helicity distribution and measurement of N spin

in general, $g_1(x_B,Q^2)$: dependence on Q^2 (= scaling violations)
calculable in perturbative QCD

interest in $g_1(x_B\,,Q^2)$ is due to its 1° Mellin moment
→ information on quark helicity; it is calculable on lattice

1° Mellin moment of g_1

$$\Gamma_1(Q^2) = \int_0^1 dx\, g_1(x,Q^2) = \frac{1}{2}\sum_{f,\bar f} e_f^2 \int_0^1 dx\,(q_f^\uparrow(x,Q^2) - q_f^\downarrow(x,Q^2)) = \frac{1}{2}\sum_{f\bar f} e_f^2\, \Delta q_f$$

$$\Delta q_f = \int_0^1 dx\,(q_f^\uparrow(x,Q^2) - q_f^\downarrow(x,Q^2))$$

exp. → A_1 (A_2~0) → g_1 (x_B,Q^2) → $\Gamma_1(Q^2)$ → Δq_f

1 relation for f ≥ 3 unknowns !

(cont'ed)

in QPM for proton : $$\Gamma_1^p = \frac{1}{2}\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right)$$

QPM : wave function of q in P↑ "induced" by $SU_f(3) \otimes SU(2)$

$$|P^\uparrow\rangle \approx \frac{1}{\sqrt{6}}\left(2u^\uparrow u^\uparrow d^\downarrow - u^\uparrow u^\downarrow d^\uparrow - u^\downarrow u^\uparrow d^\uparrow\right)$$

$\rightarrow \Gamma_1{}^p = 5/18 \sim 0.28$

$\Delta\Sigma = 1$

3 unknowns → info from axial current $A_\mu{}^a \sim \gamma_\mu \gamma_5 T^a$ in semi-leptonic decays (ex. β decay) in baryonic octet

Result:

$$\begin{aligned}\Gamma_1^p &= \int_0^1 dx\, g_1^p(x) \sim \frac{1}{12}\langle A_\mu^3\rangle\left[1+\frac{5}{3}\frac{\langle A_\mu^8\rangle}{\langle A_\mu^3\rangle}\right] = \frac{1}{12}\left|\frac{g_A}{g_V}\right|_{np}\left[1+\frac{5}{3}\frac{3F-D}{F+D}\right] \\ &= 0.17 \pm 0.01\end{aligned}$$

$$\Delta\Sigma = 3F - D = 0.60 \pm 0.12$$

from a fit to semi-leptonic decays → F= 0.47 ± 0.004 ; D=0.81± 0.003

Ellis-Jaffe sum rule ('73)
(hp.= perfect symmetry $SU_f(3)$ + $\Delta s=0$)

complicated corrections

Experiment EMC (CERN, '87)

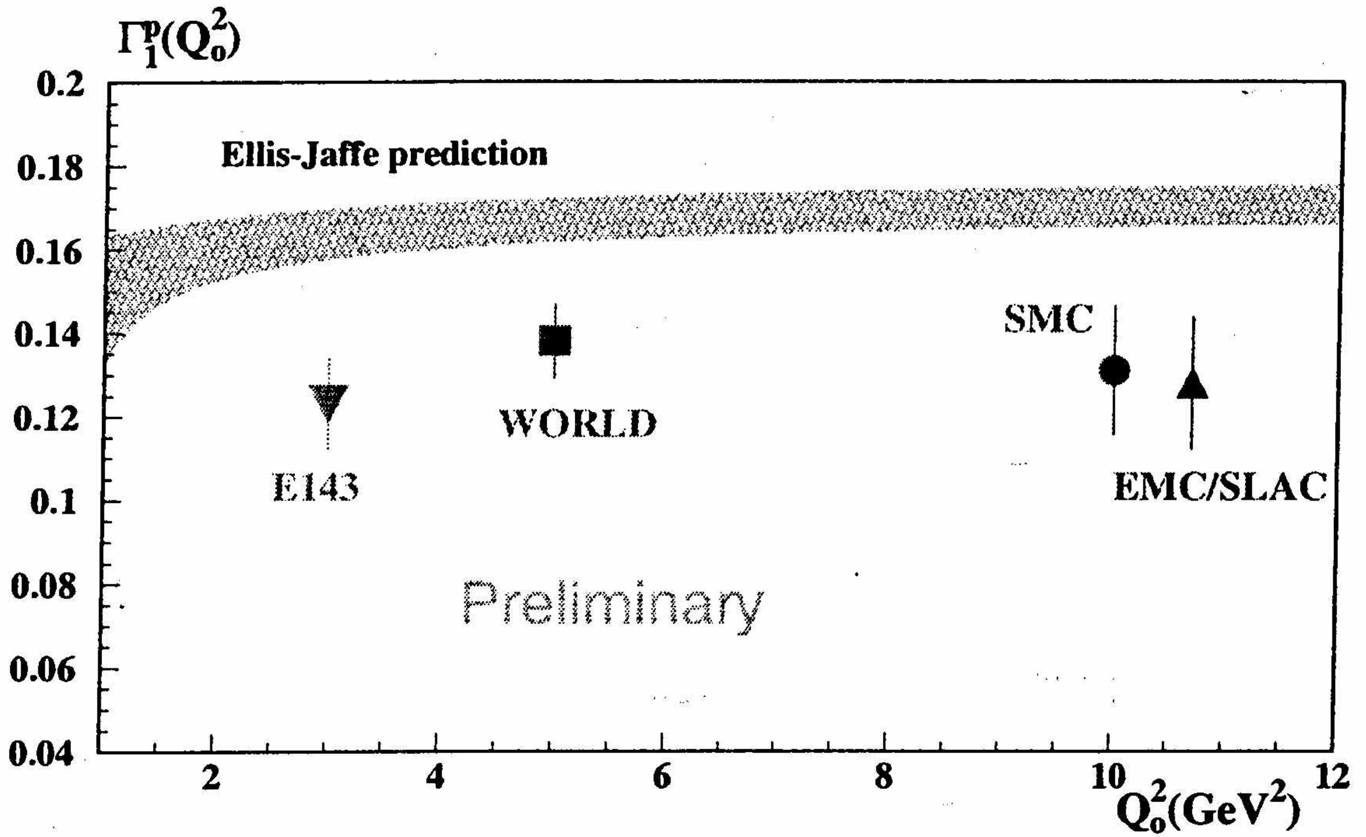
$\mu^{\uparrow}p^{\uparrow} \rightarrow \mu p$ at $Q^2 = 10.7$ GeV^2

$$A_{\parallel} = \frac{d\sigma^{\uparrow\downarrow} - d\sigma^{\uparrow\uparrow}}{d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}} \sim \frac{E - E'\epsilon}{E(1+\epsilon R)} A_1 \sim \frac{E - E'\epsilon}{E(1+\epsilon R)} \frac{g_1(x_B, Q^2)}{F_1(x_B, Q^2)}$$

$$\Gamma_1^p(10.7) = \int_{x_{min}}^{x^{max}} dx\, g_1(x, 10.7)$$
$$= 0.126 \pm 0.010 \pm 0.015$$

$R = \sigma_L/\sigma_T$
from unpolarized cross section

confirmed also from:
SMC (Cern),
E142 and E143 (SLAC)



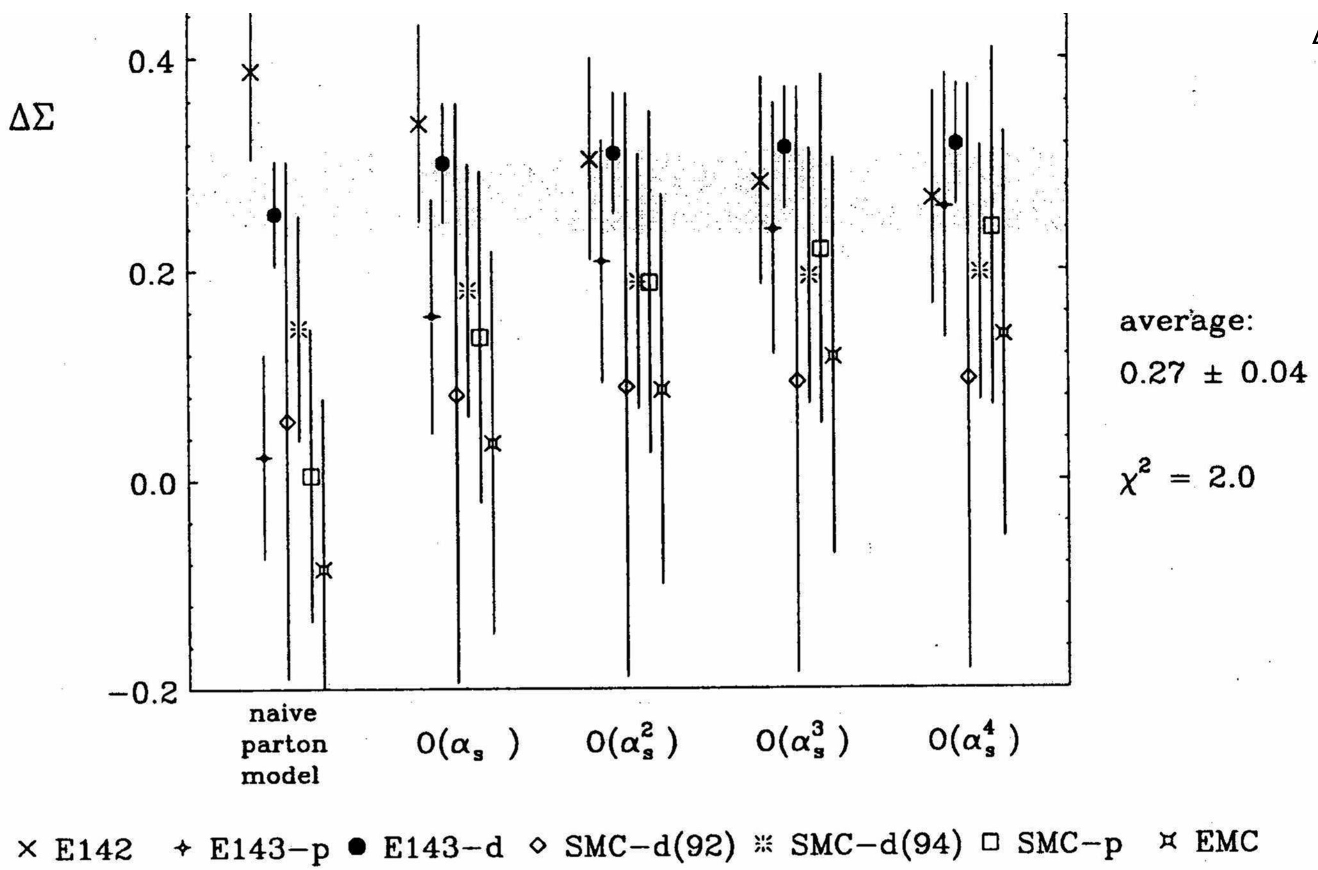
Spin crisis

$F + D + \Gamma_1^p(Q^2) \rightarrow \Delta\Sigma$ and Δu, Δd, Δs

$Q^2 = 10.7\ \mathrm{GeV}^2$
$\Delta\Sigma = 0.13 \pm 0.19$

$\Delta u = 0.78 \pm 0.10$
$\Delta d = 0.50 \pm 0.10$
$\Delta s = -0.20 \pm 0.11$

negative sea polarization



$Q^2 = 3\ \mathrm{GeV}^2$
$\Delta\Sigma = 0.27 \pm 0.04$

(cont'ed)

QPM	Ellis – Jaffe sum rule	exp.
	$SU_f(3)$ + $\Delta s=0$	
		$Q^2 = 10.7\ GeV^2$
$\Gamma_1^p \sim 0.28$	$\Gamma_1^p = 0.17 \pm 0.01$	$\Gamma_1^p = 0.126 \pm 0.010 \pm 0.015$
$\Delta\Sigma = 1$	$\Delta\Sigma = 0.60 \pm 0.12$	$\Delta\Sigma = 0.13 \pm 0.19$
		$Q^2 = 3\ GeV^2$
	discrepancy $> 2\sigma$	$\Delta\Sigma = 0.27 \pm 0.04$

Violation of $SU_f(3)$?

extrapolating $g_1(x)$ for $x \to 0$?

axial anomaly $\partial^\mu A^0_\mu = \frac{n_f \alpha_s}{2\pi} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$

$\to$ gluon contribution ?

$$\Delta q = \Delta q' - \frac{\alpha_s}{2\pi} \Delta g$$

} none of them quantitatively explains observed discrepancy

polarized Bjorken sum rule

$$\int_0^1 dx\,[g_1^p(x) - g_1^n(x)] = \frac{1}{6}\frac{G_A}{G_V}\left(1 - \frac{\alpha_s(Q^2)}{\pi} + \dots\right)$$

axial

vector

from weak couplings in β decay of N

pQCD corrections

QPM: wave function of q in P according to $SU_f(3) \otimes SU(2)$

$$A_1^p = \frac{\sum_f e_f^2 (q_f^\uparrow - q_f^\downarrow)}{\sum_f e_f^2 (q_f^\uparrow + q_f^\downarrow)}$$

$$A_1^n = \text{stesso con } u \leftrightarrow d = 0$$

$$\int_0^1 dx\,(g_1^p - g_1^n) = \int_0^1 dx\,(A_1^p F_1^p - A_1^n F_1^n) = \frac{5}{18}$$

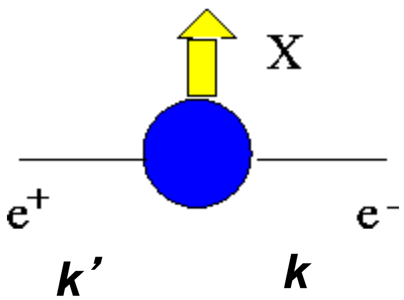
$$\Rightarrow \quad \frac{G_A}{G_V} \overset{\text{QPM}}{=} \frac{5}{3} \leftrightarrow 1.6667 \pm 0.003$$

exp. 1.267 ± 0.004

Sum rule :

QPM	+ pQCD	exp.
0.27778	0.191 ± 0.002	0.209 ± 0.003

inclusive e^+e^- annihilations



$q = k+k'$ time-like $q^2 \equiv Q^2 = s \geq 0$

$$d\sigma = \frac{1}{\mathcal{F}} |\mathcal{M}|^2 \, dR$$

$$\mathcal{F} = 4\sqrt{(k \cdot k')^2 - k^2 k'^2} \overset{\text{TRF}}{=} 2Q^2 \equiv 2s$$

$$\mathcal{M} = \bar{v}(k')\gamma_\mu u(k) \, \frac{e^2}{Q^2} \langle P_X | J^\mu(0) | 0 \rangle \qquad dR = (2\pi)^4 \delta(k + k' - P_X) \, \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0}$$

$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} L_{\mu\nu} \, H^{\mu\nu}$$

$$L_{\mu\nu} = 2\,(k_\mu k'_\nu + k'_\mu k_\nu - k \cdot k' \, g_{\mu\nu})$$

$$H^{\mu\nu} = \sum_{S_X} \langle 0 | J^\mu | P_X \rangle \langle P_X | J^\nu | 0 \rangle$$

$$\sigma = \frac{1}{2} \, \frac{1}{2Q^2} \, \frac{e^4}{Q^4} L_{\mu\nu} \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \, \delta(q - P_X) \, H^{\mu\nu} = \frac{4\pi^2\alpha^2}{Q^6} L_{\mu\nu} \, W^{\mu\nu}$$

average on initial polarizations

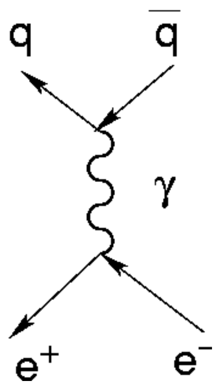
(cont'ed)

QPM picture

no hadrons in initial and final states

σ in QPM ≡ elementary σ $e^+e^- \to q\bar{q}$

only N_c ways of creating a pair by conserving color in vertex



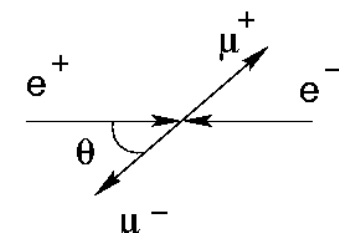
$Q^2 = s$ such that only γ are produced

$$\sigma(e^+e^- \to q\bar{q}) \equiv \sigma(e^+e^- \to \mu^+\mu^-)$$

$$\sigma(e^+e^- \to X) = N_c \sum_f e_f^2 \, \sigma(e^+e^- \to q\bar{q})$$

$$= N_c \sum_f e_f^2 \int d\Omega \, \frac{d\sigma}{d\Omega}(e^+e^- \to \mu^+\mu^-)$$

$$= N_c \sum_f e_f^2 \int d\Omega \, \frac{\alpha^2}{4Q^2}(1+\cos^2\theta) = N_c \sum_f e_f^2 \, \frac{4\pi\alpha^2}{3Q^2}$$



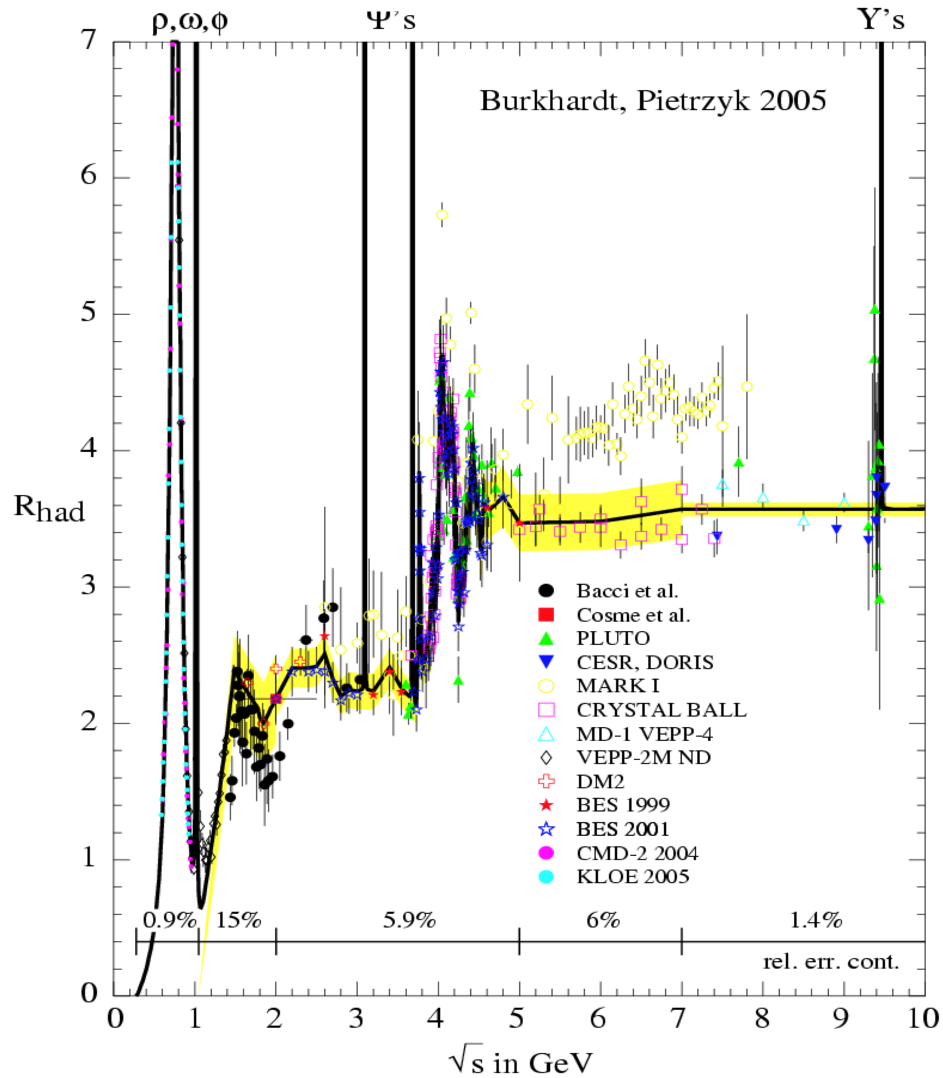
Q^2 σ (e^+e^- → *X*) scales !

hence

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_f e_f^2$$

evidence of N_c
test of gauge structures
$SU_c(3)$ and $SU_f(3)$

below *c* threshold
$R = 3\ (4/9 + 2/9) = 2$
around threshold
resonances J/ψ, ψ'
above *c* threshold
R = 2 + 3 4/9 = 3+1/3
.....



see also
Wu, Phys.Rep. **C107** 59 (84)

inclusive e⁺e⁻ annihilations

(factorization) theorem :

total cross section is finite in the limit of massless particles, i.e. it is free from “infrared” (IR) divergences

(Sterman, ’76, ’78)

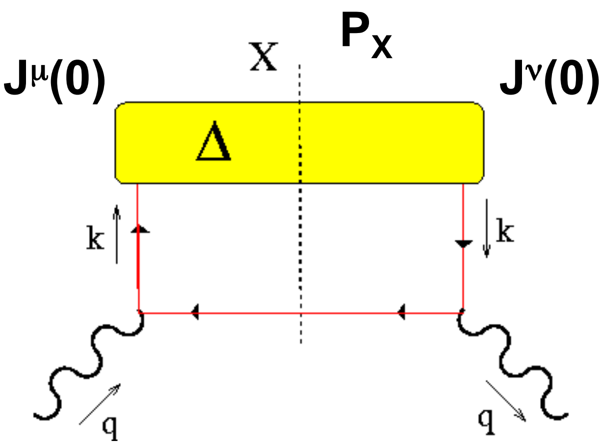
[generalization of theorem KLN (Kinoshita-Lee-Nauenberg)]

$$\sigma_{tot} = N_c \frac{4\pi\alpha^2}{3Q^2} \sum_f e_f^2 \sum_n s_n \, \alpha_s^n(Q^2)$$

$s_0 = 1$

QPM pQCD corrections

inclusive e^+e^-



$$W^{\mu\nu} = \int \frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0} (2\pi)^4 \, \delta(q - P_X) \langle 0|J^\mu(0)|P_X\rangle\langle P_X|J^\nu(0)|0\rangle$$

$$= \int d^4\xi \, e^{iq\cdot\xi} \langle 0|[J^\mu(\xi) \, , \, J^\nu(0)]|0\rangle$$

theorem: dominant contribution in Bjorken limit comes from short distances $\xi \to 0$ (on the light-cone)

but product of operators in the same space-time point is not always well defined in field theory !

(cont'ed)

Example: free neutral scalar field $\phi(x)$; free propagator $\Delta(x-y)$

$$\langle 0|\mathcal{T}\left[\phi(x)\,\phi(y)\right]|0\rangle \;=\; -i\Delta(x-y) = i\int \frac{d^4p}{(2\pi)^4}\,\frac{e^{-ip\cdot(x-y)}}{p^2-m^2+i\epsilon}$$

$$= \frac{m}{4\pi^2}\,\frac{K_1\left(m\sqrt{-(x-y)^2+i\epsilon}\right)}{\sqrt{-(x-y)^2+i\epsilon}} - \frac{i}{4\pi}\,\delta((x-y)^2) \xrightarrow{x\to y} \infty$$

K_1 modified Bessel funct. of 2nd kind

Example: interacting neutral scalar field $\phi(x)$

$$\hat{P}\,|p,n\rangle = p\,|p,n\rangle$$

$$\phi(x) = e^{i\hat{P}\cdot x}\,\phi(0)\,e^{-i\hat{P}\cdot x}$$

$$\langle 0|\phi(x)^2|0\rangle \;=\; \int\frac{d\mathbf{p}}{(2\pi)^3 2p^0}\sum_n \langle 0|\phi(0)|p,n\rangle\langle p,n|\phi(0)|0\rangle$$

$$\geq \int\frac{d\mathbf{p}}{(2\pi)^3 2p^0}\,|\langle 0|\phi(0)|p,1\rangle|^2 \equiv N\int\frac{d\mathbf{p}}{(2\pi)^3 2p^0} \;\to\; \infty$$

depends only on $p^2=m^2 \to$ it is a constant *N*

Operator **P**roduct **E**xpansion

(Wilson, '69 first hypothesis; Zimmermann, '73 proof in perturbation theory; Collins, '84 diagrammatic proof)

(operational) definition of composite operator:

$$\widehat{A}(x)\,\widehat{B}(y) \equiv \sum_{i=0}^{\infty} C_i(x-y)\,\widehat{O}_i\left(\frac{x+y}{2}\right)$$

- local operators $\hat{O}_i$ are regular for every i=0,1,2…
- divergence for $x \rightarrow y$ is reabsorbed in coefficients C_i
- terms are ordered by decreasing singularity in C_i , i=0,1,2…
- usually $\hat{O}_0 = \mathbf{I}$, but explicit expression of the expansion must be separately determined for each different process
- OPE is also an operational definition because it can be used to define a regular composite operator.
  Example : theory ϕ^4 ; the operator $\phi(x)^2$ can be defined as

$$\phi(x)^2 \equiv \lim_{x \to y} \frac{\phi(x)\,\phi(y) - C_0(x-y)}{C_1(x-y)} = \widehat{O}_1(x)$$

the Wick theorem

scalar field $$\phi(x) = \phi^+(x) + \phi^-(x) = \int \frac{d\mathbf{p}}{(2\pi)^3 2p^0} \left[a_p \, e^{-ip\cdot x} + a_p^\dagger \, e^{ip\cdot x} \right]$$

"normal" order : : = move $a^\dagger$ to left, a to right → annihilate on |0>

"time" order T = order fields by increasing times towards left

Step 1 $$\mathcal{T}\phi(x) = :\phi(x):$$

Step 2 $$[\mathcal{T}\phi(x_1)]\,\phi(x_2) = \mathcal{T}[\phi(x_1)\phi(x_2)] = :\phi(x_1):\;\phi(x_2)$$

$t_2<t_1$ $$= \phi(x_1)\phi^+(x_2) + \phi(x_1)\phi^-(x_2) = \phi(x_1)\phi^+(x_2) + \phi^-(x_1)\phi^-(x_2)$$

$$+\phi^+(x_1)\phi^-(x_2)$$

$$+\phi^-(x_2)\phi^+(x_1) + \left[\phi^+(x_1), \phi^-(x_2)\right]$$

analogously for $t_2>t_1$

$$= \langle 0| \left[\phi^+(x_1), \phi^-(x_2)\right] |0\rangle$$

$$= \langle 0|\mathcal{T}[\phi(x_1)\phi(x_2)]|0\rangle$$

hence

$$\mathcal{T}[\phi(x)\phi(y)] = :\phi(x)\phi(y): + \langle 0|\mathcal{T}[\phi(x)\phi(y)]|0\rangle$$

$$\lim_{x\to y} \qquad \lim_{x\to y}$$

$$= 1 \cdot \hat{O}_1(x) + C_0(x-y)\cdot \mathbb{I}$$

recursive generalization

analogously non interacting fermion fields

$$\mathcal{T}\left[\psi(x)\bar{\psi}(y)\right] = :\psi(x)\bar{\psi}(y): + \langle 0|\mathcal{T}\left[\psi(x)\bar{\psi}(y)\right]|0\rangle$$

general formula of Wick theorem:

$$\overset{\sqcap}{\phi_i\phi_j} \equiv \langle 0|\mathcal{T}\left[\phi(x_i)\phi(x_j)\right]|0\rangle$$

$$\mathcal{T}\left[\phi_1\phi_2\ldots\phi_n\right] = :\phi_1\phi_2\ldots\phi_n:$$

$$+\sum_{i\neq j=1}^{n} P_{ij} :\phi_1\ldots\phi_{i-1}\phi_{i+1}\ldots\phi_{j-1}\phi_{j+1}\ldots\phi_n: \overset{\sqcap}{\phi_i\phi_j}$$

$$+\sum_{i\neq j\neq k\neq l=1}^{n} :\phi_1\ldots\phi_{i-1}\phi_{i+1}\ldots\phi_{j-1}\phi_{j+1}\ldots\phi_{k-1}\phi_{k+1}\ldots\phi_{l-1}\phi_{l+1}\ldots\phi_n:$$

$$\left(P_{ijkl}\ \overset{\sqcap}{\phi_i\phi_j}\overset{\sqcap}{\phi_k\phi_l} + P_{ikjl}\ \overset{\sqcap}{\phi_i\phi_k}\overset{\sqcap}{\phi_j\phi_l} + P_{iljk}\ \overset{\sqcap}{\phi_i\phi_l}\overset{\sqcap}{\phi_j\phi_k}\right)$$

$$+\ldots$$

$P_{ij} = (-1)^m$

m= n^0 of permutations to reset indices in natural order 1, … ,i-1,i, … ,j-1,j, ... ,n

application to inclusive e^+e^- and DIS

$W^{\mu\nu}$ ⇒ $J^\mu(\xi)\, J^\nu(0)$ with J^μ e.m. current of quark

normal product : : useful to define a composite operator for $\xi \to 0$

⇒ study $\mathcal{T}\,[J^\mu(\xi)\, J^\nu(0)]$ per $\xi \to 0$ with Wick theorem

$$\mathcal{T}\,[J^\mu(\xi) J^\nu(0)] =$$

$$: \bar\psi(\xi)\gamma^\mu\psi(\xi)\ \bar\psi(0)\gamma^\nu\psi(0) : \ + \ : \bar\psi(\xi)\gamma^\mu\gamma^\nu\psi(0) : \ \overbracket{\psi(\xi)\bar\psi}(0) \ +$$

$$: \bar\psi(0)\gamma^\nu\gamma^\mu\psi(\xi) : \ \overbracket{\psi(0)\bar\psi}(\xi) \ - \mathrm{Tr}\,[\gamma^\mu\gamma^\nu] \ \ \overbracket{\psi(\xi)\bar\psi}(0) \ \ \overbracket{\psi(0)\bar\psi}(\xi)$$

$$= \mathrm{Tr}\,[\gamma^\mu\gamma^\nu] \ S_F(-\xi)\ S_F(\xi) - \ : \bar\psi(\xi)\gamma^\mu\gamma^\nu\psi(0) : \ i\,S_F(\xi)$$

$$- \ : \bar\psi(0)\gamma^\nu\gamma^\mu\psi(\xi) : \ i\,S_F(-\xi) \ + \ : \bar\psi(\xi)\gamma^\mu\psi(\xi)\ \bar\psi(0)\gamma^\nu\psi(0) :$$

$$\overbracket{\psi(\xi)\bar\psi}(0) = \langle 0|\mathcal{T}\left[\psi(\xi)\bar\psi(0)\right]|0\rangle = -i\,S_F(\xi) = i\int \frac{d^4p}{(2\pi)^4}\,\frac{e^{-ip\cdot\xi}}{\not p - m + i\epsilon}$$

divergent for $\xi \to 0$ ⇒ OPE

singularities of free fermion propagator

$$S_F(\xi) = (i\,\partial\!\!\!/ + m)\,\Delta(\xi)$$

$$\Delta(\xi) = -\lim_{\epsilon\to 0}\int \frac{d^4p}{(2\pi)^4}\,\frac{e^{-ip\cdot\xi}}{p^2-m^2+i\epsilon} = i\frac{m}{4\pi^2}\lim_{\epsilon\to 0}\frac{K_1\left(m\sqrt{-\xi^2+i\epsilon}\right)}{\sqrt{-\xi^2+i\epsilon}} + \frac{1}{4\pi}\delta(\xi^2)$$

$$\overset{\xi\to 0}{\sim} \frac{im}{4\pi^2}\lim_{\epsilon\to 0}\frac{1}{m\sqrt{-\xi^2+i\epsilon}}\,\frac{1}{\sqrt{-\xi^2+i\epsilon}} + \text{termini meno singolari}$$

$$= \frac{1}{4\pi^2 i}\lim_{\epsilon\to 0}\frac{1}{\xi^2-i\epsilon} + \text{termini meno singolari}$$

light-cone singularity

degree of singularity proportional to powers of q in Fourier transform

$$\int_{-\infty}^{\infty} dx\,\frac{e^{iq\cdot x}}{(x-i\epsilon)^\alpha} = \frac{2\pi e^{i\alpha\pi/2}}{\Gamma(\alpha)}\,\theta(q)\,q^{\alpha-1}$$

highest singularity in OPE coefficients

⇓

dominant contribution to J^μ in $W^{\mu\nu}$

(cont'ed)

$$S_F(\xi) = (i\gamma\cdot\partial + m)\,\Delta(\xi) \sim (i\gamma\cdot\partial + m)\,\frac{1}{4\pi^2 i}\,\frac{1}{\xi^2 - i\epsilon} + \ldots$$

$$= \frac{-2\gamma\cdot\xi}{(\xi^2 - i\epsilon)^2}\,\frac{i}{4\pi^2 i} + \frac{1}{4\pi^2 i}\,\frac{m}{\xi^2 - i\epsilon} + \text{termini meno singolari}$$

most singular term in $\mathcal{T}\,[J^\mu(\xi)\,J^\nu(0)]$

$$\mathrm{Tr}\,\left[S_F(-\xi)\gamma^\mu S_F(\xi)\gamma^\nu\right] \sim -\frac{4}{16\pi^4(\xi^2 - i\epsilon)^4}\,\mathrm{Tr}\,[\,\not\xi\gamma^\mu\,\not\xi\gamma^\nu] + \ldots.$$

$$= \frac{\xi^2 g^{\mu\nu} - 2\xi^\mu\xi^\nu}{\pi^4(\xi^2 - i\epsilon)^4} + \ldots.$$

less singular term in $\mathcal{T}\,[J^\mu(\xi)\,J^\nu(0)]$

$$:\bar\psi(\xi)\gamma^\mu\psi(\xi)\,\bar\psi(0)\gamma^\nu\psi(0): \;=\; \hat{O}(\xi, 0)$$ regular bilocal operator