application to inclusive e$^+$e$^-$ and DIS

$W^{\mu\nu}$ $\Rightarrow$ $J^\mu(\xi)\, J^\nu(0)$ with J^μ e.m. current of quark

normal product : : useful to define a composite operator for $\xi \to 0$

$\Rightarrow$ study $\mathcal{T}\,[J^\mu(\xi)\, J^\nu(0)]$ per $\xi \to 0$ with Wick theorem

$$\mathcal{T}\,[J^\mu(\xi) J^\nu(0)] =$$

$$: \bar{\psi}(\xi)\gamma^\mu\psi(\xi)\ \bar{\psi}(0)\gamma^\nu\psi(0) : \ + \ : \bar{\psi}(\xi)\gamma^\mu\gamma^\nu\psi(0) : \ \psi(\xi)\bar{\psi}(0) \ +$$

$$: \bar{\psi}(0)\gamma^\nu\gamma^\mu\psi(\xi) : \ \psi(0)\bar{\psi}(\xi) \ - \mathrm{Tr}\,[\gamma^\mu\gamma^\nu]\ \ \psi(\xi)\bar{\psi}(0)\ \ \psi(0)\bar{\psi}(\xi)$$

$$= \mathrm{Tr}\,[\gamma^\mu\gamma^\nu]\ S_F(-\xi)\ S_F(\xi) - \ : \bar{\psi}(\xi)\gamma^\mu\gamma^\nu\psi(0) : \ i\, S_F(\xi)$$

$$- \ : \bar{\psi}(0)\gamma^\nu\gamma^\mu\psi(\xi) : \ i\, S_F(-\xi) \ + \ : \bar{\psi}(\xi)\gamma^\mu\psi(\xi)\ \bar{\psi}(0)\gamma^\nu\psi(0) :$$

$$\psi(\xi)\bar{\psi}(0) = \langle 0|\mathcal{T}\left[\psi(\xi)\bar{\psi}(0)\right]|0\rangle = -i\, S_F(\xi) = i\int \frac{d^4p}{(2\pi)^4}\, \frac{e^{-ip\cdot\xi}}{\not{p} - m + i\epsilon}$$

divergent for $\xi \to 0$ $\Rightarrow$ OPE

(cont'ed)

$$S_F(\xi) = (i\gamma\cdot\partial + m)\,\Delta(\xi) \sim (i\gamma\cdot\partial + m)\,\frac{1}{4\pi^2 i}\,\frac{1}{\xi^2 - i\epsilon} + \ldots$$

$$= \frac{-2\gamma\cdot\xi}{(\xi^2 - i\epsilon)^2}\,\frac{i}{4\pi^2 i} + \frac{1}{4\pi^2 i}\,\frac{m}{\xi^2 - i\epsilon} + \text{termini meno singolari}$$

most singular term in $\mathcal{T}$ [J$^\mu$(ξ) J$^\nu$(0)]

$$\mathrm{Tr}\left[S_F(-\xi)\gamma^\mu S_F(\xi)\gamma^\nu\right] \sim -\frac{4}{16\pi^4(\xi^2 - i\epsilon)^4}\,\mathrm{Tr}\,[\,\not\xi\gamma^\mu\,\not\xi\gamma^\nu] + \ldots.$$

$$= \frac{\xi^2 g^{\mu\nu} - 2\xi^\mu\xi^\nu}{\pi^4(\xi^2 - i\epsilon)^4} + \ldots.$$

less singular term in $\mathcal{T}$ [J$^\mu$(ξ) J$^\nu$(0)]

$$:\bar\psi(\xi)\gamma^\mu\psi(\xi)\,\bar\psi(0)\gamma^\nu\psi(0): \;=\; \hat{O}(\xi, 0)$$ regular bilocal operator

(cont'ed)

intermediate terms

$$- : \bar{\psi}(\xi)\gamma_\mu \, iS_F(\xi)\, \gamma_\nu \psi(0) : \; - \; : \bar{\psi}(0)\gamma^\nu \, iS_F(-\xi)\, \gamma_\mu \psi(\xi) :$$

$$\sim \frac{i\xi^\lambda}{2\pi^2(\xi^2 - i\epsilon)^2} \; : \bar{\psi}(\xi)\gamma_\mu\gamma_\lambda\gamma_\nu\psi(0) - \bar{\psi}(0)\gamma_\nu\gamma_\lambda\gamma_\mu\psi(\xi) : \; + \ldots$$

$$= \frac{i\xi^\lambda}{2\pi^2(\xi^2 - i\epsilon)^2} \; \left(\sigma_{\mu\lambda\nu\rho}\, \widehat{O}^\rho_V(\xi,0) \; + \; i\,\epsilon_{\mu\lambda\nu\rho}\, \widehat{O}^\rho_A(\xi,0)\right)$$

$$\begin{aligned} \gamma_\mu\gamma_\lambda\gamma_\nu &= \left(\sigma_{\mu\lambda\nu\rho} + i\epsilon_{\mu\lambda\nu\rho}\,\gamma_5\right)\, \gamma^\rho \\ \gamma_\nu\gamma_\lambda\gamma_\mu &= \left(\sigma_{\mu\lambda\nu\rho} - i\epsilon_{\mu\lambda\nu\rho}\,\gamma_5\right)\, \gamma^\rho \\ \sigma_{\mu\lambda\nu\rho} &= g_{\mu\lambda}g_{\nu\rho} + g_{\mu\rho}g_{\nu\lambda} - g_{\mu\nu}g_{\lambda\rho} \end{aligned}$$

$$\begin{aligned} \Rightarrow \widehat{O}^\rho_V(\xi,0) &= \; : \bar{\psi}(\xi)\gamma^\rho\psi(0) - \bar{\psi}(0)\gamma^\rho\psi(\xi) : \\ \widehat{O}^\rho_A(\xi,0) &= \; : \bar{\psi}(\xi)\gamma_5\gamma^\rho\psi(0) + \bar{\psi}(0)\gamma_5\gamma^\rho\psi(\xi) : \end{aligned}$$

regular bilocal operators

(cont'ed)

summarizing :

$$\mathcal{T}\left[J_\mu(\xi)J_\nu(0)\right] = \frac{\xi^2 g_{\mu\nu} - 2\xi_\mu\xi_\nu}{\pi^4(\xi^2 - i\epsilon)^4} + \frac{i\xi^\lambda}{2\pi^2(\xi^2 - i\epsilon)^2}\,\sigma_{\mu\lambda\nu\rho}\,\widehat{O}_V^\rho(\xi,0)$$
$$-\frac{\xi^\lambda}{2\pi^2(\xi^2 - i\epsilon)^2}\,\epsilon_{\mu\lambda\nu\rho}\,\widehat{O}_A^\rho(\xi,0) + \widehat{O}_{\mu\nu}(\xi,0)$$

- $\hat{O}_{V/A}{}^{\mu}(\xi,0)$ and $\hat{O}^{\mu\nu}(\xi,0)$ are regular bilocal operators for $\xi \to 0$; bilocal → contain info on long distance behaviour
- coefficients are singular for $\xi \to 0$ (ordered in decreasing singularity); contain info on short distance behaviour
- rigorous factorization between short and long distances at any order
- formula contains the behaviour of free quarks at short distances → general framework to recover QPM results

- in inclusive e^+e^- (and also DIS) hadronic tensor displays $[J^\mu(\xi), J^\nu(0)]$ → manipulate above formula

(cont'ed)

$$\mathcal{T}\left[J^\mu(\xi)J^\nu(0)\right] - \mathcal{T}\left[J^\mu(\xi)J^\nu(0)\right]^\dagger = \epsilon(\xi^0)\ \left[J^\mu(\xi), J^\nu(0)\right]$$

$$\epsilon(x^0) = \frac{x^0}{|x^0|} \qquad J^\mu \ \text{hermitiana}$$

we have
$$\lim_{\epsilon\to 0} \frac{1}{x^2 - i\epsilon} = PV\frac{1}{x^2} + i\pi\delta(x^2)$$

$$\lim_{\epsilon\to 0} \frac{1}{(x^2 - i\epsilon)^n} = PV\frac{1}{(x^2)^n} + i\pi\,\frac{(-1)^{n-1}}{(n-1)!}\,\partial^{n-1}(x^2)$$

$$\lim_{\epsilon\to 0} \frac{1}{(x^2 - i\epsilon)^n} - \frac{1}{(x^2 + i\epsilon)^n} = 2\pi i\,\frac{(-1)^{n-1}}{(n-1)!}\,\partial^{n-1}(x^2)$$

con $\partial^n(x^2) = \dfrac{d^n}{d(x^2)^n}\,\delta(x^2)$

$$\epsilon(\xi^0)\left[J_\mu(\xi), J_\nu(0)\right] = \frac{i(2\xi_\mu\xi_\nu - \xi^2 g_{\mu\nu})}{3\pi^3}\,\partial^3(\xi^2) + \frac{\xi^\lambda}{\pi}\,\partial(\xi^2)\,\sigma_{\mu\lambda\nu\rho}\,\widehat{O}^\rho_V(\xi,0)$$
$$+ \frac{i\xi^\lambda}{\pi}\,\partial(\xi^2)\,\epsilon_{\mu\lambda\nu\rho}\,\widehat{O}^\rho_A(\xi,0) + \widehat{O}_{\mu\nu}(\xi,0) - \widehat{O}_{\nu\mu}(0,\xi)$$

application: inclusive e^+e^-

$$\sigma_{tot} = \frac{1}{2}\,\frac{e^4}{2s^3}\,L^{\mu\nu}\,W_{\mu\nu}$$

$$I_n(q) = \int d^4x\, e^{iq\cdot x}\,\epsilon(x^0)\,\partial^n(x^2) = \frac{i\pi^2}{4^{n-2}(n-1)!}\,(q^2)^{n-1}\,\epsilon(q^0)\,\theta(q^2)$$

$$\int d^4x\, e^{iq\cdot x}\,\langle 0|[J_\mu(x), J_\nu(0)]|0\rangle$$

$$\sim \int d^4x\, e^{iq\cdot x}\,\epsilon(x^0)\,\langle 0|\frac{i}{3\pi^3}\left(2x_\mu x_\nu - x^2\,g_{\mu\nu}\right)\partial^3(x^2)|0\rangle$$

$$= \frac{i}{3\pi^3}\left(g_{\mu\nu}\frac{\partial}{\partial q}\cdot\frac{\partial}{\partial q} - 2\frac{\partial}{\partial q^\mu}\frac{\partial}{\partial q^\nu}\right)\underbrace{\int d^4x\, e^{iq\cdot x}\,\epsilon(x^0)\,\partial^3(x^2)}_{I_3(q)}$$

$$= \frac{1}{6\pi}\,\epsilon(q^0)\,\theta(q^2)\,(4q_\mu q_\nu - q^2 g_{\mu\nu})$$

$$= \frac{e^4}{4\pi}\,\frac{1}{6s^3}\,L^{\mu\nu}\,(4q_\mu q_\nu - q^2 g_{\mu\nu})\,\epsilon(q^0)\,\theta(q^2) \rightarrow \frac{4\pi\alpha^2}{3s}$$

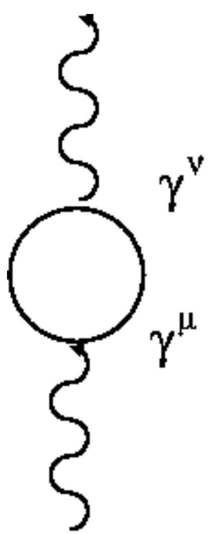
(cont'ed)

starting from quark current $\sum_f e_f^2 \sum_c : \bar{\psi}_f(x)\gamma^\mu\psi_f(x) :$

$$\longrightarrow \sigma_{tot} = N_c \frac{4\pi\alpha^2}{3s} \sum_f e_f^2$$ QPM result !

summary : OPE for free quarks at short distances is equivalent to QPM

because QPM assumes that at short distance quarks are free fermions
→ asymptotic freedom postulated in QPM is rigorously recovered in OPE



equivalent diagram :

$$W_{\mu\nu} = \int d^4x\, e^{iq\cdot x} \langle 0| \frac{i}{3\pi^3} \left(2x_\mu x_\nu - x^2 g_{\mu\nu}\right) \partial^3(x^2)|0\rangle$$

$$= \int d^4x\, e^{iq\cdot x} \langle 0|\mathrm{Tr}\left[S_F(x)\gamma^\mu S_F(-x)\gamma^\nu\right]|0\rangle$$

application: inclusive DIS

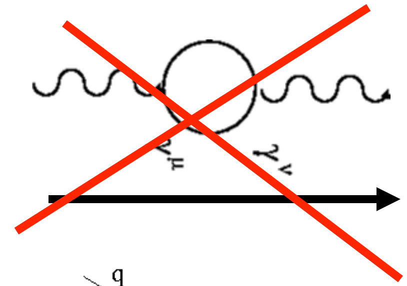
$$2MW_{\mu\nu} = \frac{1}{2\pi}\int d^4x\, e^{iq\cdot x}\,\langle P|[J_\mu(x),J_\nu(0)]|P\rangle$$

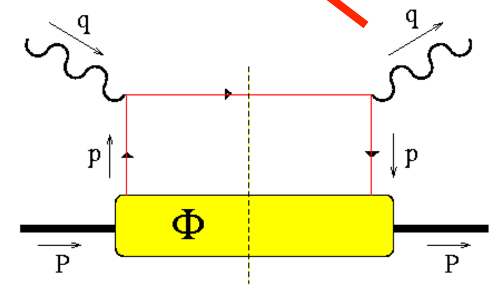
$$= \frac{i}{6\pi^4}\int d^4x\, e^{iq\cdot x}\,\left(2x_\mu x_\nu - x^2 g_{\mu\nu}\right)\partial^3(x^2)\,\langle P|P\rangle$$

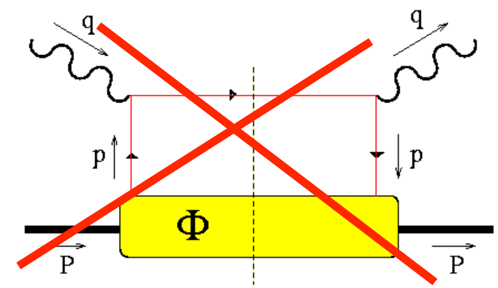
$$+\frac{1}{2\pi^2}\int d^4x\, e^{iq\cdot x}\,x^\lambda\,\epsilon(x^0)\,\partial^1(x^2)\,\langle P|\sigma_{\mu\lambda\nu\rho}\,\hat{O}_V^\rho(x,0)|P\rangle$$

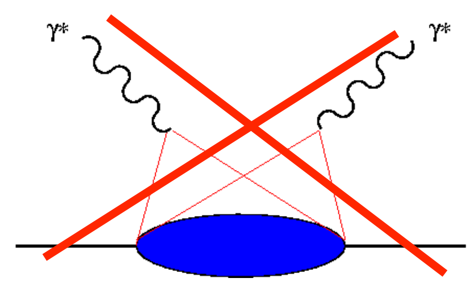
$$+\frac{1}{2\pi^2}\int d^4x\, e^{iq\cdot x}\,x^\lambda\,\epsilon(x^0)\,\partial^1(x^2)\,\langle P|i\epsilon_{\mu\lambda\nu\rho}\,\hat{O}_A^\rho(x,0)|P\rangle$$

$$+\frac{1}{2\pi}\int d^4x\, e^{iq\cdot x}\,\epsilon(x^0)\,\langle P|\hat{O}_{\mu\nu}(x,0)-\hat{O}_{\nu\mu}(0,x)|P\rangle$$









no polarization → $W_S^{\mu\nu}$

(cont'ed)

$[J^\mu(x),J^\nu(0)]$ dominated by kin. $x^2 \to 0 \Rightarrow$ expand $\hat{O}_V(x,0)$ around $x=0$
regular bilocal operator $\to$ infinite series of regular local operators

$$\psi(x) = \psi(0) + x^\mu\, \partial_\mu \psi(x)|_{x=0} + \frac{1}{2!}\, x^{\mu_1}\, x^{\mu_2}\, \partial_{\mu_1}\partial_{\mu_2}\psi(x)|_{x=0} + ...$$

$$\hat{O}_V^\rho(x,0) \;=\; \sum_{n=0}^{\infty} \frac{1}{n!}\, x^{\mu_1}...x^{\mu_n}\; \underbrace{:(\partial_{\mu_1}...\partial_{\mu_n}\,\bar{\psi}(x))\Big|_{x=0}\gamma^\rho\psi(0) - \bar{\psi}(0)\gamma^\rho(\partial_{\mu_1}...\partial_{\mu_n}\,\psi(x))\Big|_{x=0}:}_{\hat{O}^\rho_{V\;\mu_1...\mu_n}(0)}$$

then

$$\sigma_{\mu\lambda\nu\rho}\int d^4x\, e^{iq\cdot x}\, x^\lambda\, \dots \sum_{n=0}^{\infty}\frac{1}{n!}\, x^{\mu_1}...x^{\mu_n}\,\langle P|\hat{O}^\rho_{V\;\mu_1...\mu_n}(0)|P\rangle$$

$$\xrightarrow{\text{DIS}}\;\frac{F_1(x_B)}{M}\left(-g_{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)+\frac{F_2(x_B)}{\nu}\,\tilde{P}^\mu\,\tilde{P}^\nu$$

QPM result

OPE: general procedure for (non)interacting fields

$$J_\mu(x) J_\nu(0) = \sum_{\{\alpha\}} C_{\mu\nu\,\{\alpha\}}(x^2)\, x^{\mu_1} \ldots x^{\mu_{n_\alpha}}\, \hat{O}_{\mu_1 \ldots \mu_{n_\alpha}}(0)$$

light-cone expansion valid for $x^2 \sim 0$

$$\Rightarrow C_{\{\alpha\}}(x^2) \sim \frac{1}{x^{6+n_\alpha-d}}$$

n_α = spin of Ô
d = canonical dimension of Ô

$$W_{\mu\nu} \propto \int d^4x\, e^{iq\cdot x} \langle P|[J_\mu(x), J_\nu(0)]|P\rangle$$

$W_{\mu\nu}$ dimensionless

$$[d^4x] = 4$$

$$[x^{\mu_1} \ldots x^{\mu_{n_\alpha}}] = n_\alpha$$

$$\left[\langle P|P'\rangle = 2E\,(2\pi)^3\,\delta(\mathbf{P}-\mathbf{P}')\right] = 2$$

$$\left[\langle P|\hat{O}_{\mu_1 \ldots . \mu_{n_\alpha}}(0)|P\rangle = P_{\mu_1} \ldots P_{\mu_{n_\alpha}}\, M^{d-n_\alpha-2}\, c_{\hat{O}} + o\left(\frac{M^2}{Q^2}\right)\right] = -d+2$$

interacting field theory:
radiative corrections → structure of singularities from Renormalization Group Equations (RGE) for C

$$C_{\{\alpha\}}(x^2) \overset{x\to 0}{\sim} \frac{1}{x^{6+n_\alpha-d}} \left(\log^{\gamma_{\hat{O}}}(\mu_F x) + ...\right)$$

$\gamma_{\hat{O}}$ anomalous dimension of $\hat{O}$

μ_F factorization scale

N.B. dependence on μ_F cancels with similar dependence in $\hat{O}(0,\mu_F)$

(cont'ed)

$$W_{\mu\nu} \propto \lim_{\epsilon\longrightarrow 0} \int d^4x \, e^{iq\cdot x} \quad g_{\mu\nu}$$

$$\times \sum_{\{\alpha\}} \left(\frac{1}{(x^2 - i\epsilon)^{3+\frac{n_\alpha - d}{2}}} - \frac{1}{(x^2 + i\epsilon)^{3+\frac{n_\alpha - d}{2}}} \right)$$

$$\times x^{\mu_1} \ldots . x^{\mu_{n_\alpha}} P_{\mu_1} \ldots P_{\mu_{n_\alpha}} M^{d-n_\alpha-2} c_{\hat{O}}$$

$$\sim \quad g_{\mu\nu} \, c_{\hat{O}} \sum_{\{\alpha\}} c'_{\{\alpha\}} \left(\frac{M}{\sqrt{q^2}} \right)^{d-n_\alpha-2} \left(\frac{1}{x_B} \right)^{n_\alpha}$$

for x $\rightarrow$ 0 (i.e., $q^2 \rightarrow \infty$) importance of Ô determined by twist $t = d - n_\alpha$

$t \geq 2$ ($t=2 \rightarrow$ scaling in DIS regime)

summarizing

procedure for calculating $W_{\mu\nu}$:

- OPE expansion for bilocal operator in series of local operators
- Fourier transform of each term
- sum all of them
- final result written as power series in M/Q through twist
  $t = d$ (canonical dimension) - n_α (spin) ≥ 2

$$2MW_{\mu\nu} = \frac{1}{2\pi} \int d^4x \, e^{iq\cdot x} \langle P|[J_\mu(x), J_\nu(0)]|P\rangle$$

$$\sim \frac{1}{2\pi^2} \sigma_{\mu\lambda\nu\rho} \int d^4x \, e^{iq\cdot x} \, x^\lambda \, \epsilon(x^0) \, \partial^1(x^2) \, \langle P|\hat{O}_V^\rho(x,0)|P\rangle$$

$$\sim \sum_{\{\alpha\}} \int d^4x \, e^{iq\cdot x} \left[C_{\mu\nu\,\{\alpha\}}(x^2) - \left(C_{\mu\nu\,\{\alpha\}}(x^2) \right)^\dagger \right] x^{\mu_1} \ldots . x^{\mu_{n_\alpha}} \langle P|\hat{O}_{\mu_1\ldots\mu_{n_\alpha}}(0)|P\rangle$$

$$\sim c_{\hat{O}} \sum_{\{\alpha\}} c'_{\mu\nu,\,\{\alpha\}} \left(\frac{M}{\sqrt{q^2}} \right)^{d-n_\alpha-2} \left(\frac{1}{x_B} \right)^{n_\alpha} \qquad \sim \frac{1}{(x^2)^{3+\frac{n_\alpha-d}{2}}}$$

(cont'ed)

is it possible to directly work with bilocal operators skipping previous steps?
which is the twist *t* of a bilocal operator?

Example :

$$\begin{aligned}\bar{\psi}(0)\,\gamma^\mu\,\psi(x) \;&=\; \bar{\psi}(0)\,\gamma^\mu\,\psi(0) + x_\nu\,\bar{\psi}(0)\,\gamma^\mu\,\partial^\nu\,\psi(0) + \dots\\ &\equiv\; J^\mu(0) + x_\nu\,\theta^{\mu\nu}(0) + \dots\end{aligned}$$

if local → *t* =2

t =2

$$\theta^{\mu\nu} = \left(\theta^{\mu\nu} - \frac{1}{4}\,g^{\mu\nu}\,\theta^\lambda_\lambda\right) + \frac{1}{4}\,g^{\mu\nu}\,\theta^\lambda_\lambda$$

t =2 *t* =4

hence, if local version of bilocal operator has twist *t*=2
→ bilocal operator has twist *t* ≥ 2

operational definition of twist (Jaffe, 1995)

since a bilocal operator with twist t can be expanded as

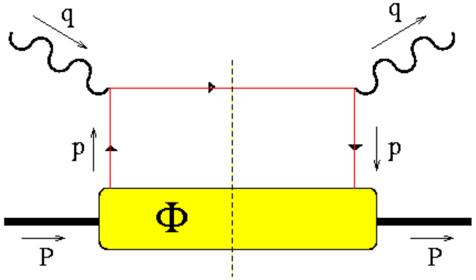
$$\left(\frac{M}{Q}\right)^{t-2}, \left(\frac{M}{Q}\right)^{t+2-2} \ldots., \quad t \geq 2$$

operational definition of twist for a regular bilocal operator

=

the leading power in *M/Q* at which the operator matrix element contributes to the considered deep-inelastic process in short distance limit

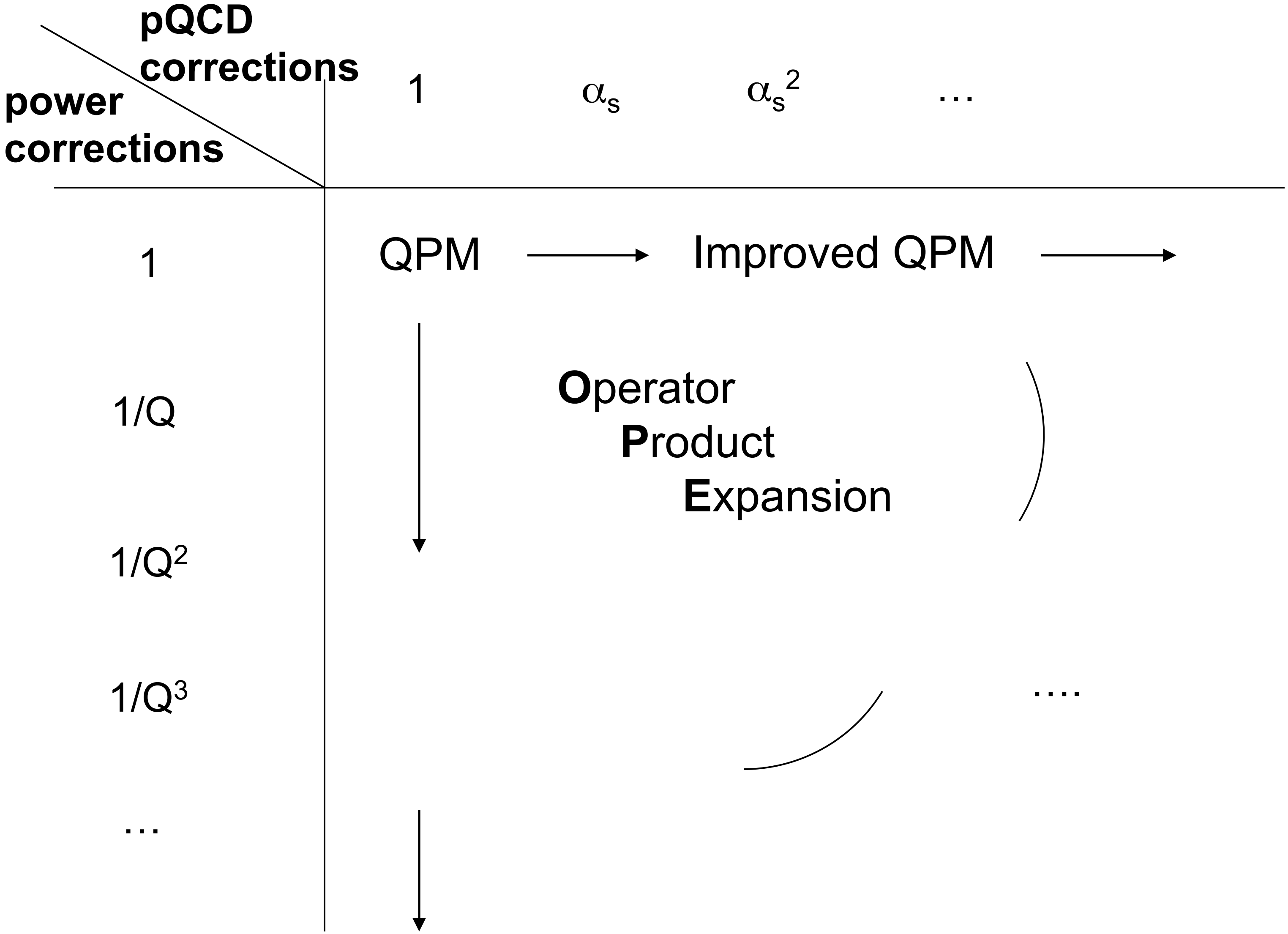
(⇔ in DIS regime)



power series parametrizes the bilocal operator Φ

N.B. – the necessary powers of *M* are determined by decomposing the matrix element in Lorentz tensors and making a dimensional analysis

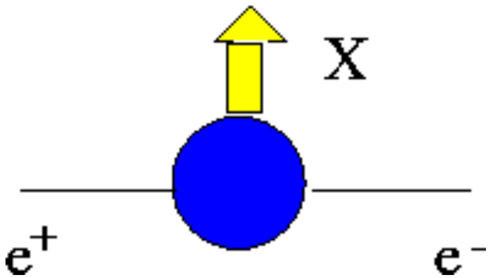
- definition does not coincide with $t = d$ – spin, but this is more convenient and it allows to directly estimate the level of suppression as *1/Q*



N.B. for the moment only
inclusive e^+e^- and DIS

OPE valid only for inclusive e^+e^- and DIS

inclusive e^+e^-



$$W^{\mu\nu} = \int d^4\xi \, e^{iq\cdot\xi} \langle 0| \left[J^\mu(\xi) \, , \, J^\nu(0) \right] |0\rangle$$

$q^\mu \overset{c.m.}{=} (q^0, \mathbf{0})$ regime DIS: $Q^2 \to \infty \;\Rightarrow\; q^0 \to \infty$

causalità $\Rightarrow$ [..] definito su $\xi^2 \geq 0$

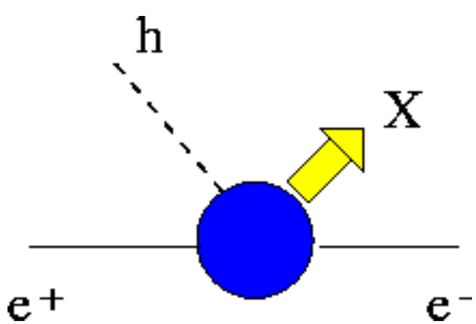
contributo principale all'integrale da $q \cdot \xi$ finito

$\Rightarrow \xi^0 \sim 0 \Rightarrow \xi \sim 0$

composite operator at short distance → OPE

semi-inclusive e^+e^-

$$W^{\mu\nu} = \frac{1}{4\pi} \sum_X \int d^4\xi \, e^{iq\cdot\xi} \langle 0| J^\mu(\xi) |P_h X\rangle \langle P_h X| J^\nu(0) |0\rangle$$



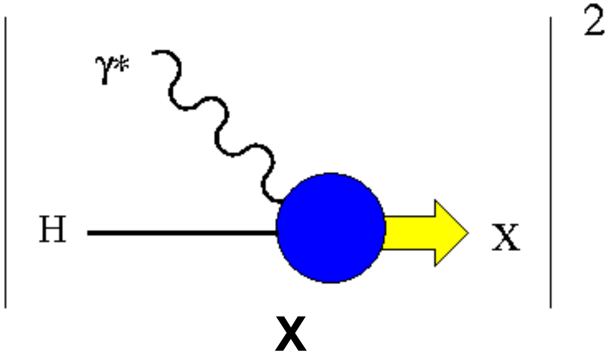
hadron rest frame $P_h^\mu = (M_h, \mathbf{0})$

$q{\cdot}\xi$ finite → $W^{\mu\nu}$ dominated by $\xi^2 \sim 0$

but ket $|P_h>$ prevents closure $\sum_X$

→ OPE cannot be applied

inclusive DIS

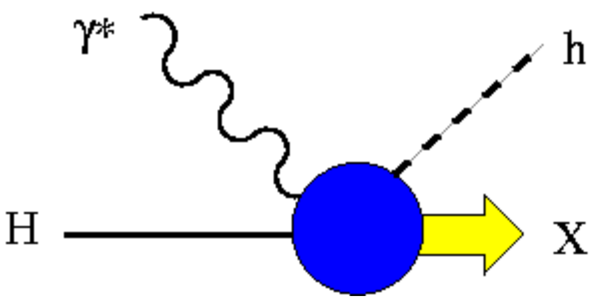


$$2MW^{\mu\nu} = \frac{1}{2\pi}\int d^4\xi\, e^{iq\cdot\xi}\,\langle P|\,[J^\mu(\xi)\,,\; J^\nu(0)]\,|P\rangle$$

in DIS limit $\Rightarrow$ ($x_B = -q^2/2P\cdot q$ finite) $\Leftrightarrow$ ($\nu \rightarrow \infty$)
$q\cdot\xi$ finite in DIS limit $\rightarrow \xi^0 \sim 0 \rightarrow \xi^\mu \sim 0$

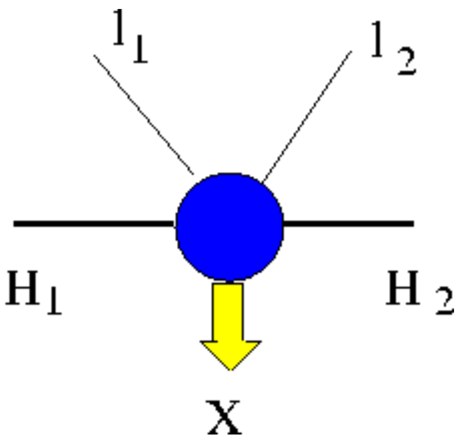
semi-inclusive DIS

$$2MW^{\mu\nu} \propto \sum_X \int d^4\xi\, e^{iq\cdot\xi}\,\langle P|J^\mu(\xi)|P_h X\rangle\langle P_h X|J^\nu(0)|P\rangle$$



ket $|P_h>$ prevents closure $\sum_X$
$\rightarrow$ OPE cannot be applied

Drell-Yan



$$W^{\mu\nu} = \frac{1}{2}\, s \int d^4\xi\, e^{iq\cdot\xi}\, \langle P_1 P_2 | J^\mu(\xi)\, J^\nu(0) | P_1 P_2 \rangle$$

$q{\cdot}\xi$ finite → dominance of $\xi^2 \sim 0$

but < .. > is not limited in any frame since $s=(P_1+P_2)^2 \sim 2P_1 \cdot P_2 \geq Q^2$ and in $Q^2 \to \infty$ limit both P_1, P_2 are not limited
$W^{\mu\nu}$ gets contributes outside the light-cone!

which are the dominant diagrams for processes where OPE cannot be applied ?
is it possible to apply the OPE formalism (factorization) also to semi-inclusive processes?

classify dominant contributions in various hard processes

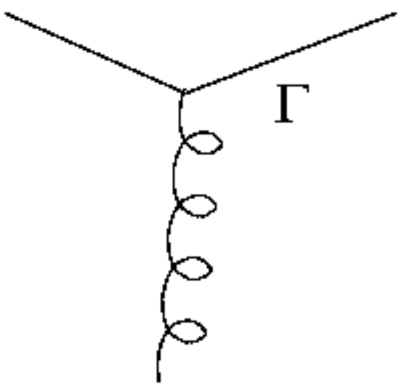
preamble :

- free quark propagator at short distance $S_F(x)$

$$S_F(x) = (i\gamma\cdot\partial + m)\,\Delta(x) \sim (i\gamma\cdot\partial + m)\,\frac{1}{4\pi^2 i}\,\frac{1}{x^2 - i\epsilon} \;+ ...$$

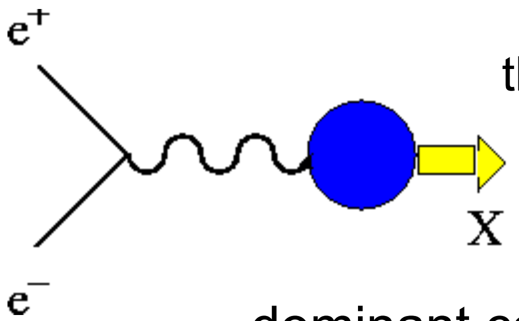
$$= \frac{-2\gamma\cdot x}{(x^2 - i\epsilon)^2}\,\frac{i}{4\pi^2 i} + ... \sim \frac{1}{x^3} + \text{termini meno singolari}$$

- interaction with gluons does not increase singularity



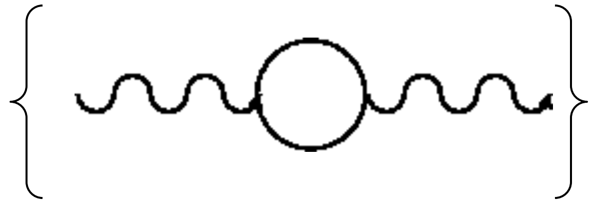
$$\sim \int \frac{d^4y}{(2\pi)^4}\,S_F(x-y)\,\Gamma\,S_F(x) \;\sim\; \frac{1}{x^2}$$

inclusive e^+e^-



optical theorem

$= Im$

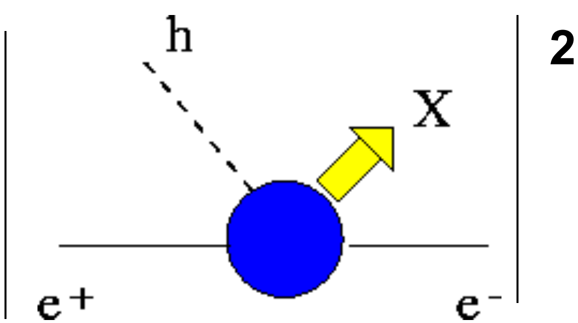


$\sim \ \mathrm{Tr}\left[S_F(x)\right]^2 \sim \dfrac{1}{x^6}$

dominant contribution at short distance → σ_{tot} in QPM

radiative corrections → ~ $(\log x^2\mu_R^2)^n$ → recover OPE results

semi-inclusive e^+e^-



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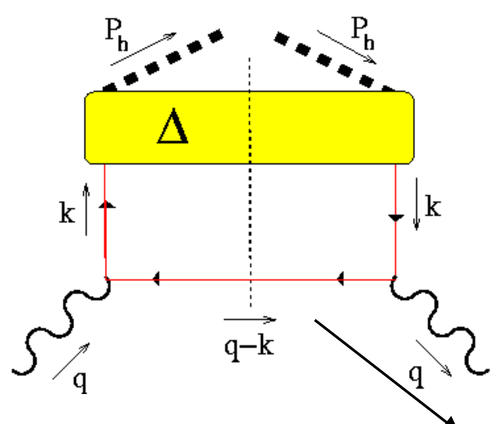


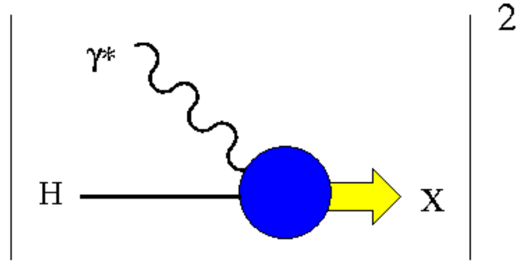
dominant diagram at short distance because $\sim \ S_F(x) \sim \dfrac{1}{x^3}$

radiative corrections → ~ $(\log x^2\mu_R^2)^n$

factorization between hard vertex and soft fragmentation

(cont'ed)

<u>inclusive DIS</u>



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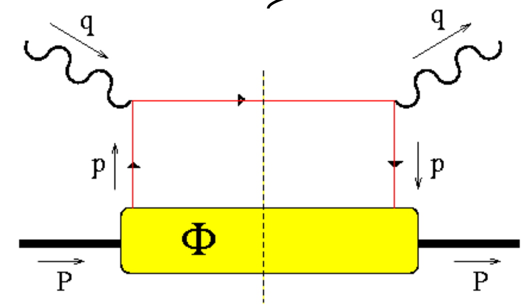
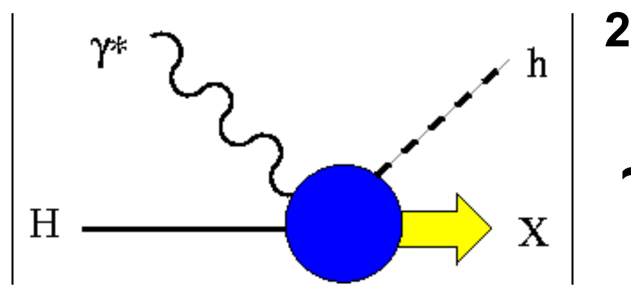


dominant diagram at short distance because $\sim S_F(x) \sim \dfrac{1}{x^3}$

radiative corrections → ~ $(\log x^2\mu_R{}^2)^n$ hence recover OPE result

<u>semi-inclusive DIS</u>



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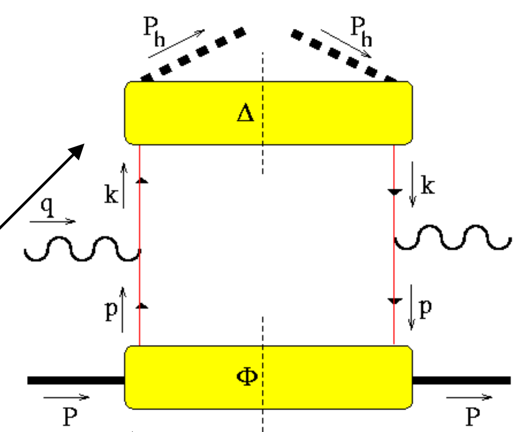

from semi-inclusive e^+e^-

factorization between hard e.m.
vertex and distribution and fragmentation
functions (from soft matrix elements)

from inclusive DIS

pQCD corrections: 1, α_s, α_s^2, …

power corrections: 1, 1/Q, $1/Q^2$, $1/Q^3$, …

QPM → IQPM →

Operator **P**roduct **E**xpansion

convolution approach

diagrammatic approach

….

systematic studies of power corrections (↔ OPE for inclusive DIS and e^+e^-) (Ellis,Furmanski,Petronzio, ’82)

convolution with factorized hard scattering (Efremov,Teryaev,Jaffe, Ji,Ralston,Soper,Qiu, Sterman,Collins,Leader Anselmino…)

(cont'ed)

for all DIS or e^+e^- processes (either inclusive or semi-inclusive) the dominant contribution to hadronic tensor comes from light-cone kin.

- definition and proprerties of light-cone variables
- quantized field theory on the light-cone
- Dirac algebra on the light-cone