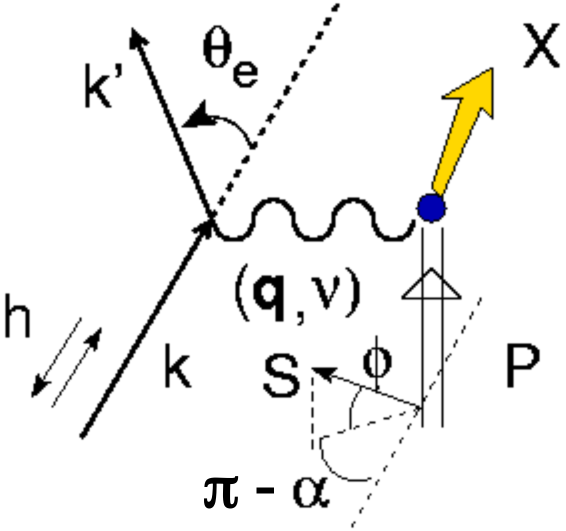
Quark Parton Model (QPM)

main failure: Momentum sum rule

partons with no charge (= gluons) carry around half of *N* momentum, but they are not included in QPM!

Introduce polarization: inclusive DIS



recall: if S=0 → parity violation from weak decays

→ current *V-A* → $W^{\mu\nu}_A$

if S ≠ 0 → two 4-vectors P,q + one 4-pseudovector S

→ richer structure of hadronic tensor

build S^μ such that $S^2 = -1$ and $S\cdot P = 0$

$$S^\mu = \frac{S\cdot q}{P\cdot q}\left(P^\mu - \frac{M^2}{P\cdot q}q^\mu\right) + S_\perp^\mu = \frac{\lambda}{M}\left(P^\mu - \frac{M^2}{P\cdot q}q^\mu\right) + S_\perp^\mu$$

helicity $\lambda = M\frac{S\cdot q}{P\cdot q}$

$$S\cdot P = 0 \rightarrow S_\perp \cdot P = 0$$

$$S^2 \sim -(\lambda^2 + \mathbf{S}_\perp^2) = -1$$

hadronic tensor

S = ½ → $W^{\mu\nu}$ is at most linear in S, because it is a 2x2 matrix in spin space

⇒ it can be expanded on basis of Dirac σ matrices

$$W^{\mu\nu} = \sum_{\alpha\alpha'} W^{\mu\nu}_{\alpha\alpha'} \rho_{\alpha\alpha'} = \frac{1}{2} \sum_{\alpha\alpha'} W^{\mu\nu}_{\alpha\alpha'} (1 + \mathbf{P} \cdot \sigma)_{\alpha\alpha'}$$

target spin density matrix

polarization vector

$$P_i = \frac{N_+ - N_-}{N_+ + N_-} = < \sigma_i > = \mathrm{Tr}(\rho\sigma_i)$$

- S^μ coplanar with scattering plane → $\phi = 0$
- hermitean tensor
- parity invariance
- time-reversal invariance
- current conservation

polarized DIS hadronic tensor

$W^{\mu\nu} = W^{\mu\nu}_S + W^{\mu\nu}_A$

$$W_S^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1 + \frac{\tilde{P}^\mu \tilde{P}^\nu}{M^2} W_2$$

$$\tilde{P}^\mu = P^\mu - \frac{P \cdot q}{q^2} q^\mu$$

$$W_A^{\mu\nu} = \frac{i}{M^2} \epsilon^{\mu\nu\rho\sigma} q_\rho \left[A_1 S_\sigma + A_2 P_\sigma\right]$$

scalar

pseudoscalar

$$\begin{aligned} W_A^{\mu\nu} &= \frac{i}{M^2} \epsilon^{\mu\nu\rho\sigma} q_\rho S_\sigma \left[\underline{G_1(\nu, Q^2)} + \frac{P \cdot q}{M^2} \underline{G_2(\nu, Q^2)}\right] \\ & \quad - \frac{i}{M^2} \epsilon^{\mu\nu\rho\sigma} q_\rho P_\sigma \frac{S \cdot q}{M^2} G_2(\nu, Q^2) \\ &= i \epsilon^{\mu\nu\rho\sigma} q_\rho P_\sigma \lambda \frac{G_1(\nu, Q^2)}{M^3} \\ & \quad + i \epsilon^{\mu\nu\rho\sigma} q_\rho S_{\perp\sigma} \frac{1}{M^2} \left[G_1(\nu, Q^2) + \frac{P \cdot q}{M^2} G_2(\nu, Q^2)\right] \end{aligned}$$

scattering amplitude

lepton polarized with helicity h=±

lepton tensor : $L_{\mu\nu} = L_{\mu\nu}{}^{S} \pm L_{\mu\nu}{}^{A}$

$$\begin{aligned} L^S_{\mu\nu} &= 2k_\mu k'_\nu + 2k_\nu k'_\mu - 2k\cdot k' g_{\mu\nu} \\ L^A_{\mu\nu} &= h2i\varepsilon_{\mu\nu\rho\sigma}k^\rho q^\sigma \end{aligned}$$

$L_{\mu\nu}{}^{S}\ W^{\mu\nu}{}_{S} \rightarrow$

$$\frac{d\sigma^0}{dE'd\Omega} = \frac{4\alpha^2}{Q^4}E'^2\left(2\sin^2\frac{\theta_e}{2}W_1 + \cos^2\frac{\theta_e}{2}W_2\right)$$

$L_{\mu\nu}{}^{A}\ W^{\mu\nu}{}_{A} \leftarrow$

$$\begin{aligned} L^A_{\mu\nu} i\varepsilon^{\mu\nu\rho\sigma}q_\rho S_\sigma &= 8EE'\sin^2\frac{\theta_e}{2}S\cdot(k+k') \\ L^A_{\mu\nu}(-i)\varepsilon^{\mu\nu\rho\sigma}q_\rho P_\sigma &= -8EE'\sin^2\frac{\theta_e}{2}P\cdot(k+k') \end{aligned}$$

$$L^A_{\mu\nu}W^{\mu\nu}_A = \frac{8EE'}{M^2}\sin^2\frac{\theta_e}{2}\left[\left(G_1 + \frac{P\cdot q}{M^2}G_2\right)S\cdot(k+k') - \frac{S\cdot q}{M^2}G_2 P\cdot(k+k')\right]$$

cross section

$$\begin{cases} k &= (E, 0, 0, E) \\ k' &= (E', E' \sin\theta_e, 0, E' \cos\theta_e) \\ \widehat{S} &= (0, \sin\alpha\cos\phi, \sin\alpha\sin\phi, \cos\alpha) \end{cases}$$

coplanar → $\phi = 0$

$$\begin{aligned} S\cdot(k+k') &= -E'(\cos\theta_e\cos\alpha+\sin\theta_e\sin\alpha)-E\cos\alpha \\ P\cdot(k+k') &= M(E+E') \\ S\cdot q &= E'(\cos\theta_e\cos\alpha+\sin\theta_e\sin\alpha)-E\cos\alpha \end{aligned}$$

$$\frac{d\Delta\sigma^h}{dE'd\Omega} = -h\,\frac{2\alpha^2}{Q^2}\,\frac{1}{M^2}\,\frac{E'}{E}\left\{\underline{\cos\alpha}\left[(E+E'\cos\theta_e)\,G_1-\frac{Q^2}{M}G_2\right] + E'\sin\theta_e\,\underline{\sin\alpha}\,(G_1+\frac{2E}{M}G_2)\right\}$$

$\alpha = 0$ ⇔ S || k

$\alpha = \pi/2$ ⇔ S ⊥ k

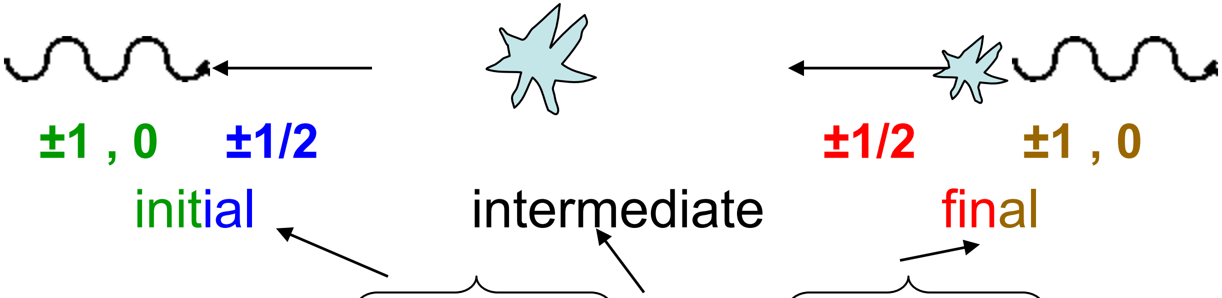
(cont'ed)

$$\frac{d\sigma^{o}}{dE'\,d\Omega},\ \frac{d\Delta\sigma^{h\pm}}{dE'\,d\Omega} \Longrightarrow F_1,\ F_2,\ G_1,\ G_2$$

why 4 structure functions F_1, F_2, G_1, G_2 ?

total cross section for γ^* absorption : σ_{tot} (γ^* N)
optical theorem : σ_{tot} (γ^* N) ∝ Im [f(θ_e=0) Compton]

$$f(\theta_e = 0, \vec{q}, \vec{N})$$



1	+1	+1/2	+3/2	+1	+1/2
2	+1	-1/2	+1/2	+1	-1/2
3	+1	-1/2	+1/2	0	+1/2
4	0	+1/2	+1/2	+1	-1/2
5	0	+1/2	+1/2	0	+1/2

related by time-reversal
→ 4 independent structures

helicity asymmetries

rearrangement of 4 independent combinations

$$
\begin{array}{rcccccc}
\left[(1,\tfrac{1}{2}) \to (1,\tfrac{1}{2})\right] + \left[(1,-\tfrac{1}{2}) \to (1,-\tfrac{1}{2})\right] & \equiv & W_T & = & W_1 & = & \sigma^T_{3/2} + \sigma^T_{1/2} \\
(0,\tfrac{1}{2}) \to (0,\tfrac{1}{2}) & \equiv & W_L & = & \left(1+\frac{\nu^2}{Q^2}\right) W_2 - W_1 & = & \sigma^L_{1/2} \\
\left[(1,\tfrac{1}{2}) \to (1,\tfrac{1}{2})\right] - \left[(1,-\tfrac{1}{2}) \to (1,-\tfrac{1}{2})\right] & \equiv & W_{TT} & = & \frac{1}{M^3}(-\nu M G_1 + Q^2 G_2) & = & \sigma^T_{3/2} - \sigma^T_{1/2} \\
(1,-\tfrac{1}{2}) \to (0,\tfrac{1}{2}) & \equiv & W_{LT} & = & \frac{\sqrt{Q^2}}{M^3}(M G_1 + \nu G_2) & = & \sigma^{LT}_{1/2}
\end{array}
$$

γ^* helicity → $\sigma^{\lambda_{\gamma^*}}_{J_z}$

intermediate ↗

asymmetries for scattering from γ^*

$$A_1 = \frac{\sigma^T_{1/2} - \sigma^T_{3/2}}{\sigma^T_{1/2} + \sigma^T_{3/2}} = -\frac{W_{TT}}{W_T} = \frac{\nu M G_1 - Q^2 G_2}{M^3 W_1} \qquad 1 \geq |A_1|$$

$$A_2 = \frac{W_{LT}}{W_T} = \frac{\sqrt{Q^2}(M G_1 + \nu G_2)}{M^3 W_1} \qquad R = \frac{\sigma_L}{\sigma_T} \geq |A_2| = \frac{\sigma_{LT}}{\sigma_T}$$

Accesso sperimentale alle asimmetrie

S || k → $\alpha = 0$

misura sperimentale accede a

$$A_{\parallel} = \frac{d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}}{d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}} = \frac{E - E'\epsilon}{E(1+\epsilon R)} A_1 + \frac{\epsilon Q}{E(1+\epsilon R)} A_2$$

$$A_{\perp} = \frac{d\sigma^{\uparrow\leftarrow} - d\sigma^{\uparrow\rightarrow}}{d\sigma^{\uparrow\leftarrow} + d\sigma^{\uparrow\rightarrow}} = \frac{E - E'\epsilon}{E(1+\epsilon R)} \sqrt{2\epsilon(1+\epsilon)}\, A_2 - \frac{\epsilon Q}{E(1+\epsilon R)} \sqrt{\frac{(1+\epsilon)^3}{2\epsilon}}\, A_1$$

S ⊥ k → $\alpha = \pi/2$

polarizz. lineare trasversa di γ^* $\epsilon = \left[1 + 2\frac{q^2}{Q^2} \tan^2\frac{\theta_e}{2}\right]^{-1}$

$$R = \frac{W_L}{W_T} = \left(1 + \frac{\nu^2}{Q^2}\right)\frac{W_2}{W_1} - 1$$

inversione

$$A_1 = \frac{2(E - E'\epsilon)\, E\,(1+\epsilon R)}{2(E - E'\epsilon)^2 + \epsilon(1+\epsilon)\, Q^2} A_{\parallel} - \sqrt{\frac{2\epsilon}{1+\epsilon}}\, \frac{QE(1+\epsilon R)}{2(E - E'\epsilon)^2 + \epsilon(1+\epsilon)\, Q^2} A_{\perp}$$

$$A_2 = \frac{E\,(1+\epsilon R)\, Q\,(1+\epsilon)}{2(E - E'\epsilon)^2 + \epsilon(1+\epsilon)\, Q^2} A_{\parallel} + \sqrt{\frac{2}{\epsilon(1+\epsilon)}}\, \frac{(E - E'\,\epsilon)\, E(1+\epsilon R)}{2(E - E'\epsilon)^2 + \epsilon(1+\epsilon)\, Q^2} A_{\perp}$$

misura di Q^2, ε, R, $A_{\parallel}$, $A_{\perp}$ → A_1, A_2

DIS limit

$\nu, Q^2 \to \infty$ with fixed x_B ; if $Q^2\, \sigma_{Jz}^{\lambda}$ scales, then

scaling :

$$MW_1(\nu,Q^2) \to F_1(x_B) \qquad \frac{\nu}{M} G_1(\nu,Q^2) \to \tilde{G}_1(x_B)$$

$$\nu W_2(\nu,Q^2) \to F_2(x_B) \qquad \frac{\nu^2}{M^2} G_2(\nu,Q^2) \to \tilde{G}_2(x_B)$$

(see expressions of A_1 and A_2)

scaling in helicity asymmetries :

$$A_1 = \frac{\nu M G_1(\nu,Q^2) - Q^2\, G_2(\nu,Q^2)}{M^3\, W_1(\nu,Q^2)} \to \frac{\tilde{G}_1(x_B)}{F_1(x_B)} - \frac{Q^2}{\nu^2} \frac{\tilde{G}_2(x_B)}{F_1(x_B)} \to \frac{\tilde{G}_1(x_B)}{F_1(x_B)}$$

$$A_2 = Q \frac{M G_1(\nu,Q^2) + \nu\, G_2(\nu,Q^2)}{M^3\, W_1(\nu,Q^2)} \to \sqrt{\frac{2 M x_B}{\nu}} \frac{\tilde{G}_1(x_B) + \tilde{G}_2(x_B)}{F_1(x_B)} \to 0$$

QPM picture

$$\frac{d\Delta\sigma^h}{dx_B\,dy} = \frac{2M\nu\pi}{E'}\,\frac{d\Delta\sigma^h}{dE'\,d\Omega} = \frac{2M\nu\pi}{E'}\,h\,\frac{\alpha^2}{Q^4}\,\frac{E'}{E}\,L^A_{\mu\nu}\,W^{\mu\nu}_A$$

$$= h\,\frac{4\pi\alpha^2}{Q^2}\left[\lambda\,(2-y)\,\tilde{G}_1 - |\mathbf{S}_\perp|\,\sqrt{1-y}\,\frac{Q}{E}\,(\tilde{G}_1+\tilde{G}_2)\right]$$

then :

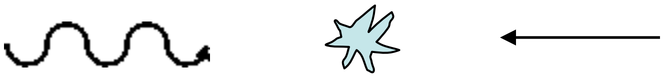
- write elementary cross section for process $\vec{e}\,\vec{q} \to e'\,q$
- write down convolution in QPM factorization hypothesis
  → deduce structure functions in terms of
  partonic densities

alternatively

alternative method

γ^* +1 $J_z=3/2$ $\sigma^T_{3/2}$ +1/2 $P^\uparrow$

-1 $J_z=1/2$ $\sigma^T_{1/2}$ +1/2



γ^* ± 1 $J_z=1/2$ ±1/2 $q^{\uparrow\downarrow}$

because $L_z = 0$ (collinear process)

→ conservation of angular momentum

then $\gamma^{*\uparrow}\, q^\downarrow \to q^\uparrow$ $\xcancel{\gamma^{*\uparrow}\, q^\uparrow}$

$\gamma^{*\downarrow}\, q^\uparrow \to q^\downarrow$ $\xcancel{\gamma^{*\downarrow}\, q^\downarrow}$

$$\left.\begin{aligned}\sigma^T_{3/2} &\leftrightarrow \gamma^{*\uparrow} P^\uparrow \propto \sum_f e_f^2\, q_f^\downarrow \\ \sigma^T_{1/2} &\leftrightarrow \gamma^{*\downarrow} P^\uparrow \propto \sum_f e_f^2\, q_f^\uparrow\end{aligned}\right\} \longrightarrow A_1 = \frac{\sigma^T_{1/2} - \sigma^T_{3/2}}{\sigma^T_{1/2} + \sigma^T_{3/2}} = \frac{\sum_{f,\bar f} e_f^2\, (q_f^\uparrow - q_f^\downarrow)}{\sum_{f,\bar f} e_f^2\, (q_f^\uparrow + q_f^\downarrow)}$$

$$= \frac{\tilde G_1(x_B)}{F_1(x_B)} \equiv \frac{g_1(x_B)}{f_1(x_B)}$$

helicity distribution $$g_1(x_B) = \frac{1}{2}\sum_{f,\bar f} e_f^2\, [q_f^\uparrow(x_B) - q_f^\downarrow(x_B)]$$

transverse polarization distribution

similarly

$$\tilde{G}_1(x_B)+\tilde{G}_2(x_B) \equiv g_1(x_B)+g_2(x_B) = \frac{1}{2Mx_B}\sum_{f,\bar{f}} e_f^2\, m_f\, [q_f^{\rightarrow}(x_B)-q_f^{\leftarrow}(x_B)]$$

it holds

$$-\frac{g_1(x)}{x} = \frac{\partial}{\partial x}[g_1(x)+g_2(x)]$$

Wandzura−Wilczek relation

$$g_2(x) = \int_x^1 \frac{dy}{y} g_1(y) - g_1(x)$$

Burkhardt−Cottingham sum rule

$$\int_0^1 dx g_2(x) = 0$$

and in general

$$\int_0^1 dx\, x^{J-1}\left[\frac{J-1}{J} g_1(x) + g_2(x)\right] = 0$$

(cont’ed)

if $p_T \neq 0$ $\gamma^{*\uparrow} q^{\uparrow}$, $\gamma^{*\downarrow} q^{\downarrow}$ allowed

example: take 1 flavor only with $q^{\uparrow}$ in $\sigma_{J_z}^{\lambda}$ $(\gamma^* q^{\uparrow})$

$p_T = 0$

$$A_1 = \frac{\sigma^T_{1/2} - \sigma^T_{3/2}}{\sigma^T_{1/2} + \sigma^T_{3/2}} = \frac{\sum_{f,\bar{f}} e_f^2 \, (q_f^{\uparrow} - q_f^{\downarrow})}{\sum_{f,\bar{f}} e_f^2 \, (q_f^{\uparrow} + q_f^{\downarrow})} \sim \frac{q_f^{\uparrow}}{q_f^{\uparrow}} = 1$$

$p_T \neq 0$

$$A_1 = \frac{\sigma^T_{1/2} - \sigma^T_{3/2}}{\sigma^T_{1/2} + \sigma^T_{3/2}} = 1 - \frac{p_T^2}{E(E+m)} \sim 1$$

in DIS regime, transverse motion of partons seems suppressed. We will see that it is not always true, in particular in connection with transverse polarized parton / parent nucleon