q_f(x) = \Phi^{[\gamma^+\gamma_5]} = \int \frac{d\xi^-}{2\pi}\,e^{-ixP^+\xi^-}\,\langle P|\bar{\psi}_f(\xi^-)\,\gamma^+\gamma_5\,\psi_f(0)|P\rangle$$

$\mathrm{Tr}\,[\gamma^+\gamma^i\,\gamma_5\ldots] \rightarrow$ $$\delta q_f(x) = \Phi^{[i\sigma^{i+}\gamma_5]} = \int \frac{d\xi^-}{2\pi}\,e^{-ixP^+\xi^-}\,\langle P|\bar{\psi}_f(\xi^-)\,i\sigma^{i+}\gamma_5\,\psi_f(0)|P\rangle$$

Trace of bilocal operator → partonic density

$$\Phi^{[\gamma^+]}(x) = \int dp^- d\mathbf{p}_\perp \, \mathrm{Tr}\left[\Phi(p,P)\,\gamma^+\right]\Big|_{p^+=xP^+}$$

$$= \sqrt{2}\sum_n |\langle n|\phi_f(0)|P\rangle|^2 \,\delta(P^+ - xP^+ - P_n^+) \equiv q_f(x)$$

LC "good" components

probability density
of annihilating in |P>
a quark with momentum xP^+

similarly for antiquark

$$\Phi^{[\gamma^+]}(x) + \bar{\Phi}^{[\gamma^+]}(x) = \int dp^- d\mathbf{p}_\perp \, \mathrm{Tr}\left[\Phi(p,P,S)\,\gamma^+ - \bar{\Phi}(p,P,S)\,\gamma^+\right]\Big|_{p^+=xP^+}$$

$$= q_f(x) + \bar{q}_f(x)$$

= probability of finding a (anti)quark with flavor f and fraction x of longitudinal (light-cone) momentum P^+ of hadron in general : $$\Phi^{[\Gamma]}(x,S) = \int dp^- d\mathbf{p}_\perp \, \mathrm{Tr}\left[\Phi(p,P,S)\,\Gamma\right]\Big|_{p^+=xP^+}$$

leading-twist projections (involve "good" components of ϕ)

$$\Phi^{[\gamma^+]}(x,S) = q(x)$$
$$\Phi^{[\gamma^+\gamma_5]}(x,S) = \lambda\,\Delta q(x)$$
$$\Phi^{[i\sigma^{i+}\gamma_5]}(x,S) = S_T^i\,\delta q(x)$$

twist 3 projections

(involve "good" ϕ and "bad" χ components)

$$\Phi^{[\mathbb{1}]}(x,S) = \frac{M}{P+}\,e(x)$$
$$\Phi^{[\gamma^i\gamma_5]}(x,S) = \frac{M}{P+}\,S_T^i\,g_T(x)$$
$$\Phi^{[i\sigma^{+-}\gamma_5]}(x,S) = \frac{M}{P+}\,\lambda\,h_L(x)$$

Example: $$\int dp^- d\mathbf{p}_\perp \, \mathrm{Tr}\left[\Phi(p,P,S)\,\mathbb{1}\right]\Big|_{p^+=xP^+} = \frac{M}{P+}\int\frac{d\xi^-}{2\pi}\,e^{-ixP^+\xi^-}\,\langle P|\underbrace{\bar\psi(\xi^-)\,\psi(0)}|P\rangle$$

$$\psi^\dagger\gamma^0\psi = \overline{\begin{matrix}\phi & \chi\end{matrix}}\begin{pmatrix}0 & \sigma_3\\ \sigma_3 & 0\end{pmatrix}\begin{vmatrix}\phi\\ \chi\end{vmatrix} \sim \phi^\dagger\sigma_3\chi \to \phi^\dagger\sigma_3(i\,\not{D}+m)\phi$$

quark-gluon correlator suppressed

not a density → no probabilistic interpretation

probabilistic interpretation at leading twist

helicity (chirality) projectors $P_{R/L} = \frac{1 \pm \gamma_5}{2}$ $[P_{R/L}, P_\pm] = 0$

$$\Phi^{[\gamma^+]} \to \bar{\psi}\,\gamma^+\,\psi \to \psi^\dagger\,P_+\,\psi \to \phi^\dagger\,\phi = \phi^\dagger\,(P_R + P_L)^\dagger\,(P_R + P_L)\,\phi$$

$$= \phi^\dagger(P_R^\dagger P_R + P_L^\dagger P_L)\,\phi = \bar{R}R + \bar{L}L$$

momentum distribution

$$\Phi^{[\gamma^+\gamma_5]} \to \bar{\psi}\,\gamma^+\gamma_5\,\psi \to \psi^\dagger\,P_+\,\gamma_5 P_+ \psi \to \phi^\dagger\,(P_R - P_L)\,\phi$$

$$= \phi^\dagger(P_R^\dagger P_R - P_L^\dagger P_L)\,\phi = \bar{R}R - \bar{L}L$$

$[P_\pm, \gamma_5] = 0$

helicity distribution

$$\Phi^{[i\sigma^{i+}\gamma_5]} \to \bar{\psi}\,i\sigma^{i+}\gamma_5\,\psi \ldots \to \phi^\dagger(P_L^\dagger\gamma^i P_R - P_R^\dagger\gamma^i P_L)\,\phi$$

?

(cont'ed)

projector of transverse polarization $P_{\uparrow/\downarrow} = \dfrac{1 \pm \gamma^i \gamma_5}{2}$ (from helicity basis to transversity basis)

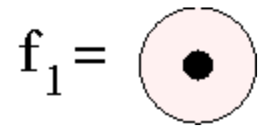
$$\Phi^{[i\sigma^{i+}\gamma_5]} \to \bar{\psi}\, i\sigma^{i+}\gamma_5\, \psi \ldots \to \phi^\dagger (P_\uparrow P_\uparrow - P_\downarrow P_\downarrow)\, \phi$$

→ δq is "net" distribution of transverse polarization !

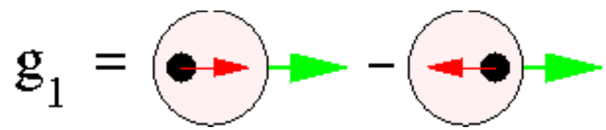
more usual and "comfortable" notations:

$$\Phi^{[\gamma^+]}(x,S) \quad = q(x) \quad \longrightarrow \quad f_1^q(x)$$

unpolarized quark q **leading twist**

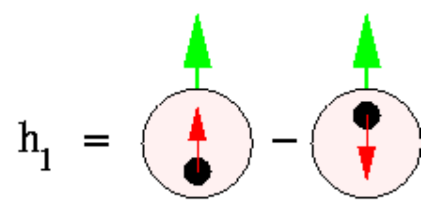


$$\Phi^{[\gamma^+\gamma_5]}(x,S) \quad = \lambda\, \Delta q(x) \quad \longrightarrow \quad \lambda\, g_1^q(x)$$



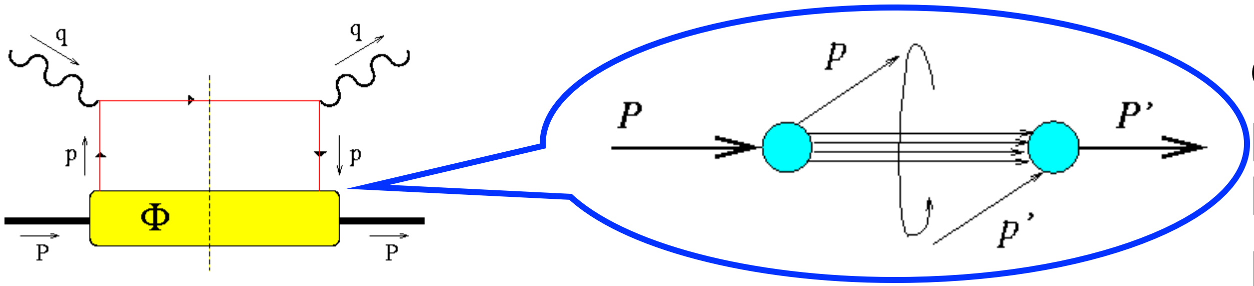
long. polarized quark $\vec{q}$

$$\Phi^{[i\sigma^{i+}\gamma_5]}(x,S) \quad = S_T^i\, \delta q(x) \quad \longrightarrow \quad S_T^i\, h_1^q(x)$$



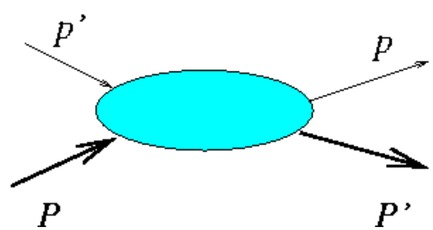
$q^\uparrow$ **transv. polarized quark**

need for 3 **P**arton **D**istribution **F**unctions al leading twist



target with helicity P
emits
parton with helicity p
hard scattering
parton with helicity p'
reabsorbed in
hadron with helicity P'

discontinuity in *u* channel of forward scattering amplitude parton-hadron



$\rightarrow A_{Pp,P'p'}$

at leading twist only "good" components $|\psi\rangle \sim \begin{vmatrix} \phi_+ \\ \phi_- \\ o(1/Q) \\ o(1/Q) \end{vmatrix}$

process is collinear modulo *o*(1/Q)

⇒ helicity conservation P+p' = p+P'

(cont'ed)

invariance for parity transformations $\rightarrow A_{Pp,P'p'} = A_{-P-p.-P'-p'}$
invariance for time-reversal $\rightarrow A_{Pp,P'p'} = A_{P'p',Pp}$

constraints $\rightarrow$ 3 $A_{Pp,P'p'}$ independent

	P	p	$\rightarrow$	P'	p'
1)	+	+		+	+
2)	+	-		+	-
3)	+	+		-	-

$(+,+) \rightarrow (+,+) + (+,-) \rightarrow (+,-) \equiv f_1 \; \bar{R}R + \bar{L}L$

$(+,+) \rightarrow (+,+) - (+,-) \rightarrow (+,-) \equiv g_1 \; \bar{R}R - \bar{L}L$

$(+,+) \rightarrow (-,-) \equiv h_1 \; \bar{L}R$

helicity basis $h_1 \sim \phi^\dagger P_L^\dagger \gamma_i P_R \phi$

transversity basis $h_1 \sim \phi^\dagger (P_\uparrow^\dagger P_\uparrow - P_\downarrow^\dagger P_\downarrow)\phi$

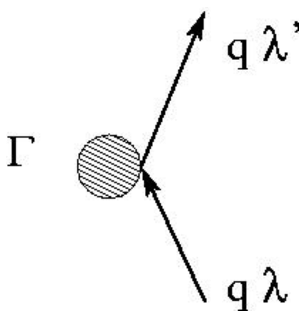
$$\langle \uparrow |\ldots| \uparrow\rangle - \langle \downarrow |\ldots| \downarrow\rangle \propto \langle +|\ldots|-\rangle + \langle -|\ldots|+\rangle$$

$$\begin{cases} |\uparrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle) \\ |\downarrow\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |-\rangle) \end{cases}$$

for "good" components ($\Leftrightarrow$ twist 2) helicity = chirality hence h_1 does not conserve chirality (chiral odd)

QCD conserves helicity at leading twist

massless quark spinors $\lambda = \pm 1$

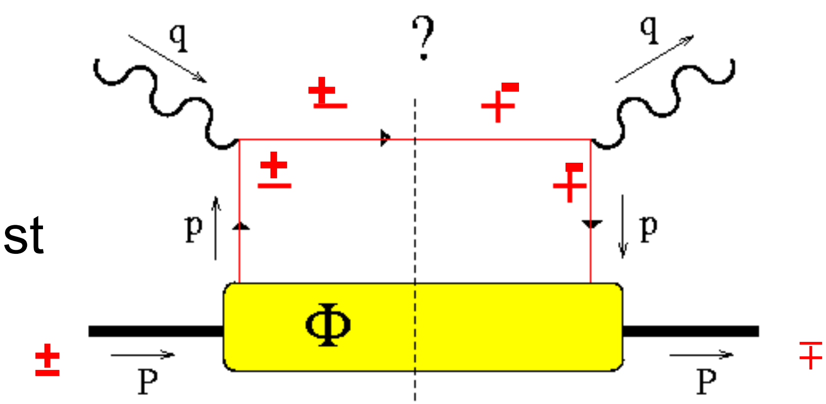


$$\begin{aligned} M &\sim \bar{u}_{\lambda'}\,\Gamma\, u_\lambda \\ &\sim \bar{u}_{\lambda'}\,(1-\lambda'\gamma_5)\,(1-\lambda\gamma_5)\,\Gamma\, u_\lambda \\ &\sim \delta_{\lambda\lambda'}\,\bar{u}_{\lambda'}\,\Gamma\, u_\lambda + o\left(\frac{m_q}{E_q}\right) \end{aligned}$$

$$\frac{1+\lambda\gamma_5}{2}u_\lambda = u_\lambda$$

$$\bar{u}_\lambda \frac{1-\lambda\gamma_5}{2} = \bar{u}_\lambda$$

QCD conserves helicity at leading twist
→ h_1 suppressed in inclusive DIS



different properties between f_1, g_1 and h_1

for inclusive DIS in QPM, correspondence between PDF's and structure fnct's

$$f_1(x) \;\rightarrow\; F_1(x_B) = \frac{1}{2}\sum_f e_f^2\,[f_1^f(x_B) + \bar{f}_1^f(x_B)] \longrightarrow \frac{1}{2}\sum_{f\bar{f}} e_f^2\,[q_f^{\uparrow}(x_B) + q_f^{\downarrow}(x_B)]$$

$$g_1(x) \;\rightarrow\; G_1(x_B) = \frac{1}{2}\sum_f e_f^2\,[g_1^f(x_B) + \bar{g}_1^f(x_B)] \longrightarrow \frac{1}{2}\sum_{f\bar{f}} e_f^2\,[q_f^{\uparrow}(x_B) - q_f^{\downarrow}(x_B)]$$

but h_1 has no counterpart at structure function level, because for inclusive polarized DIS, in $W_A^{\mu\nu}$ the contribution of G_2 is suppressed with respect to that of G_1: it appears at twist 3

$$W_A^{\mu\nu} \;=\; i\epsilon^{\mu\nu\rho\sigma}\, q_\rho\, S_\sigma \left[M G_1(\nu, Q^2) + \frac{P\cdot q}{M} G_2(\nu, Q^2)\right] - i\epsilon^{\mu\nu\rho\sigma}\, q_\rho\, P_\sigma\, \frac{S\cdot q}{M} G_2(\nu, Q^2)$$

for several years h_1 has been ignored; common belief that transverse polarization would generate only twist-3 effects, confusing with g_T in G_2

$$\Phi^{[\gamma^i\gamma_5]}(x,S) = \frac{M}{P+} S_T^i\, g_T(x) \;\longrightarrow\; g_1(x) + g_2(x) = \sum_f \frac{e_f^2\, m_f}{2Mx}\,[q_f^{\rightarrow}(x) - q_f^{\leftarrow}(x)]$$

in reality, this bias is based on the misidentification of transverse spin of hadron (appearing at twist 3 in hadron tensor) and distribution of transverse polarization of partons in transversely polarized hadrons, that does not necessarily appear only at twist 3:

	$\Phi^{[\Gamma]}$	long. pol.	$\Phi^{[\Gamma]}$	transv. pol.
twist 2	$\gamma^+ \gamma_5$	g_1	$i\,\sigma^{i+}\gamma_5$	h_1
twist 3	$i\,\sigma^{+-}\gamma_5$	h_L	$\gamma^i \gamma_5$	g_T

perfect "crossed" parallel between t=2 and t=3 for both helicity and transversity

moreover, h_1 has same relevance of f_1 and g_1 at twist 2. In fact, on helicity basis f_1 and g_1 are diagonal whilst h_1 is not,

$$f_1 \sim \phi^\dagger (P_R^\dagger P_R + P_L^\dagger P_L)\phi \quad g_1 \sim \phi^\dagger (P_R^\dagger P_R - P_L^\dagger P_L)\phi \quad h_1 \sim \phi^\dagger P_L^\dagger P_R \phi$$

but on transversity basis the situation is reversed:

$$f_1 \sim \phi^\dagger (P_\uparrow^\dagger P_\uparrow + P_\downarrow^\dagger P_\downarrow)\phi \quad g_1 \sim \phi^\dagger P_\downarrow^\dagger P_\uparrow \phi \quad h_1 \sim \phi^\dagger (P_\uparrow^\dagger P_\uparrow - P_\downarrow^\dagger P_\downarrow)\phi$$

h_1 is badly known because it is suppressed in inclusive DIS
theoretically, we know its evolution equations up to NLO in α_s
there are model calculations, and lattice calculations of its first Mellin moment (= tensor charge).

(Barone & Ratcliffe, *Transverse Spin Physics*, World Scientific (2003))

only recently first extraction of parametrization of h_1 with two independent methods by combining data from semi-inclusive reactions:

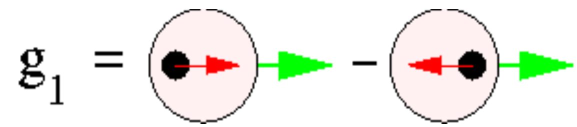
(Anselmino *et al*., Phys. Rev. D**75** 054032 (2007); hep-ph/0701006
updated in arXiv:1303.3822 [hep-ph]

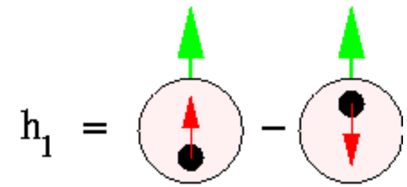
Bacchetta, Courtoy, Radici, Phys. Rev. Lett. **107** 012001 (2011)
JHEP 1303 (2013) 119)

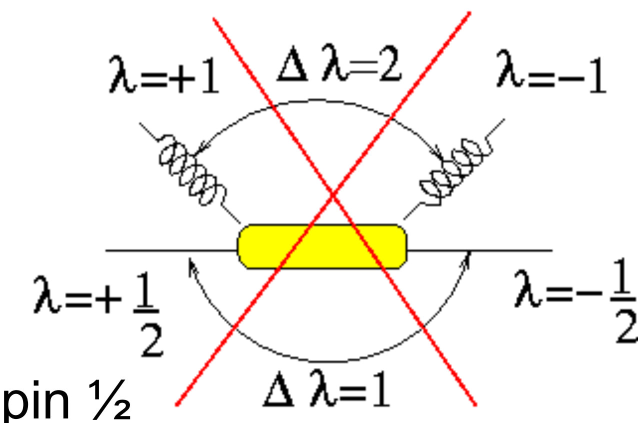
1. h_1 has very different properties from g_1
2. need to define best strategies for extracting it from data

chiral-odd h_1 → interesting properties with respect to other PDF

- g_1 and h_1 (and all PDF) are defined in IFM
  i.e. boost $Q \to \infty$ along z axis
  but boost and Galileo rotations commute in
  nonrelativistic frame → $g_1 = h_1$
  any difference is given by relativistic effects
  → info on relativistic dynamics of quarks







- for gluons we define
  G(x) = momentum distribution
  ΔG(x) = helicity distribution
  but we have no "transversity" in hadron with spin ½
  → evolution of h_1^q decoupled from gluons !

(cont'ed)

$$\langle PS|\overline{q}^f\gamma^\mu\gamma_5 q^f|PS\rangle\Big|_{Q^2} = 2\lambda P^\mu \int dx \left[g_1^f(x,Q^2) + g_1^{\overline{f}}(x,Q^2)\right] = 2\lambda P^\mu\, g_A$$

axial charge

$$\langle PS|\overline{q}^f i\sigma^{\mu\nu}\gamma_5 q^f|PS\rangle\Big|_{Q^2} = 2S^{[\mu}P^{\nu]} \int dx \left[h_1^f(x,Q^2) - h_1^{\overline{f}}(x,Q^2)\right] = 2S^{[\mu}P^{\nu]}\, g_T(Q^2)$$

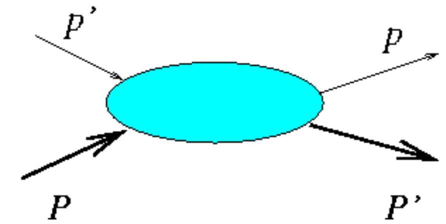
tensor charge
(not conserved)

- axial charge from C(harge)-even operator
  tensor charge from C-odd → it does not take
  contributions from quark-antiquark pairs of Dirac sea

summary: evolution of $h_1^q(x,Q^2)$ is very different from other PDF because it does not mix with gluons → evolution of non-singlet object
moreover, tensor charge is non-singlet, C-odd and not conserved
→ h_1 is best suited to study valence contribution to spin

- relations between PDF's

$A_{Pp,P'p'}$



$[(+,+) \rightarrow (+,+)] + [(+,-) \rightarrow (+,-)] \equiv f_1$

$[(+,+) \rightarrow (+,+)] - [(+,-) \rightarrow (+,-)] \equiv g_1$

$(+,+) \rightarrow (-,-) \equiv h_1$

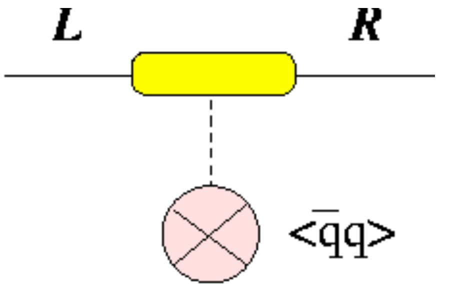
by definition $\rightarrow$ $f_1 \geq |g_1|, |h_1|$, $f_1 \geq 0$

$|(+,+) \pm (-,-)|^2 = A_{++,++} + A_{--,--} \pm 2\,\mathrm{Re}A_{++,--} \geq 0$

invariance for parity transformations $\rightarrow A_{Pp,P'p'} = A_{-P-p.-P'-p'}$

$A_{++,++} = \frac{1}{2}(f_1 + g_1) \geq |A_{++,--}| = |h_1|$ $\rightarrow$ Soffer inequality valid for every x and Q^2 (at least up to NLO)

(cont'ed)



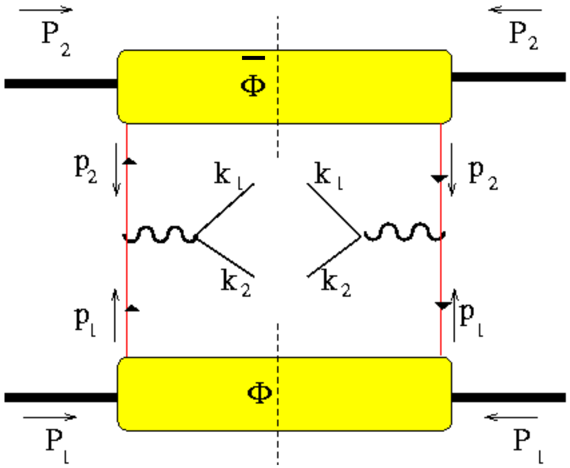
h_1 does not conserve chirality (chiral odd)
h_1 can be determined by soft processes related to chiral symmetry breaking of QCD
(role of nonperturbative QCD vacuum?)

in helicity basis cross section must be chiral-even
hence h_1 must be extracted in elementary process where it appears with a chiral-odd partner
further constraint is to find this mechanism at leading twist

<u>how to extract transversity from data ?</u>

how to extract transversity from data ?

the most obvious choice: polarized Drell-Yan $p^{\uparrow}p^{\uparrow} \rightarrow l^{+}l^{-}X$



$$\bar{\Phi}(x,S) = \int dp^{-} d\mathbf{p}_T \, \bar{\Phi}(p,P,S)\Big|_{p^{+}=xP^{+}} \longrightarrow [\bar{f}_1(x) + \lambda\, \bar{g}_1(x)\,\gamma_5 + \bar{h}_1(x)\,\gamma_5\, \not{S}_T]\, \not{P}$$

$$\Phi(x,S) = \int dp^{-} d\mathbf{p}_T \, \Phi(p,P,S)\Big|_{p^{+}=xP^{+}} \longrightarrow [f_1(x) + \lambda\, g_1(x)\,\gamma_5 + h_1(x)\,\gamma_5\, \not{S}_T]\, \not{P}$$

Single-Spin Asymmetry (SSA)

$$A_{TT} = \frac{d\sigma(p^\uparrow p^\uparrow) - d\sigma(p^\uparrow p^\downarrow)]}{d\sigma(p^\uparrow p^\uparrow) + d\sigma(p^\uparrow p^\downarrow)}$$

$$= |\mathbf{S}_{T_1}|\,|\mathbf{S}_{T_2}|\,\frac{\sin^2\theta\,\cos(2\phi - \phi_{S_1} - \phi_{S_2})}{1+\cos^2\theta}\,\frac{\sum_{f,\bar f} e_f^2\, h_1^f(x_1)\, \bar h_1^f(x_2)}{\sum_{f,\bar f} e_f^2\, f_1^f(x_1)\, \bar f_1^f(x_2)}$$

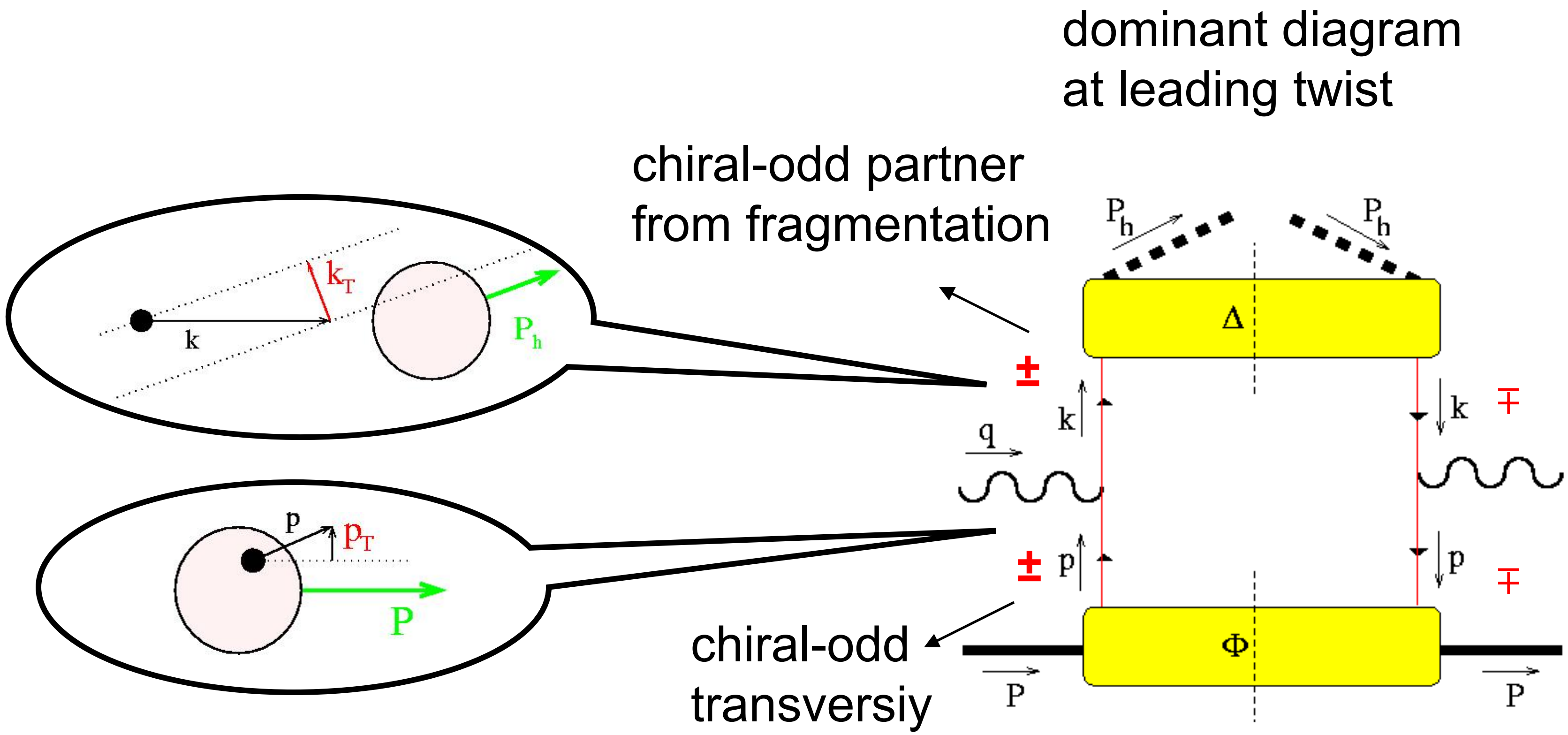
but = transverse spin distribution of antiquark in polarized proton
$\rightarrow$ antiquark from Dirac sea is suppressed

and simulations suggest that Soffer inequality, for each Q^2, bounds A_{TT} to very small numbers (~ 1%)

better to consider $p^\uparrow \bar p^\uparrow \rightarrow l^+ l^- X$ (recent proposal PAX at GSI - Germany)
but technology still to be developed

otherwise …. need to consider semi-inclusive reactions

alternative: semi-inclusive DIS (SIDIS)



in SIDIS $\{P,q,P_h\}$ not all collinear;
convenient to choose frame where $\mathbf{q}_T \neq 0$

→ sensitivity to transverse momenta of partons in hard vertex

→ more rich structure of Φ → Transverse Momentum Distributions (TMD)