inclusive (in)elastic cross section for Dirac particle with structure

general result :
$$\frac{d\sigma}{d\Omega} = \sigma_{\mathrm{Mott}} \frac{E'}{E} \left[A(\nu, Q^2) + B(\nu, Q^2) \tan^2 \frac{\theta_e}{2} \right]$$

procedure :

- 2 independent "hadronic" 4-vectors P, q
- tensor basis (reflecting parity and time-reversal invariance): $b_1 = g^{\mu\nu}$, $b_2 = q^\mu q^\nu$, $b_3 = P^\mu P^\nu$,
  $b_4 = (P^\mu q^\nu + P^\nu q^\mu)$, $b_5 = (P^\mu q^\nu - P^\nu q^\mu)$,
  $b_6 = \varepsilon_{\mu\nu\rho\sigma}\, q^\rho P^\sigma$
- hadronic tensor $W^{\mu\nu} = \sum_i c_i\, (q^2, P\cdot q)\; b_i$
- current conservation $q_\mu W^{\mu\nu} = W^{\mu\nu} q_\nu = 0$
- linear system with c_6 undetermined (=0), c_5=0, c_1 and c_3 dependent from c_2 and c_4
- final result :

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) q^2\, c_2(q^2, P\cdot q) + \frac{\widetilde{P}^\mu \widetilde{P}^\nu}{M^2} \left(-\frac{M^2\, q^2}{P\cdot q} \right) c_4(q^2, P\cdot q)$$

$$\widetilde{P}^\mu = P^\mu - \frac{P\cdot q}{q^2}\, q^\mu$$

(cont'ed)

- structure $\varepsilon_{\mu\nu\rho\sigma}\, q^\rho P^\sigma$ forbidden by parity invariance
- structure $(P^\mu q^\nu - P^\nu q^\mu)$ forbidden by time-reversal invariance
- hermiticity $W^{\mu\nu} = (W^{\nu\mu})^* \Rightarrow c_{2,4}$ real functions

$$L_{\mu\nu} W^{\mu\nu} = 4EE' \cos^2\frac{\theta_e}{2}\left(W_2 + 2W_1 \tan^2\frac{\theta_e}{2}\right)$$

$$\boxed{\begin{aligned}\frac{d\sigma}{dE'\,d\Omega} &= \frac{\alpha^2}{Q^4}\frac{E'}{E} L_{\mu\nu} W^{\mu\nu} \\ &= \sigma_{\mathrm{Mott}}\left[W_2(\nu,Q^2) + 2W_1(\nu,Q^2)\tan^2\frac{\theta_e}{2}\right]\end{aligned}}$$

Summary

inclusive scattering off free Dirac particle with structure

inelastic

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\text{Mott}} \left[W_2(\nu, Q^2) + 2W_1(\nu, Q^2)\ \tan^2\frac{\theta_e}{2}\right]$$

elastic

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\text{Mott}} \left[\left(F_1^2 + \tau F_2^2\right)\right.$$

$$\left. +2\tau\,(F_1 + F_2)^2\ \tan^2\frac{\theta_e}{2}\right]\ \delta\left(\nu - \frac{Q^2}{2M}\right)$$

$$W_2^{\text{el}} \leftrightarrow (F_1^2 + \tau F_2^2)\delta\left(\nu - \frac{Q^2}{2M}\right)$$

$$2W_1^{\text{el}} \leftrightarrow 2\tau(F_1 + F_2)^2\delta\left(\nu - \frac{Q^2}{2M}\right)$$

pointlike elastic

$F_1 \rightarrow 1$
$F_2 \rightarrow 0$

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\text{Mott}} \left(1 + 2\tau\ \tan^2\frac{\theta_e}{2}\right)\ \delta\left(\nu - \frac{Q^2}{2M}\right)$$

$$W_2^{\text{el}} \leftrightarrow \delta\left(\nu - \frac{Q^2}{2M}\right)$$

$$W_1^{\text{el}} \leftrightarrow \tau\delta\left(\nu - \frac{Q^2}{2M}\right)$$

DIS regime

$$Q^2 \longrightarrow \infty$$

$$x_B = \frac{Q^2}{2P \cdot q} \quad \text{fissato}$$

Bjorken variable

TRF : $\nu \to \infty$ as much as Q^2

since $Q^2 = -\nu^2 + \mathbf{q}^2 \geq 0$ then
$|\mathbf{q}| \to \infty$ as much as Q^2

frame dependent

frame independent

Scaling

$$W_2^{\text{el}} \;\leftrightarrow\; \delta\left(\nu - \frac{Q^2}{2M}\right) \qquad \nu W_2^{\text{el}} \;\leftrightarrow\; \delta\left(1 - \frac{Q^2}{2M\nu}\right) \equiv \delta(1 - x_B) \equiv F_2(x_B)$$

$$W_1^{\text{el}} \;\leftrightarrow\; \tau\delta\left(\nu - \frac{Q^2}{2M}\right) \qquad 2MW_1^{\text{el}} \;\leftrightarrow\; \frac{Q^2}{2M\nu}\delta\left(1 - \frac{Q^2}{2M\nu}\right) \equiv x_B\,\delta(1 - x_B) \equiv 2F_1(x_B)$$

DIS regime: x_B fixed, the response does not depend on Q^2 $\rightarrow$ scaling

Experimental observation of scaling

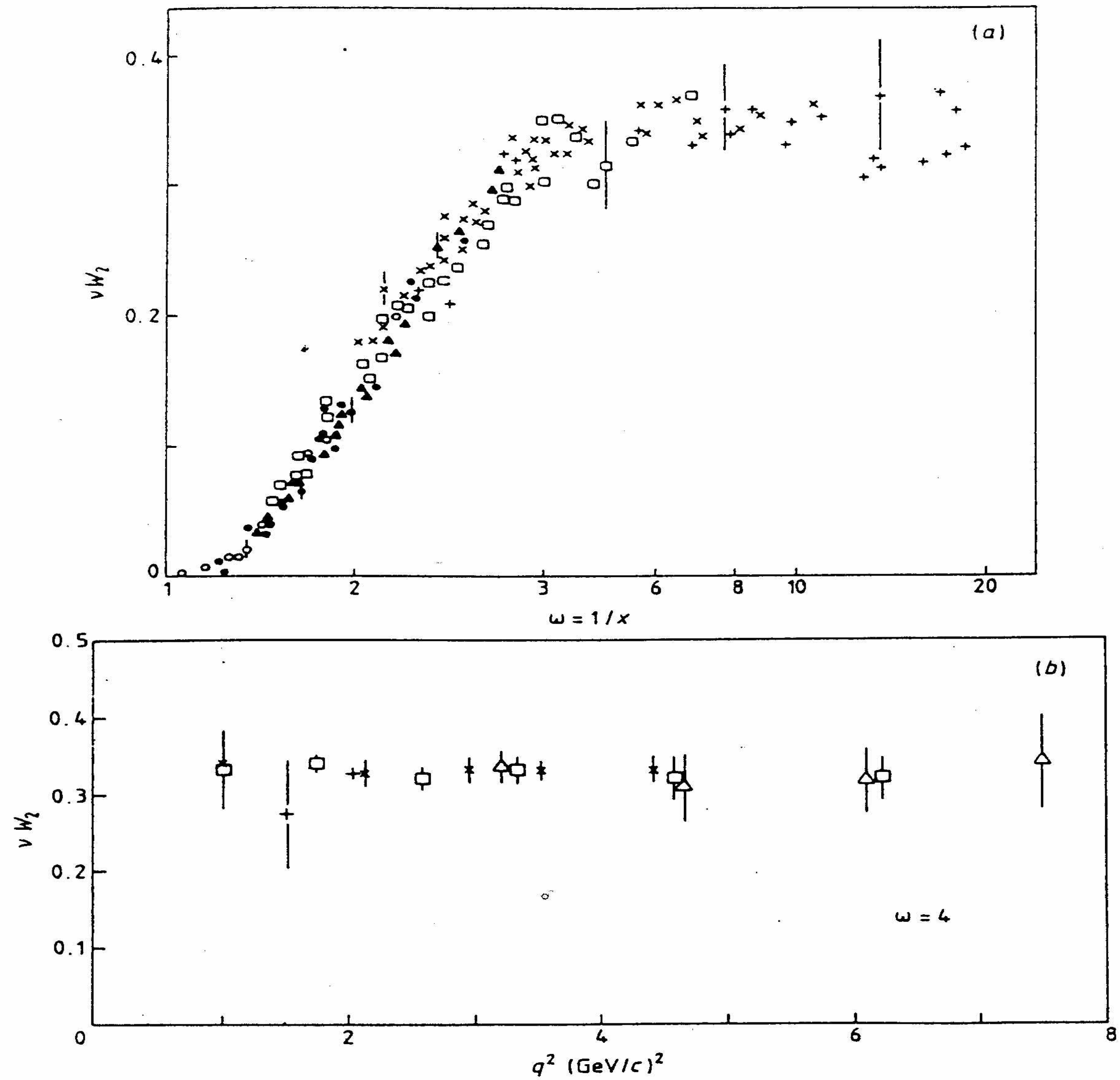
=

in DIS kinematics (i.e., $Q^2,\nu \rightarrow \infty$, x_B fixed) scattering can be represented as the incoherent sum of elastic scatterings off pointlike spin ½ constituents inside the target ⇒ origin of partons

N.B. Analogue of Rutherford experiment on scattering of α particles off atoms

$\nu\ W_2$

Aitchison & Hey



$1/x$

Q^2

Figure 4.2 Bjorken scaling: the structure function νW_2 (*a*) plotted against $\omega = 1/x$ for different q^2 values (Miller *et al* 1972) (*b*) plotted against q^2 for a single value of $x = 0.25$ ($\omega = 4$) (Friedman and Kendall 1972).

Nachtmann

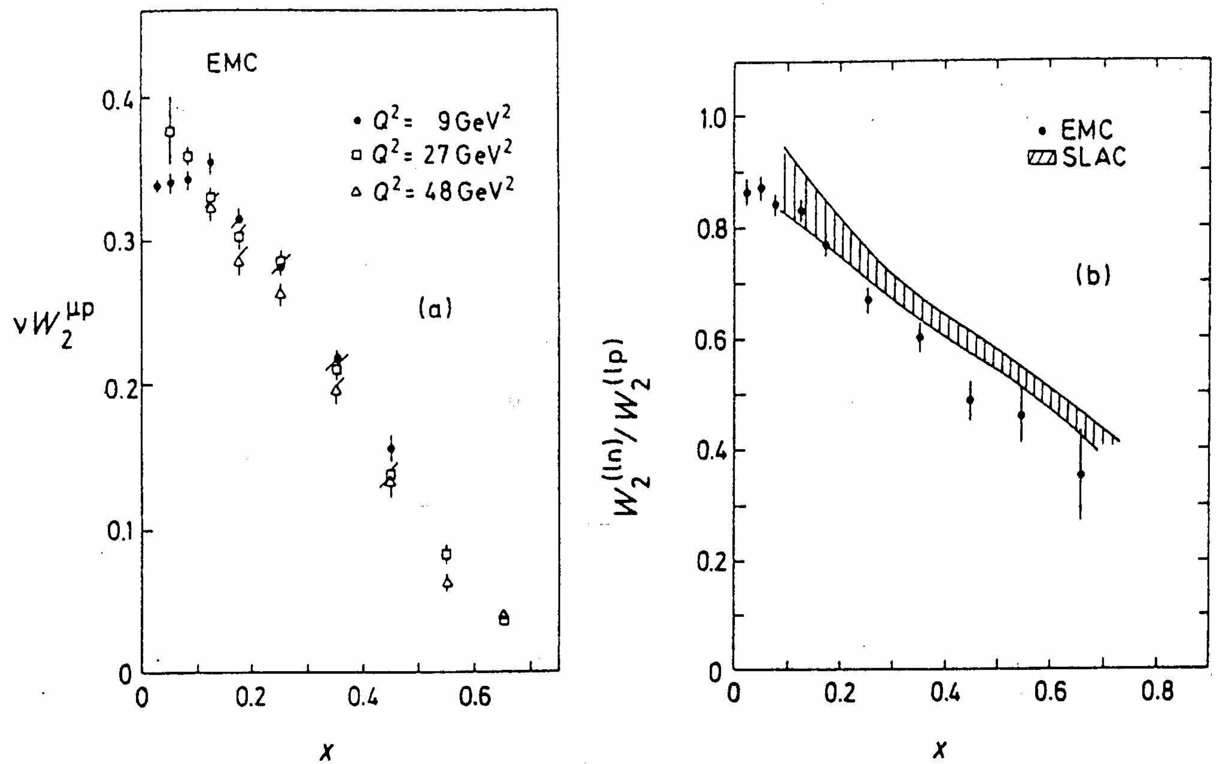
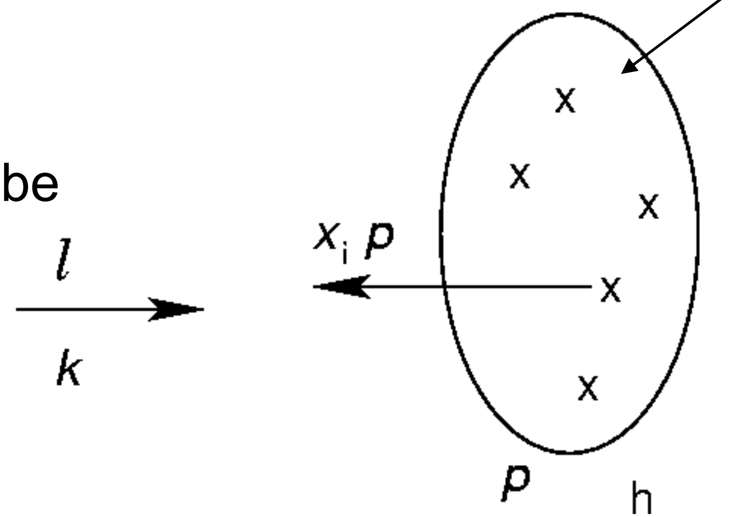


Figure 18.9 Data from the European Muon Collaboration (EMC) for the structure function $\nu W_2^{(\mu p)}(\nu, Q^2)$ of the proton as a function of $x = Q^2/(2M\nu)$ for various Q^2 values. Exact Bjorken scale invariance would demand that the data points for the same x but different Q^2 should lie on top of one another (a). Part (b) shows the ratio of the neutron and proton structure functions $W_2^{(\mathrm{ln})}(\nu, Q^2)$ and $W_2^{(\mathrm{lp})}(\nu, Q^2)$ ($\mathrm{l} = \mathrm{e}, \mu$) as a function of x. The shaded band represents the SLAC data obtained from electron scattering in the interval $2 \leq Q^2 \leq 20$ GeV^2. The points correspond to preliminary EMC data from muon scattering in the interval $10 \leq Q^2 \leq 80$ GeV^2 (after Drees 1983 and Dydak 1983).

The Quark Parton Model (QPM)

leptonic probe



partons

target =
{ partons i=1..n as virtual particles with momentum $x_i\, p$, $0 \le x_i \le 1$}

hadronic target

every virtual state has average life $\tau_i > 0$ in rest frame of h

in c.m. frame
Lorentz contraction
time dilatation

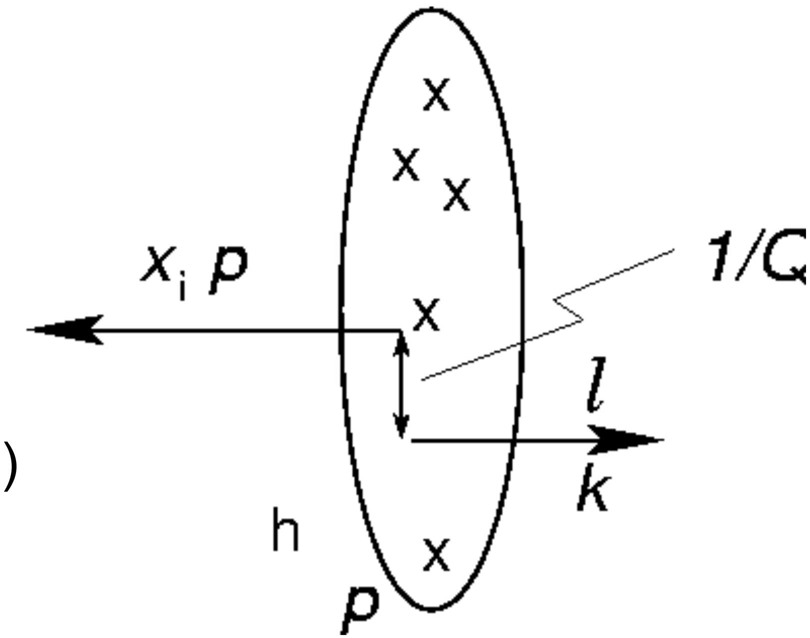
$$\tau_i \to \frac{\tau_i}{\sqrt{1-\frac{v^2}{c^2}}}$$

lepton l crosses target h during time

$$t \overset{Q^2\to\infty}{\longrightarrow} 0$$

lepton sees a “frozen” configuration of partons

because of Heisenberg principle
l and parton exchange γ^* only if impact parameter (transverse separation between two trajectories) is < $1/Q$



Probability of finding another parton j ≠ i nearby

=

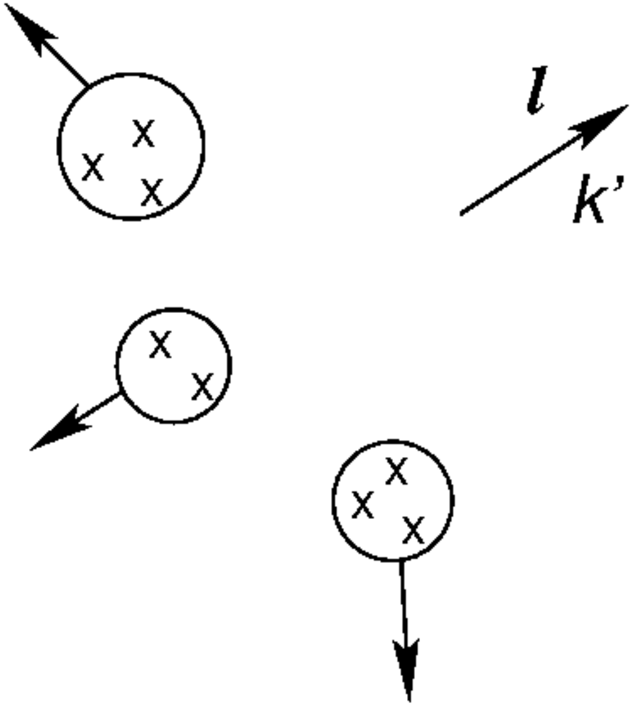
$$\frac{\text{area of hard scattering } l \text{ - parton}}{\text{impact area of target}}$$

$$\sim \frac{\frac{1}{Q^2}}{\pi R_h^2} \stackrel{Q^2\to\infty}{\longrightarrow} 0$$

lepton $\boldsymbol{l}$ detected in final state

debris of target h recombine
in not observed hadrons (Σ_X)

hadronization happens on longer
time scale than hard scattering
$\boldsymbol{l}$ – parton

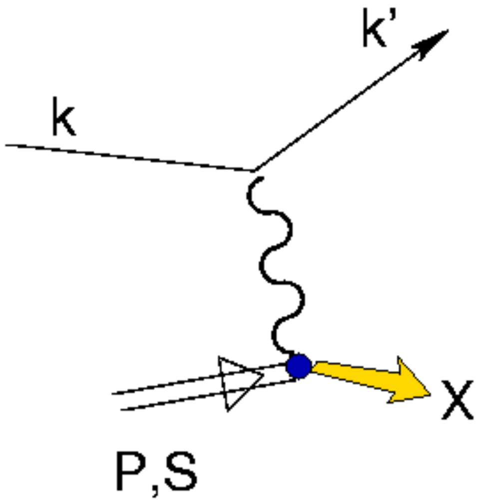


factorization between hard scattering $\boldsymbol{l}$ – parton and soft processes
that lead to form hadrons without color

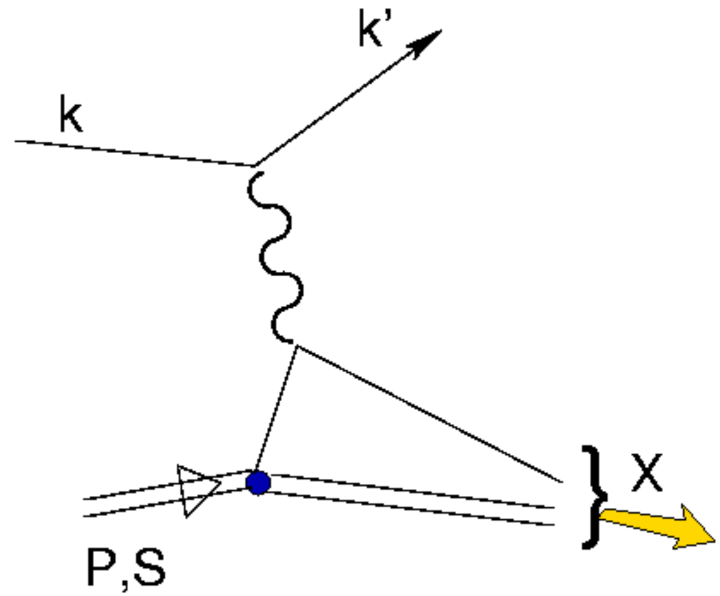
high energy: $Q^2 \to \infty$, DIS regime

the parton is almost on its mass shell and it lives longer than $1/Q$

Born approximation for hard scattering l - parton



$$= \sum_{\text{partons}}$$



generalization of Impulse Approximation (IA)

QPM

- for $Q^2 \to \infty$ in DIS, hard scattering $\boldsymbol{l}$ – parton in Born approximation
- partons live in “frozen” virtual state → almost on mass shell
- factorization between hard scattering and soft processes among partons

convolution between elementary process (hard scattering) and probability distribution of partons with flavor *f* in hadron h

$$\frac{d\sigma}{dE' d\Omega}(P,q) = \sum_f \int_0^1 dx \, \frac{d\sigma^{\text{el}}}{dE' d\Omega}(xP,q) \, \phi_f(x)$$

elastic scattering $\boldsymbol{l}$ – parton calculable in QED

unknown probability of finding parton *f* with fraction *x* of hadron momentum *P*

Remarks :

- factorization between hard scattering and probability distribution
  ⇒ cross section proportional to parton density

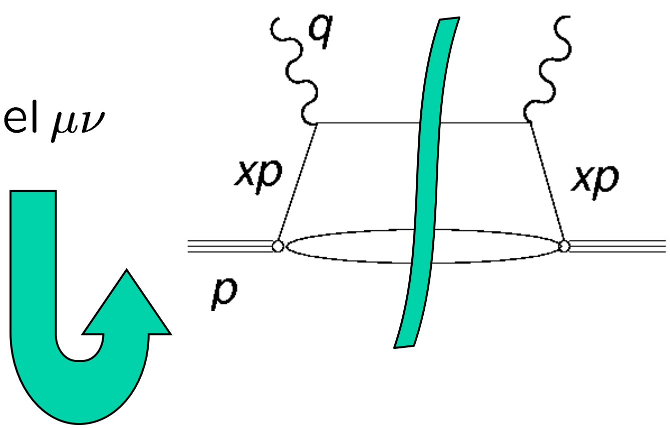
- hard scattering calculable in QED;
  probability distribution extracted from comparison with exp. data

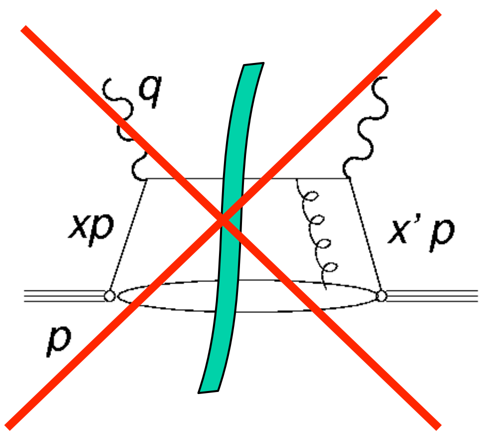
- in Born approximation, hard scattering off free partons

  ⇒ asymptotic freedom $\alpha_s \stackrel{Q^2\to\infty}{\longrightarrow} 0$

  ⇒ incoherent sum of hard scatterings

$$\frac{d\sigma^{\text{el}}}{dE' d\Omega} \sim L_{\mu\nu} W^{\text{el}\,\mu\nu}$$





Calculation of $(W^{\text{el}})^{\mu\nu}$

elastic scattering off pointlike particle (Dirac fermion)

$$\frac{d\sigma^{\text{el}}}{dE' d\Omega} = \frac{\alpha^2}{Q^4}\,\frac{E'}{E}\,L_{\mu\nu}\,W^{\text{el}\,\mu\nu}$$

$$\begin{aligned} 2m\,W^{\text{el}\,\mu\nu} &= \frac{1}{2\pi}\int \frac{d\mathbf{p}'}{(2\pi)^3 2p'^0}\,(2\pi)^4\delta(p+q-p')\,H^{\text{el}\,\mu\nu} \\ &= \delta(2xP\cdot q - Q^2)\,H^{\text{el}\,\mu\nu} = \frac{1}{2M\nu}\,\delta(x-x_B)\,H^{\text{el}\,\mu\nu} \end{aligned}$$

$H^{\text{el}\,\mu\nu}$ for pointlike Dirac particle $\leftrightarrow$ $L^{\mu\nu}$, but ….

$$\begin{aligned} H^{\text{el}\,\mu\nu} &= e_f^2\,\frac{1}{2}\,\text{Tr}\left[\,(\not{p}'+m)\,\gamma^\mu\,(\not{p}+m)\,\gamma^\nu\,\right] \\ &= e_f^2\,2\left[p'^{\,\mu}p^\nu + p'^{\,\nu}p^\mu + g^{\mu\nu}\,(m^2 - p'\cdot p)\right] \end{aligned}$$

elementary scattering amplitude

$$L_{\mu\nu} = 2\,(k_\mu k'_\nu + k_\nu k'_\mu - k \cdot k'\, g_{\mu\nu})$$

$$\begin{aligned}
L_{\mu\nu} H^{\mathrm{el}\,\mu\nu} &= e_f^2\, 8 \left[p' \cdot k'\, p \cdot k + p' \cdot k\, p \cdot k' - m^2\, k \cdot k' \right] \\
&\overset{\mathrm{TRF}}{=} e_f^2\, 8 \left[2x^2 M^2 E E' + x M E\, k' \cdot q + \right. \\
&\qquad \left. x M E'\, k \cdot q - m^2\, k \cdot k' \right] \\
&= e_f^2\, 16 E E' m^2 \cos^2 \frac{\theta_e}{2} \left[1 + \frac{Q^2}{2m^2} \tan^2 \frac{\theta_e}{2} \right]
\end{aligned}$$

$$\begin{aligned}
p' &= p + q \\
p &= x\, P \\
m^2 &= x^2\, M^2
\end{aligned}$$

$$\begin{aligned}
q &= k - k' \\
k^2 &= k'^{\,2} \sim 0
\end{aligned}$$

elementary elastic cross section

$$\frac{d\sigma^{\text{el}}}{dE' d\Omega} = \frac{\alpha^2}{Q^4}\frac{E'}{E} L_{\mu\nu} W^{\text{el}\,\mu\nu}$$

$$= \frac{\alpha^2}{Q^4}\frac{E'}{E}\frac{1}{2m}\frac{1}{2M\nu}\,\delta(x - x_B)\, L_{\mu\nu}\, H^{\text{el}\,\mu\nu}$$

$$x_B = \frac{Q^2}{2P\cdot q}$$

$$\frac{Q^2}{2p\cdot q} = \frac{x_B}{x}$$

$$= \frac{4\alpha^2}{Q^4} E'^2 \cos^2\frac{\theta_e}{2}\, e_f^2\, \frac{2m x_B}{Q^2}\,\delta(x - x_B)\left[1 + \frac{Q^2}{2m^2}\tan^2\frac{\theta_e}{2}\right]$$

$$= \sigma_{\text{Mott}}\left[e_f^2\,\delta(x - x_B)\,\frac{x}{\nu} + e_f^2\,\delta(x - x_B)\,\frac{x_B}{m}\tan^2\frac{\theta_e}{2}\right]$$

Recall :

elastic scattering off pointlike fermions

inclusive (in)elastic scattering

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\text{Mott}} \left(1 + 2\tau \tan^2\frac{\theta_e}{2}\right) \delta\left(\nu - \frac{Q^2}{2M}\right)$$

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\text{Mott}} \left\{W_2 + 2W_1 \tan^2\frac{\theta_e}{2}\right\}$$

$$W_2^{\text{el}} \leftrightarrow \frac{1}{\nu}\delta\left(1 - \frac{Q^2}{2M\nu}\right) = \frac{\delta(1-x_B)}{\nu} \equiv \frac{F_2(x_B)}{\nu}$$

$$2W_1^{\text{el}} \leftrightarrow \frac{Q^2}{2M^2\nu}\delta\left(1 - \frac{Q^2}{2M\nu}\right) = \frac{x_B}{M}\delta(1-x_B) \equiv \frac{2}{M}F_1(x_B)$$

$$\frac{d\sigma}{dE'd\Omega} = \sigma_{\text{Mott}} \left\{\frac{F_2}{\nu} + \frac{2F_1}{M}\tan^2\frac{\theta_e}{2}\right\} = \sum_f \int_0^1 dx\, \frac{d\sigma^{\text{el}}}{dE'\,d\Omega}\,\phi_f(x)$$

structure functions

$$\frac{d\sigma}{dE'd\Omega} = \sum_f \int_0^1 dx \, \frac{d\sigma^{\text{el}}}{dE'\,d\Omega}\,\phi_f(x)$$

$$= \sigma_{\text{Mott}} \sum_f e_f^2 \int_0^1 dx\,\delta(x - x_B)\,\phi_f(x)\,\left[\frac{x}{\nu} + \frac{x_B}{m}\tan^2\frac{\theta_e}{2}\right]$$

$$= \sigma_{\text{Mott}} \left[\frac{1}{\nu}F_2 + \frac{2}{M}F_1 \tan^2\frac{\theta_e}{2}\right]$$

$$\boxed{F_1(x_B) = \frac{1}{2}\sum_f e_f^2\,\phi_f(x_B) \qquad F_2(x_B) = x_B \sum_f e_f^2\,\phi_f(x_B)}$$

Callan-Gross relation

Callan and Gross, P.R.L. **22** (69) 156

$$\boxed{2x_B\,F_1(x_B) = F_2(x_B)}$$

longitudinal and transverse components of inclusive response

Generalization of polarization vector for γ^*

$$\varepsilon^\mu_\pm = \mp\frac{1}{\sqrt{2}}\left(\varepsilon^\mu_x \pm i\,\varepsilon^\mu_y\right)$$
$$\varepsilon_0 = \frac{1}{\sqrt{Q^2}}\,(|\mathbf{q}|,\, 0,\, 0,\, \nu)$$

with

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} = \sum_\lambda (-)^\lambda\, \varepsilon^{\mu *}_\lambda \varepsilon^\nu_\lambda$$

scattering amplitude

$$\ell_\mu J^\mu = \ell_\mu\, \tilde{g}^{\mu\nu} J_\nu = \sum_\lambda (\ell_\mu \varepsilon^{\mu *}_\lambda)\,(J_\nu (-)^\lambda \varepsilon^\nu_\lambda) \equiv \sum_\lambda \ell_\lambda J_\lambda$$

$$\frac{d\sigma}{dE' d\Omega} = \sigma_{\mathrm{Mott}}\left[W_2 + 2\,W_1\,\tan^2\frac{\theta_e}{2}\right] = \sigma_{\mathrm{Mott}}\frac{Q^2}{\nu^2+Q^2}\left[W_L + \left(1 + 2\,\frac{\nu^2+Q^2}{Q^2}\,\tan^2\frac{\theta_e}{2}\right) W_T\right]$$

(cont'ed)

ratio :

$$R = \frac{W_L}{W_T} = \frac{-W_1 + \left(1 + \frac{\nu^2}{Q^2}\right) W_2}{W_1}$$

$$= \frac{F_2}{F_1} \frac{1}{2x_B} \left(1 + \frac{2Mx_B}{\nu}\right) - 1 \overset{\nu,Q^2\to\infty}{\longrightarrow} 0$$

osservazione sperimentale

Atwood *et al.*, P.L. **B64** 479 ('76)

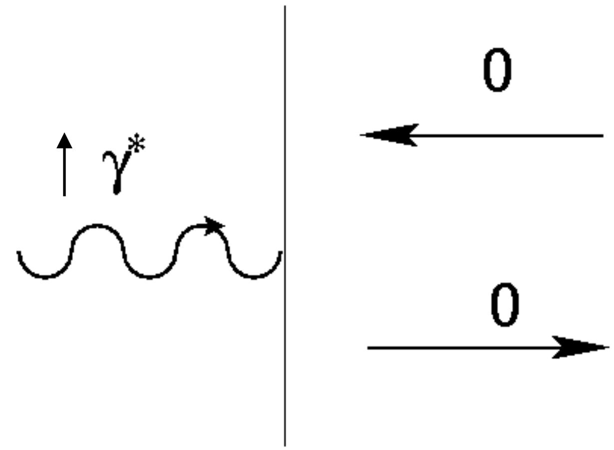
$$2\,x_B\,F_1(x_B) = F_2(x_B) \quad \Longleftrightarrow \quad R = \frac{W_L}{W_T} \to 0$$

What does that mean ?

Scattering in Breit frame

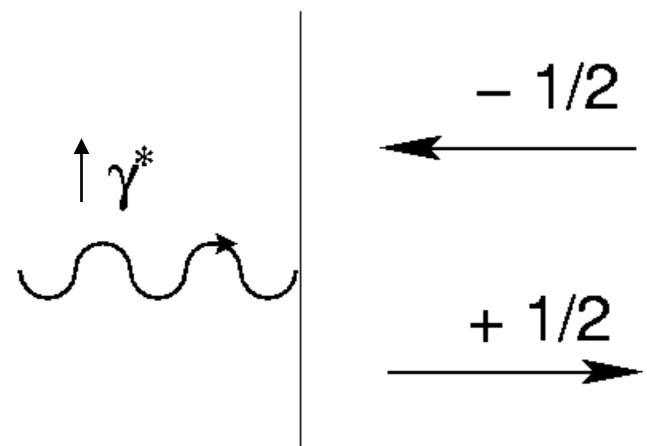
scalar particle (spin 0)

transverse polarization of γ^* carries L_z=1
$\Rightarrow$ cannot be balanced $\Rightarrow$ $W_T \to 0$



Dirac particle (spin ½)

e.m. interaction conserves helicity
$\Rightarrow$ change $\Delta h = \pm 1$ compensates $L_z = 1$
of transverse polarization of γ^*
$\Rightarrow$ longitudinal polarization of γ^* does not
$\Rightarrow$ $W_L \to 0$



Callan-Gross $\Longleftrightarrow$ partons have spin ½

Early '70 : - systematic tests of QPM
- "frame" QPM inside QCD

$$F_1(x_B) = \frac{1}{2} \sum_f e_f^2 \, \phi_f(x_B) \qquad F_2(x_B) = x_B \sum_f e_f^2 \, \phi_f(x_B)$$

DIS on N={ p,n } $\rightarrow$ access parton densities in N

Example:
suppose p = { uud } and n = { ddu } i.e. 2 flavor u,d and $\bar{u}, \bar{d} \sim 0$

4 unknowns : $u_p(x_B)$, $d_p(x_B)$, $u_n(x_B)$, $d_n(x_B)$

2 measures : $F_2^p(x_B)$, $F_2^n(x_B)$ in $e^- + N \rightarrow e^{-\prime} + X$

isospin symmetry of strong interaction :

$$\begin{cases} u_p(x_B) = d_n(x_B) \\ d_p(x_B) = u_n(x_B) \end{cases} \rightarrow \text{2 relations}$$

closed system

Definitions

$q_f(x)$ probability distribution of parton (quark) with flavor *f* and fraction *x* of parent hadron momentum

$\bar{q}_f(x)$ ditto for antiparton (antiquark)

$$\sum_f \Big(q_f(x) + \bar{q}_f(x) \Big) \equiv \Sigma(x)$$ (flavor) singlet distribution

$q_f^v(x)$ "valence" parton (quark) distribution

valence quark = quark that determines quantum # of parent hadron

if every virtual antiquark is linked to a virtual quark (vacuum polarization → pair production ~ quarkonium) then "valence" = all remaining quarks after having discarded all virtual ones

(cont'ed)

$q_f^{sea}(x)$ (Dirac) "sea" parton (quark) distribution

"sea" quark does not determine quantum # of parent hadron

if we suppose that hadron has charge = 0 (and, consequently, also valence quarks have charge =0), the remaining contribution to structure functions in DIS comes from "sea" parton distributions.

hence $$q_f(x) \equiv q_f^v(x) + q_f^{sea}(x)$$

we assume $$q_f^{sea}(x) = \overline{q}_f^{sea}(x)$$

$$\boxed{q_f^v(x) = q_f(x) - \overline{q}_f(x)}$$

$e_N = \sum_{f,\bar{f}} e_f \int_0^1 dx\, q_f(x)$ $\rightarrow$ normalization
$$2 = \int_0^1 dx\, [u(x) - \bar{u}(x)] \equiv \int_0^1 dx\, u^v(x)$$
$$1 = \int_0^1 dx\, \left[d(x) - \bar{d}(x)\right] \equiv \int_0^1 dx\, d^v(x)$$

DIS $e^- + p \to e^{-\prime} + X$
$e^- + n \to e^{-\prime} + X$

in Born approximation and at scale Q^2 such that exchange of only γ^*, not $W^\pm$, Z^0

2 flavors : $f = u,d$

isospin symmetry : $u_p = d_n$
$d_p = u_n$

$$2F_1(x_B) = \frac{F_2(x_B)}{x_B} = \sum_{f,\bar{f}} e_f^2 \, q_f(x_B)$$

$$= \begin{cases} \text{protone} & \frac{4}{9}\left[u_p(x_B) + \bar{u}_p(x_B)\right] + \frac{1}{9}\left[d_p(x_B) + \bar{d}_p(x_B)\right] \\ \\ \text{neutrone} & \frac{4}{9}\left[u_n(x_B) + \bar{u}_n(x_B)\right] + \frac{1}{9}\left[d_n(x_B) + \bar{d}_n(x_B)\right] \\ & = \frac{4}{9}\left[d_p(x_B) + \bar{d}_p(x_B)\right] + \frac{1}{9}\left[u_p(x_B) + \bar{u}_p(x_B)\right] \end{cases}$$

$$4 \geq \quad \frac{F_2^{e^-n}}{F_2^{e^-p}} = \frac{\left[u(x_B) + \bar{u}(x_B)\right] + 4\left[d(x_B) + \bar{d}(x_B)\right]}{4\left[u(x_B) + \bar{u}(x_B)\right] + \left[d(x_B) + \bar{d}(x_B)\right]} \quad \geq \frac{1}{4}$$

experimentally $\quad 1 \xleftarrow{x_B \to 0} \frac{\nu W_2^{e^-n}}{\nu W_2^{e^-p}} = \frac{F_2^{e^-n}}{F_2^{e^-p}} \xrightarrow{x_B \to 1} \frac{1}{4}$

Close, *An introduction to quarks and partons,* Fig. 11.3

data from

Bloom, in *Proc. of 6th Int. Symp. On Electron and Photon Interactions,* Bonn (’73)

Bodek *et al*., P.L. **B51** 417 (’74)

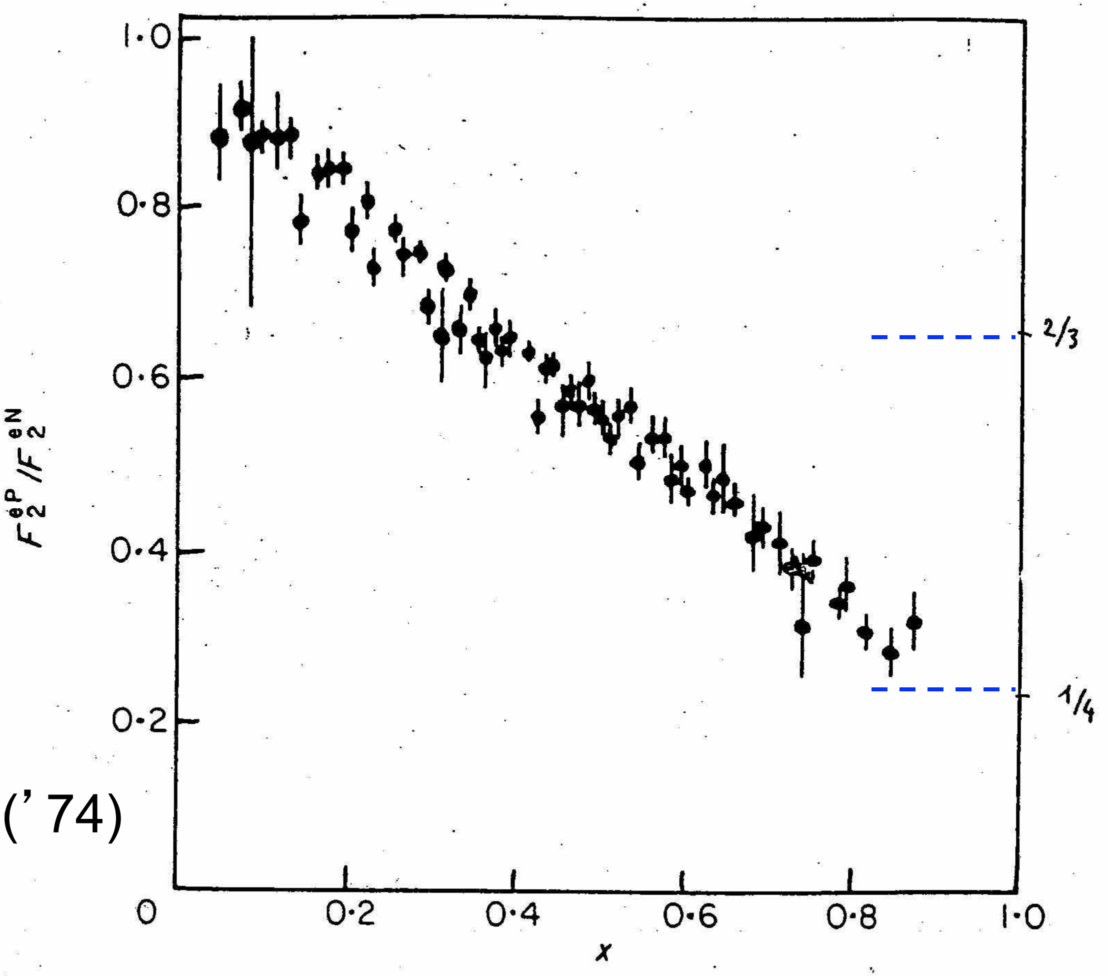


FIG. 11.3. The ratio $\nu W_2^{eN}/\nu W_2^{eP}$ as a function of x.

$$\frac{F_2^{e^-n}}{F_2^{e^-p}} = \frac{[u(x_B)+\bar{u}(x_B)] + 4\left[d(x_B)+\bar{d}(x_B)\right]}{4\,[u(x_B)+\bar{u}(x_B)] + \left[d(x_B)+\bar{d}(x_B)\right]}$$

recall : $$q_f(x_B) = q_f^v(x_B) + q_f^{sea}(x_B)$$

assume : $$\bar{u}^v(x_B) = \bar{d}^v(x_B) = 0$$ (reasonable in *N* !)

assume : $$u^{sea}(x_B) = \bar{u}^{sea}(x_B) = d^{sea}(x_B) = \bar{d}^{sea}(x_B) \equiv K(x_B)$$

Dirac “sea” is symmetric
(can be dangerous!)

$$1 \xleftarrow{x_B \to 0} \frac{F_2^{e^-n}}{F_2^{e^-p}} = \frac{u^v(x_B)+4d^v(x_B)+10K(x_B)}{4u^v(x_B)+d^v(x_B)+10K(x_B)} \xrightarrow{x_B \to 1} \frac{1}{4}$$

dominance of “sea” (K)
(pair production is
flavor independent)

u^v_p dominant
(d^v_n “)

naïve Hp: $u_p^v = 2\, d_p^v$ (|e_u|=2|

$$\Rightarrow \frac{F_2^{e^-n}}{F_2^{e^-p}} \to \frac{2}{3} \neq \frac{1}{4} \quad !$$

using same hypothesis:

$$\nu W_2^{e^- p} - \nu W_2^{e^- n} = F_2^{e^- p} - F_2^{e^- n} = \frac{1}{3} x_B \left[u^v(x_B) - d^v(x_B) \right]$$

non-singlet distribution
→ information on valence partons

Close, *An Introduction to quarks and partons,* Fig. 11.6

Bloom, in *Proc. of 6th Int. Symp. On Electron and Photon Interactions,* Bonn ('73)

Bodek *et al.*, P.L. **B51** 417 ('74)

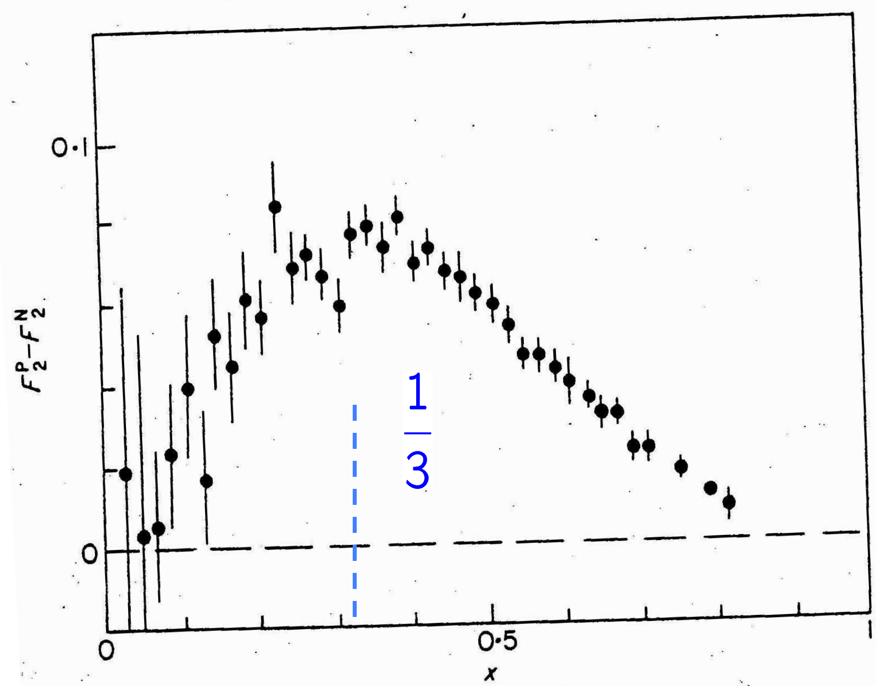


FIG. 11.6. $\nu W_2(\mathrm{ep}) - \nu W_2(\mathrm{en})$ data as a function of x.

Other tests of QPM

Gottfried sum rule

$$\int_0^1 \frac{dx}{x}\left(F_2^{e^-p} - F_2^{e^-n}\right) = \frac{1}{3}\int_0^1 dx\,(u^v - d^v) + \frac{1}{3}\int_0^1 dx\,(U^{sea} - D^{sea})$$

$$\sim \frac{1}{3}(n_u - n_d) = \frac{1}{3}$$

$U^{\text{sea}} = D^{\text{sea}}$

NMC coll., P.R.L. **66** 2712 (91)
Arneodo, P.Rep. **240** 301 (94)

exp. 0.240 ± 0.016

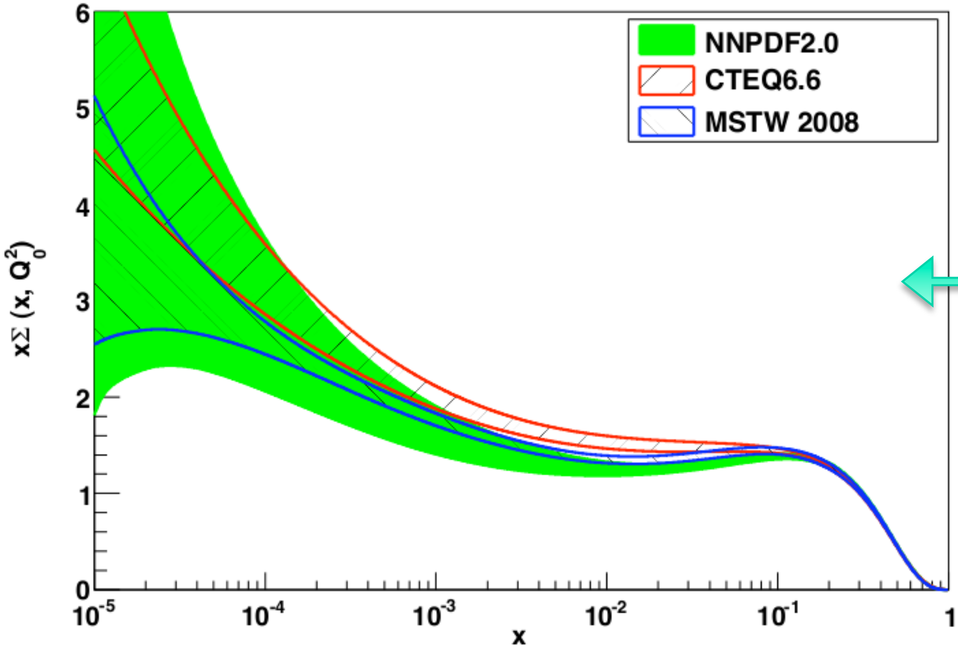
$\boxed{\text{QCD corrections} \Rightarrow U^{\text{sea}} \neq D^{\text{sea}}}$ $d\bar{d} > u\bar{u}$

$$\left\{\begin{aligned}
&\frac{F_2^{e^-n}}{F_2^{e^-p}} = \frac{u^v(x_B) + 4d^v(x_B) + 10K(x_B)}{4u^v(x_B) + d^v(x_B) + 10K(x_B)} \\
&F_2^{e^-p} - F_2^{e^-n} = \frac{1}{3}\, x_B\ [u^v(x_B) - d^v(x_B)] \\
&\frac{F_2^{e^-p} + F_2^{e^-n}}{x_B} = \frac{1}{9}\ [5\,(u^v(x_B) + d^v(x_B)) + 20K(x_B)]
\end{aligned}\right.$$

3 equations for 3 unknowns : $u^v(x_B)$, $d^v(x_B)$, $K(x_B)$

$$\Downarrow$$

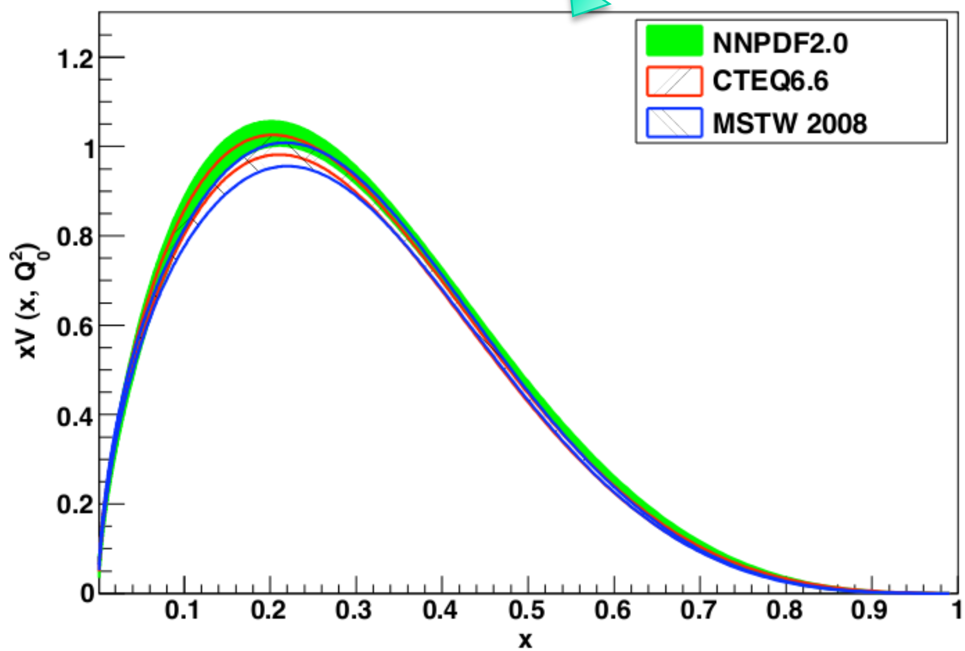
informations on valence and “sea” distributions



updated info on

singlet $\Sigma_q\ q+\bar{q}$

valence $\Sigma_q\ q-\bar{q}$



Early '70: towards the e-weak sector of Standard Model

- so far, we considered only flavors = *u,d* . In mesonic and baryonic spectra evidence for third flavor "strangeness" *s*
  BNL, 1974: discovery of resonance J/ψ : the charmonium $c\bar{c}$

- evidence of strangeness changing weak processes : $K^{\pm} \to \mu^{\pm}\, \nu$

- CERN, 1973: evidence of weak "neutral" currents in processes
  $$\nu\,(e^-) + p \to \nu\,(e^-) + p$$

- first ideas (~ '60) about unification of electromagnetic and weak interactions

(Feynmann, Gell-Mann, Glashow, Weinberg..)

but partons are eigenstates of strong interaction, not of weak one

$$|\mathrm{parton}\rangle_{weak} = \sum_f V_f\,|\mathrm{parton}_f\rangle_{strong} \qquad V_f\,\varepsilon\,\mathrm{SU}(\mathrm{N}_f)$$

$$\downarrow$$

birth of electroweak sector of SM

Nobel 1979: Glashow, Weinberg, Salam

birth of electroweak sector of SM

(only a summary)

- first hypothesis (Feynmann Gell-Mann, '58 ; Glashow, '61) :
  charged weak interactions ($W^{\pm}$) linked to isovector e.m. interaction (γ) by isospin rotation; left-handed leptons and quarks are organized in doublets of the weak isospin T following $SU(2)_T$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \quad \begin{pmatrix} ? \\ s_\theta \end{pmatrix}_L$$

where $d_\theta = d \cos \theta_C + s \sin \theta_C$; $s_\theta = - d \sin \theta_C + s \cos \theta_C$ — θ_C Cabibbo angle

d , s eigenstates of strong interaction

d_θ ,s_θ eigenstates of weak interaction

Comments: • need fourth flavor, the charm (discovered in '74)

• left-handed transitions between ν and e^-/μ^-, between quarks, via $W^{\pm}$

d_θ , s_θ justify reactions of the kind $K^{\pm} \rightarrow \mu^{\pm}\, \nu$

(cont'ed)

- hypothesis of weak charge Y (Glashow, '61): new symmetry $U(1)_Y$

quarks have e.m. charge $e_f = Y + \frac{1}{2} T_3$

weak charge $Y = \frac{1}{2}(B + S)$

summary of quantum numbers

	B	S	Y	T_3	e_f
u	$\frac{1}{3}$	0	$\frac{1}{6}$	1	$\frac{2}{3}$
d	$\frac{1}{3}$	0	$\frac{1}{6}$	-1	$-\frac{1}{3}$
s	$\frac{1}{3}$	-1	$-\frac{1}{3}$	0	$-\frac{1}{3}$

- electroweak theory: fermions interact through gauge bosons ***W*** , B

$$\mathcal{L}_{\text{weak}} = g\bar{\psi}\,\frac{\mathbf{T}}{2}\,\psi\;\cdot\mathbf{W} + g'\bar{\psi}\,Y\,\psi\,B^0$$

g , g' unknown couplings

invariance under $SU(2)_T \otimes U(1)_Y$ and massless gauge fermions / bosons

$\Rightarrow$ theory renormalizable and non-abelian, because $[W_i, W_j] = i\,\varepsilon_{ijk}\,W_k$

but $m_W \neq 0$! Otherwise we would see it in β / K decays

(cont'ed)

- ('t Hooft, '71) : non-abelian theories keep renormalizabilitty if masses are dynamically generated by spontaneous breaking of gauge symmetry (Goldstone mechanism, '64; Higgs, '64…)

- spontaneous symmety breaking implies ***W***, *B* → *W* ±, *Z*0, *A*
  in particular

$$A = \cos\theta_W B + \sin\theta_W W_3$$
$$Z^0 = -\sin\theta_W B + \cos\theta_W W_3$$

θ_W Weinberg angle

$$\mathcal{L}_{\text{weak}} = g\bar{\psi}\,\frac{\mathbf{T}}{2}\,\psi\,\cdot\mathbf{W} + g'\bar{\psi}\,Y\,\psi\,B^0$$

$$e_f = Y + \frac{T_3}{2}$$
$$g' = g\tan\theta_W$$

$$\mathcal{L}_{\text{weak}} = g\bar{\psi}\,\frac{T^{\mp}}{2}\,\psi\,\cdot W^{\pm}$$
$$+g\sin\theta_W\bar{\psi}\,e_f\,\psi\,A + \frac{g}{\cos\theta_W}\,\bar{\psi}\,\left(\frac{T_3}{2} - e_f\sin^2\theta_W\right)\,\psi\,Z^0$$

$g\sin\theta_W \equiv e$

$$+e\bar{\psi}\,e_f\,\psi\,A + \frac{2e}{2\sin\theta_W\cos\theta_W}\,\bar{\psi}\,\left(\frac{T_3}{2} - e_f\sin^2\theta_W\right)\,\psi\,Z^0$$

e.m. current → *A* ≡ γ　　　weak neutral current

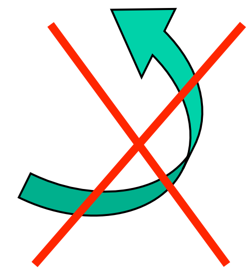
Spontaneous symmetry breaking : the Goldstone mechanism

Example: field theory for scalar particle ϕ

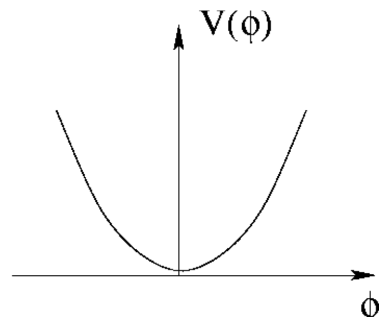
$$\mathcal{L}(\phi) = \frac{1}{2}(\partial_\mu\phi)^2 - V(\phi)$$

$$V(\phi) = \mu^2\phi^2 + \lambda\phi^4 \quad , \qquad \lambda > 0$$

symmetry
$\mathcal{L}(-\phi) = \mathcal{L}(\phi)$

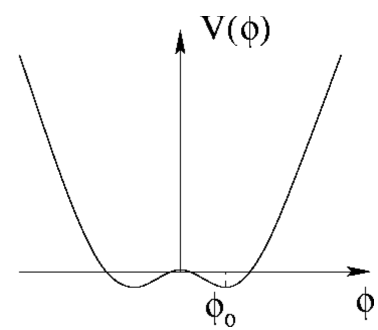


$\mu^2 > 0$ $\quad \phi_0 = 0$



vuoto ≡ $\dfrac{\partial V}{\partial \phi} = 0$

$\mu^2 < 0$ $\quad \phi_0 = \pm\sqrt{\dfrac{-\mu^2}{2\lambda}}$



new field $\qquad \phi' = \phi - \phi_0 \; ; \qquad \phi'_0 = 0$

$$V(\phi') = \mu^2(\phi'+\phi_0)^2 + \lambda(\phi'+\phi_0)^4 = -2\mu^2\phi'^2 + o(\phi'^3)$$

$$m_{\phi'} = \sqrt{-4\mu^2}$$

$$\mathcal{L}(\phi') = \frac{1}{2}\left(\partial_\mu\phi'\right)^2 + 2\mu^2\phi'^2 + \ldots. = \underbrace{\frac{1}{2}\left(\partial_\mu\phi'\right)^2 - \frac{1}{2}m_\phi^2\phi'^2}_{\mathcal{L}_{free}(\phi')} + \ldots.$$

Summary

electroweak sector of Standard Model

=

non-abelian renormalizable theory

of unified e.m. and weak interactions

in gauge symmetry $SU(2)_T \otimes U(1)_Y$

Predictions:

- need a fourth flavor, the charm
- 4 gauge bosons: γ , $W^{\pm}$, Z^0
- γ coupled to conserved current → massless (ok with QED)
- ratio $\dfrac{\text{weak strength}}{\text{e.m. strength}}$ fits with Fermi coupling

constant $G_F = \dfrac{e^2}{4\sqrt{2} M_W^2 \sin^2\theta_W}$ with $M_W \sim 75$ GeV

moreover $M_W^2 = M_Z^2 \cos^2\theta_W \rightarrow M_Z \geq M_W$

- charged weak currents: $W^{\pm}$ induce transitions

$\nu \leftrightarrow e^-$, $u \leftrightarrow d$, $u \leftrightarrow s$ (change of strangeness) , ….

- weak neutral currents: $\nu + p \rightarrow \nu + p$, …

Experimental confirmations:

- quark charm produces resonance J/ψ (BNL, 1974)

- gauge bosons $W^{\pm}$, Z^0 observed in UA1 exp. (CERN, 1983)

Nobel 1984: Rubbia, van der Meer

- from Particle Data Group: $M_W = 80.22 \pm 0.0026$ GeV

$$M_Z = 91.187 \pm 0.007 \text{ GeV}$$

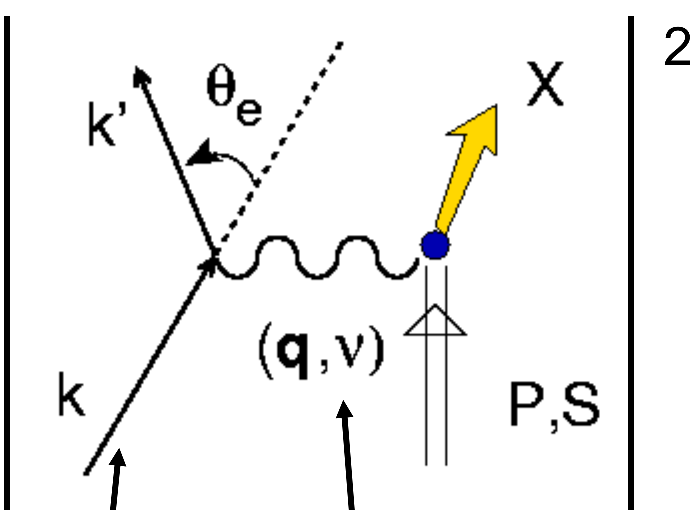
$$\sin^2 \theta_W (M_Z) = 0.2319 \pm 0.0005$$

- justify charged weak currents with change in strangeness $K^{\pm} \rightarrow \mu^{\pm} \nu$

- weak neutral currents observed at CERN in 1973

Benvenuti *et al*., PRL **32** 800 (74)
Hasert et al., PL **B46** 138 (73)

Deep Inelastic Scattering



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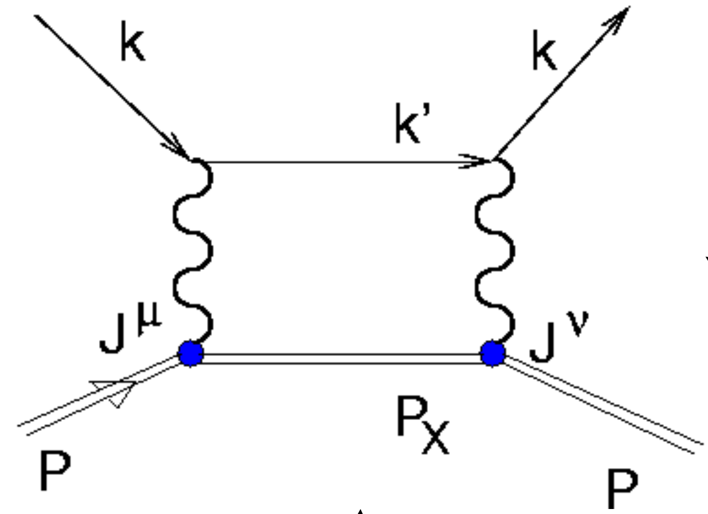


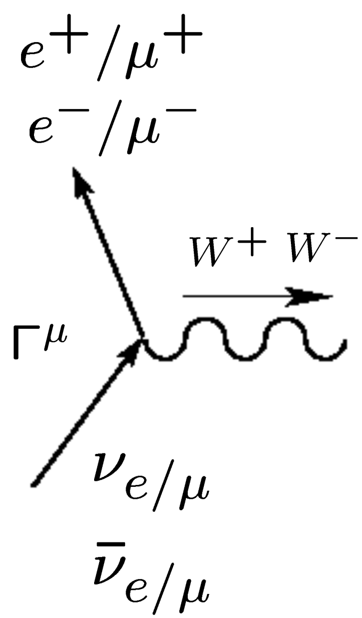
$e^{\pm}, \mu^{\pm},$
$\nu_{e/\mu}, \bar{\nu}_{e/\mu}$

$\gamma^{*}, W^{\pm}, Z^{0}$

leptonic tensor $L^{\mu\nu}$

hadronic tensor $W^{\mu\nu}$

leptonic tensor



e.m. interaction (e^- / μ^- left-handed) → exchange γ

$$\Gamma^\mu = e\gamma^\mu$$

neutrino beam (left-handed) → exchange W^+
(but also in inverse reactions like $e^+/\mu^+ \to \bar{\nu}_{e/\mu}$)

$$\Gamma^\mu = \frac{e}{2\sqrt{2}\sin\theta_W}\frac{T_3(=+1)}{2}\gamma^\mu(1-\gamma_5)$$

V - A

antineutrino beam (right-handed) → exchange W^-
(but also in $e^-/\mu^- \to \nu_{e/\mu}$)

$$\Gamma^\mu = \frac{e}{2\sqrt{2}\sin\theta_W}\frac{T_3(=-1)}{2}\gamma^\mu(1+\gamma_5)$$

V + A

V-V *A-A*

$$\begin{aligned} L^{\mu\nu} &= \mathrm{Tr}\left[\Gamma^\mu\ \not{k}'\,\Gamma^\nu\ \not{k}\right] \\ &= \frac{e^2}{8\sin^2\theta_W}\frac{1}{4}\left\{\mathrm{Tr}\left[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta\right]\ k'_\alpha k_\beta + \mathrm{Tr}\left[\gamma^\mu\gamma_5\gamma^\alpha\gamma^\nu\gamma_5\gamma^\beta\right]\ k'_\alpha k_\beta\right. \\ &\mp \left.\mathrm{Tr}\left[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma_5\gamma^\beta\right]\ k'_\alpha k_\beta \mp \mathrm{Tr}\left[\gamma^\mu\gamma_5\gamma^\alpha\gamma^\nu\gamma^\beta\right]\ k'_\alpha k_\beta\right\} \end{aligned}$$

V-A

(cont'ed)

$$\mathrm{Tr}\left[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta\right] = \mathrm{Tr}\left[\gamma^\mu\gamma_5\gamma^\alpha\gamma^\nu\gamma_5\gamma^\beta\right] = 4(g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha} - g^{\mu\nu}g^{\alpha\beta})$$
$$\mathrm{Tr}\left[\gamma^\mu\gamma_5\gamma^\alpha\gamma^\nu\gamma^\beta\right] = \mathrm{Tr}\left[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma_5\gamma^\beta\right] = 4i\epsilon^{\mu\nu\alpha\beta}$$

$$L^{\mu\nu} = \frac{e^2}{8\sin^2\theta_W} 2\left(k'^\mu k^\nu + k'^\nu k^\mu - k\cdot k' g^{\mu\nu} \mp i\epsilon^{\mu\nu\alpha\beta}k'_\alpha k_\beta\right)$$

$$\equiv L^{\mu\nu(S)} \pm L^{\mu\nu(A)}$$

antisymmetric part of tensor keeps memory of interference between weak vector and axial currents

vector boson propagator

approximated as $$\left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) \frac{1}{q^2 - M_W^2} \sim -\frac{g^{\mu\nu}}{q^2 - M_W^2}$$

because $\frac{q^\mu q^\nu}{q^2} \sim \left(\frac{m_e}{M_W}\right)^2 \sim 0$

hadronic tensor

- 2 independent vectors *P, q*
- tensor basis: $b_1=g^{\mu\nu}$, $b_2=q^\mu q^\nu$, $b_3=P^\mu P^\nu$,
  $b_4=(P^\mu q^\nu + P^\nu q^\mu)$, $b_5=(P^\mu q^\nu - P^\nu q^\mu)$,
  $b_6= \varepsilon_{\mu\nu\rho\sigma}\, q^\rho P^\sigma$
- hadronic tensor $W^{\mu\nu} = \Sigma_i\, c_i\, (q^2, P\cdot q)\; b_i$
- Hermiticity $\rightarrow$ c_i are real
- time-reversal invariance $\rightarrow$ $c_5 = 0$
- weak current is not conserved: $q_\mu W^{\mu\nu} \neq 0 \rightarrow c_6 \neq 0$
- c_1 and c_3 dependent on c_2 and c_4

$$W^{\mu\nu} = \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right) q^2\, c_2(q^2, P\cdot q) + \frac{\widetilde{P}^\mu \widetilde{P}^\nu}{M^2}\left(-\frac{M^2 q^2}{P\cdot q}\right) c_4(q^2, P\cdot q)$$

$$+i\epsilon^{\mu\nu\rho\sigma}\,\frac{P_\rho q_\sigma}{M^2}\, c_6(q^2, P\cdot q)$$

$$\underbrace{W_1 \qquad\qquad W_2}_{W^{(S)\mu\nu}}$$

$$\boxed{W_3}$$

$$\boxed{W^{(A)\mu\nu}}$$

scattering amplitude

$$\begin{aligned} L_{\mu\nu} &= L_{\mu\nu}^{(S)} \pm L_{\mu\nu}^{(A)} \\ W^{\mu\nu} &= W^{(S)\mu\nu} + W^{(A)\mu\nu} \end{aligned}$$

$$\begin{aligned} L_{\mu\nu} W^{\mu\nu} &= L_{\mu\nu}^{(S)} W^{(S)\mu\nu} \\ &\pm L_{\mu\nu}^{(A)} W^{(A)\mu\nu} \end{aligned}$$

$$L_{\mu\nu} W^{\mu\nu} \overset{TRF}{\propto} \frac{e^4}{64 \sin^4 \theta_W} 4EE' \cos^2 \frac{\theta_e}{2}$$

$$\times \left[W_2 + 2W_1 \tan^2 \frac{\theta_e}{2} \pm \frac{E + E'}{M} W_3 \tan^2 \frac{\theta_e}{2} \right]$$

interference *VA* → antisymmetry between leptons / antileptons
parity violating contribution

cross section

$$
\begin{aligned}
\frac{d\sigma^{\nu/\bar{\nu}}}{dE'd\Omega} &= \frac{\alpha^2}{64\sin^4\theta_W}\,\frac{E'}{E}\,\frac{1}{(Q^2+M_W^2)^2}\,L_{\mu\nu}\,W^{\mu\nu} \\
&= \frac{\alpha^2}{64\sin^4\theta_W}\,\frac{E'}{E}\,\frac{1}{(Q^2+M_W^2)^2}\,4EE'\cos^2\frac{\theta_e}{2} \\
&\quad\times\left[W_2 + 2W_1\tan^2\frac{\theta_e}{2} \pm \frac{E+E'}{M}W_3\,\tan^2\frac{\theta_e}{2}\right]
\end{aligned}
$$

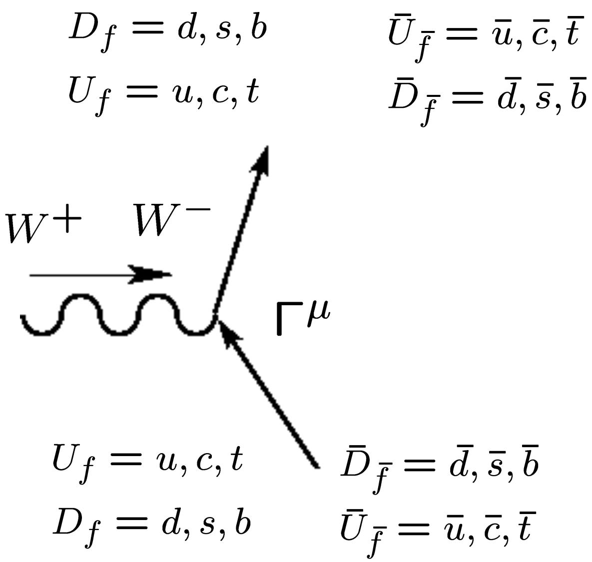
DIS limit: scaling in elastic dσ

$$
W_1 \to \frac{F_1}{M} \qquad W_2 \to \frac{F_2}{\nu} \qquad W_3 \to \frac{F_3}{\nu}
$$

$$
\begin{aligned}
&= \frac{G_F^2}{8\pi^2}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 E'^2\,\cos^2\frac{\theta_e}{2} \\
&\quad\times\left[\frac{F_2}{\nu} + 2\frac{F_1}{M}\tan^2\frac{\theta_e}{2} \pm \frac{E+E'}{M\nu}F_3\,\tan^2\frac{\theta_e}{2}\right]
\end{aligned}
$$

$$
G_F = \frac{e^2}{4\sqrt{2}M_W^2\sin^2\theta_W}
$$

elementary electroweak vertex with charged currents



e.m. interaction → exchange γ

$$\Gamma^\mu = e\gamma^\mu$$

quark (left-handed)

$$\Gamma^\mu = \frac{e}{2\sqrt{2}\sin\theta_W} \frac{T_3(=+1)}{2} \gamma^\mu (1-\gamma_5) \sum_{f'} V_{ff'}$$

antiquark (right-handed)

$$\Gamma^\mu = \frac{e}{2\sqrt{2}\sin\theta_W} \frac{T_3(=-1)}{2} \gamma^\mu (1+\gamma_5) \sum_{\bar{f}'} V_{\bar{f}\bar{f}'}$$

$$\binom{U_f}{D_f} \equiv \binom{u}{d} \binom{c}{s} \binom{t}{b}$$

$$V_{U_f D_f} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \in SU_f(3)$$

$$\sim \begin{pmatrix} \cos\theta_C & \sin\theta_C & 0 \\ -\sin\theta_C & \cos\theta_C & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \sum_{f'} \left| V_{U_f D_{f'}} \right|^2 = 1$$

elementary hadronic tensor

$$2mW^{\mathrm{el}\,\mu\nu} = \frac{1}{2\pi}\int \frac{d\mathbf{p}'}{(2\pi)^3 2p'^0}(2\pi)^4\delta(p'-p-q)\,H^{\mathrm{el}\,\mu\nu}$$
$$= \frac{1}{2M\nu}\,\delta\,(x-x_B)\;H^{\mathrm{el}\,\mu\nu}$$

$$H^{\mathrm{el}\mu\nu} = \frac{e_f^2}{4}\mathrm{Tr}\,[\,(x\,\not{P}+\not{q}+m)\,\gamma^\mu\,(1\mp\gamma_5)\,(x\,\not{P}+m)\,\gamma^\nu\,(1\mp\gamma_5)]$$
$$\times\sum_{f'}\left|V_{U_f D_{f'}}\right|^2$$
$$= H^{\mathrm{el}(S)\mu\nu}\pm H^{\mathrm{el}(A)\mu\nu}$$

then $$L_{\mu\nu}\,H^{\mathrm{el}\mu\nu} = L_{\mu\nu}^{(S)}\,H^{\mathrm{el}(S)\mu\nu}\pm L_{\mu\nu}^{(A)}\,H^{\mathrm{el}(A)\mu\nu}$$

structure functions

$$\frac{d\sigma}{dE'd\Omega}(P,q) = \sum_{f,\bar{f}} \int_0^1 dx\, \frac{d\sigma^{\text{el}}}{dE'd\Omega}(xP,q)\ \phi_f(x)$$

$$\Downarrow$$

$$F_2(x_B) = x_B \sum_f \left[\phi_f(x_B) + \bar{\phi}(x_B)\right]$$

$$F_2^{\nu} \sim 2x_B \left[d(x_b) + s(x_b) + \bar{u}(x_B) + \bar{c}(x_B)\right]$$

$$F_2^{\bar{\nu}} \sim 2x_B \left[\bar{d}(x_b) + \bar{s}(x_b) + u(x_B) + c(x_B)\right]$$

$$2x_B F_1(x_B) = F_2(x_B)$$

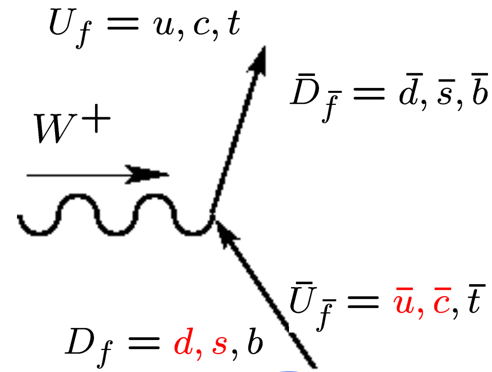
$$\boxed{F_3(x_B) = \sum_f \left[\phi_f(x_B) - \bar{\phi}(x_B)\right]}$$

$$F_3^{\nu} \sim 2\left[d(x_b) + s(x_b) - \bar{u}(x_B) - \bar{c}(x_B)\right]$$

$$F_3^{\bar{\nu}} \sim 2\left[-\bar{d}(x_b) - \bar{s}(x_b) + u(x_B) + c(x_B)\right]$$

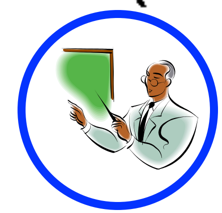
flavor non-singlet
asymmetry right-/left- handed
(*V/A*)

Esempio : $\nu_{e/\mu} + p \to e^-/\mu^- + X$

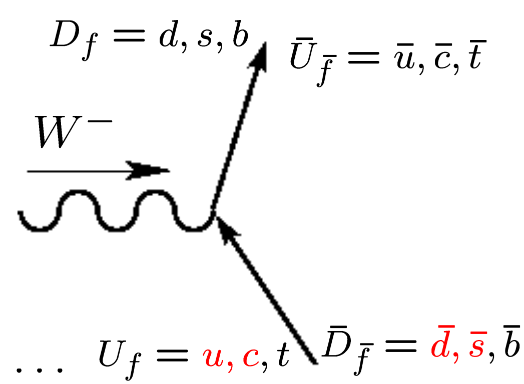


$$J^\mu_{W^+} \propto \bar{u}\,\gamma^\mu(1-\gamma_5)\,[d\cos\theta_C + s\sin\theta_C]$$
$$+\ \bar{c}\,\gamma^\mu(1-\gamma_5)\,[s\cos\theta_C - d\sin\theta_C] + \text{antiquarks}\ldots$$

$$\frac{F_2(x_B)}{x_B} = 2F_1(x_B) \sim 2\,[d(x_B) + s(x_B) + \bar{u}(x_B) + \bar{c}(x_B)] + \ldots$$
$$F_3(x_B) \sim 2\,[d(x_B) + s(x_B) - \bar{u}(x_B) - \bar{c}(x_B)] + \ldots$$



$\bar{\nu}_{e/\mu} + p \to e^+/\mu^+ + X$



$$J^\mu_{W^-} \propto [\bar{d}\cos\theta_C + \bar{s}\sin\theta_C]\,\gamma^\mu(1-\gamma_5)\,u$$
$$+\ [\bar{s}\cos\theta_C - \bar{d}\sin\theta_C]\,\gamma^\mu(1-\gamma_5)\,c$$
$$+\ \overline{(\bar{u})}\,\gamma^\mu(1+\gamma_5)\,[\bar{d}\cos\theta_C + \bar{s}\sin\theta_C]\ldots$$

$$\left[V^{-1} = V^\dagger\right]$$

$$\frac{F_2(x_B)}{x_B} = 2F_1(x_B) = 2\left[\bar{d}(x_B) + \bar{s}(x_B) + u(x_B) + c(x_B)\right] + \ldots$$
$$F_3(x_B) = 2\left[-\bar{d}(x_B) - \bar{s}(x_B) + u(x_B) + c(x_B)\right] + \ldots$$

SU$_f$(3) $\rightarrow$ 12 unknowns: $u_p, d_p, s_p, \bar{u}_p, \bar{d}_p, \bar{s}_p$
$u_n, d_n, s_n, \bar{u}_n, \bar{d}_n, \bar{s}_n$

8 observables: $F_2^{W^+p}, F_2^{W^-p}, F_3^{W^+p}, F_3^{W^-p}$
$F_2^{W^+n}, F_2^{W^-n}, F_3^{W^+n}, F_3^{W^-n}$

isospin invariance: $u_p \equiv d_n \qquad d_p \equiv u_n$
(2 relations)

isospin symmetry of “sea” : $\boxed{\bar{u} = \bar{d}}$
(2 relations)

closed system: from DIS (anti)neutrino – nucleon it is possible to extract (anti)quark distributions for the three flavors

Various experimental tests of QPM

1) (anti)neutrino DIS on isoscalar nuclei (Z=$N \rightarrow$ # u = # d quarks)

$$\frac{\sigma(\nu A)}{\sigma(\bar{\nu} A)} \sim 3$$

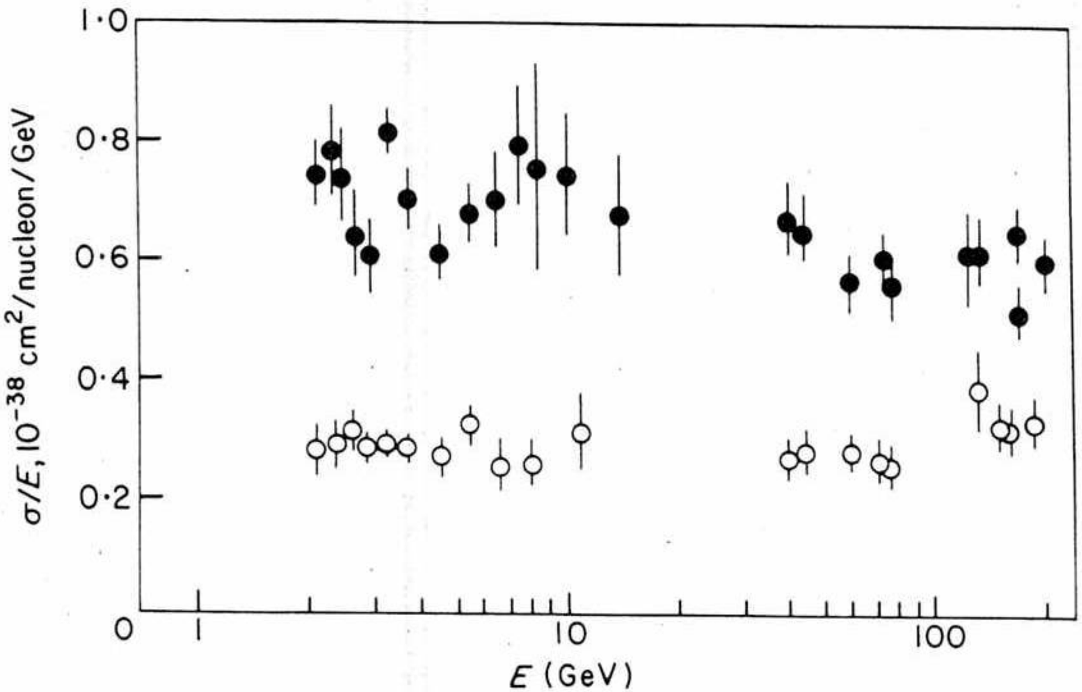


FIG. 11.12. $\sigma^{\bar{\nu}}/E$ and σ^{ν}/E for $E \leqslant 200$ GeV.

(Gargamelle coll.)
Perkins, Contemp. Phys. **16** 173 (75)

2) DIS (anti)neutrino-proton

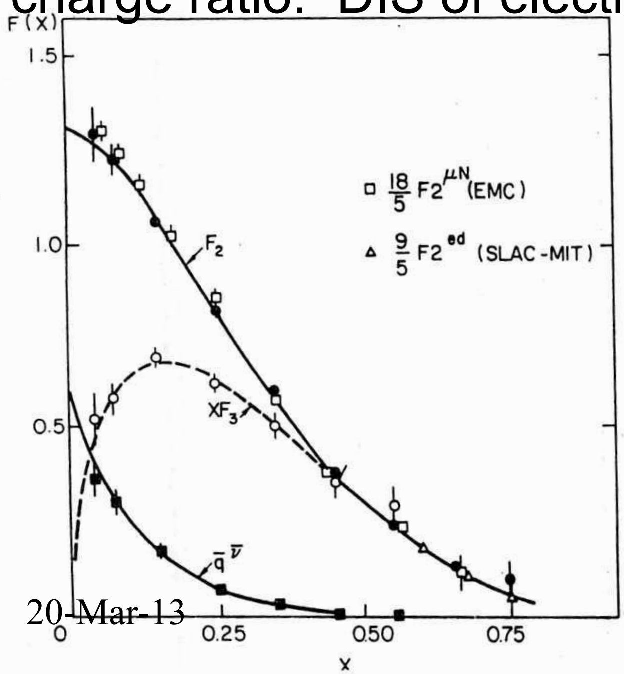
data: neutrino suppressed w.r.t. antineutrino in elastic limit $(\nu \to 0)$

$$\frac{\left.\dfrac{d\sigma^{\nu A}}{dx_B\,dy}\right|_{y=0}}{\left.\dfrac{d\sigma^{\bar\nu A}}{dx_B\,dy}\right|_{y=0}} \xrightarrow[x_B \to 1]{A=p} 0$$

for $x_B \to 1$ u-dominance consistent with

$$1 \xleftarrow{x_B \to 0} \frac{F_2^{e^- n}}{F_2^{e^- p}} \xrightarrow{x_B \to 1} \frac{1}{4}$$

3) charge ratio: DIS of electrons and (anti)neutrino on isoscalar nuclei



$$0 \xleftarrow{x_B \to 0} \frac{x_B F_3}{F_2} \xrightarrow{x_B \to 1} 1$$

$$\frac{F_2^{e^- p} + F_2^{e^- n}}{F_2^{\nu p} + F_2^{\nu n}} \sim \frac{5}{18}$$

Collab. BCDMS, P.L. **B195** 91 (87)
" " , " **B237** 592 (90)
" " , " " 599 (90)
" CCFR , Z. Phys. **C26** 1 (84)

Sum rules

Adler $$\int_0^1 \frac{dx}{2x}\left(F_2^{\bar{\nu}p} - F_2^{\nu p}\right) = n_u - n_d + n_c - n_s = 1$$

exp. 1.01 ± 0.20

Allasia *et al.*, P.L. **B135** 231 (84)
Z. Phys. **C28** 321 (85)

Gross-Lewellin Smith $$\int_0^1 \frac{dx}{2}\left(F_3^{\bar{\nu}p} + F_3^{\nu p}\right) = n_u + n_d + n_c + n_s = 3$$

$n_q = q - \bar{q}$ ⇔ excess of 3 quarks over antiquarks in *p*

exp. 2.50 ± 0.08

discrepancy !

Mishra, Proc. of SLAC
Summer Institute
(SLAC, Stanford, 1991) p. 407

(cont'd)

Momentum sum rule

$$\int_0^1 dx\left[\frac{9}{2}(F_2^{e^-p}+F_2^{e^-n})-\frac{3}{4}(F_2^{\nu p}+F_2^{\nu n})\right] \sim \int_0^1 dx\, x(u+\bar{u}+d+\bar{d}+s+\bar{s})$$

$\theta_c \sim 0$

$$= 1-\varepsilon$$

exp. $\to \varepsilon \sim 0.5$!

or using e⁻ DIS only:

$$\frac{9(1+\delta)}{5+2\delta}\int_0^1 dx\,(F_2^{e^-p}+F_2^{e^-n}) = \int_0^1 dx\, x(u+\bar{u}+d+\bar{d}+s+\bar{s}) = 1-\varepsilon$$

$$\delta = \frac{\int_0^1 dx x(s+\bar{s})}{\int_0^1 dx x(u+\bar{u}+d+\bar{d})}$$

exp. data for $F_2^{p/n}$ +
$SU_f(3)$ symmetry for q^{sea} +
extraction $u(x)$, $d(x)$, $s(x)$

$0 \lesssim \delta \lesssim 0.06$
$\Rightarrow \varepsilon \approx (0.54 \div 0.56) \pm 0.04$

partons with no charge (= gluons) carry around half of *N* momentum, but they are not included in QPM!