# Schema

- riassunto precedente lezione
- fattorizzazione e universalita` nel QPM: dal DIS al Drell-Yan (DY); definizioni di cinematica e invarianti per DY
- scaling e distribuzione angolare universale (1+cos$^2\theta$) in DY
- deviazioni dallo scaling e dalla distribuzione (1+cos$^2\theta$): effetti di QCD
- DIS inclusivo polarizzato: la distribuzione di elicita`
- regole di somma. L'esperimento EMC e la "spin crisis"

- correnti neutre → bosone vettore di gauge = $\gamma$ o $Z^0$ ; interferenza al crescere di $Q^2$ ; evidenze sperimentali e test del QPM in reazioni $\nu \to \nu$ e $\bar{\nu} \to \bar{\nu}$

- QPM = convoluzione di sezione d'urto elementare dipendente dal processo, calcolabile in QED, con distribuzione partonica universale, incognita e deducibile da confronto con i dati

- $e^+e^-$ inclusivo: distribuzione angolare della reazione elementare $e^+e^- \to \mu^+\mu^-$ ; test delle simmetrie $SU_f(3)$ e $SU_c(3)$

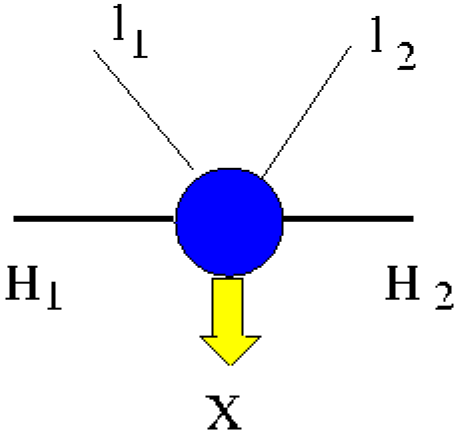
- $e^+e^-$ semi-inclusivo: frammentazione di un partone in un adrone → nuova incognita ; universalita` → confronta DIS semi-inclusivo

- fenomenologia di DIS semi-inclusivo con fasci di $e^-$ e $\nu$ + fenomenologia di $e^+e^-$ semi-inclusivo (scaling, ..) → informazioni su frammentazione (valence/sea dominance per $x_B \to$ 1/0 ..)

- $e^+e^-$ semi-inclusivo in due adroni → sezione d'urto di jet
adrone in stato finale si muove in un jet che rappresenta la direzione del quark di frammentazione rispetto all'asse z

# Drell - Yan

(Drell & Yan, P.R.L. **25** (70) 316)



adroni in annichilazione $H_{1/2}$ con momento $P_{1/2}$

leptoni prodotti $l_{1/2}$ con momento $k_{1/2}$

energia disponibile nel c.m. degli adroni

$$s = (P_1 + P_2)^2$$

massa invariante della coppia di leptoni

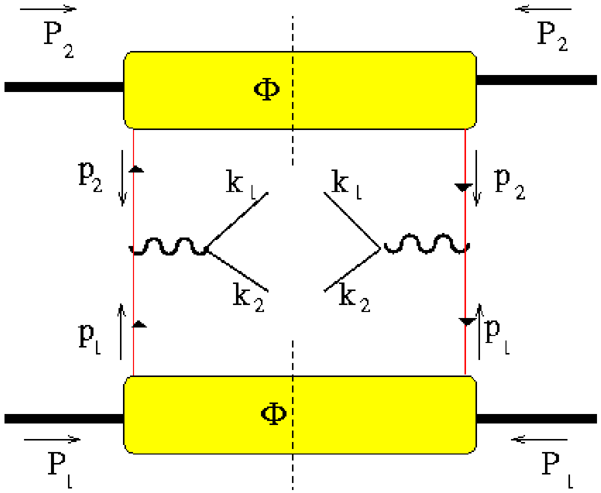
$$M^2 \equiv q^2 = (k_1 + k_2)^2$$

$q^2 = Q^2 \geq 0$ time-like

regime DIS : $q^2, s \to \infty$ con $\tau = q^2 / s$ fissato $1 \geq \tau \geq 0$

la coppia di leptoni non interagisce con la coppia di adroni iniziali
→ e` manifestazione del decadimento dei bosoni di gauge intermedi
prodotti dalla annichilazione adronica

bosoni di gauge a spin 1 con $Q^2 \geq 0 \to$ risonanze mesoniche vettoriali
→ decadimento → produzione di coppie leptoniche con $p_T$



$$\begin{aligned} p_1 &= x_1 P_1 \\ p_2 &= x_2 P_2 \end{aligned} \quad \text{con} \quad \begin{aligned} x_1 &= \frac{Q^2}{2P_1\cdot q} \\ x_2 &= \frac{Q^2}{2P_2\cdot q} \end{aligned} \quad 1 \geq x_{1/2} \geq 0$$

$$\tau = \frac{q^2}{s} = \frac{M^2}{s} = x_1 x_2 \qquad 1 \geq \tau \geq 0$$

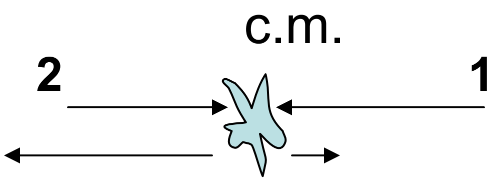
$$x_F = x_1 - x_2 \qquad 1 \geq x_F \geq -1$$

$$x_{1/2} = \frac{1}{2}\left(\pm x_F + \sqrt{x_F^2 + \frac{4M^2}{s}}\right)$$

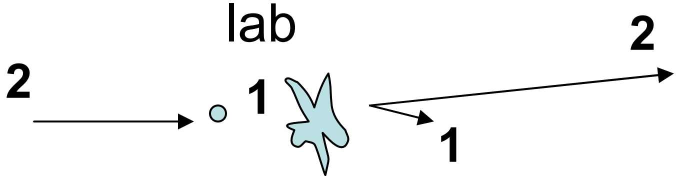
$x_{1/2}$ = frazione del momento longitudinale →
$x_F$ = momento longitudinale della coppia nel c.m. rispetto al momento longitudinale massimo possibile

supponiamo $H_2$ = fascio e $H_1$ = target

$x_F \to -1 \Leftrightarrow x_2 \to 1$
$x_1 \to 0$



angoli indietro



angoli in avanti

$x_F \to 1 \Leftrightarrow x_2 \to 0$
$x_1 \to 1$

situazione rovesciata

# “solite” formule, applicate al processo DY

$$d\sigma = \frac{1}{F}\,|\mathcal{M}|^2\,dR$$

$$\begin{aligned}
F &= 4\sqrt{(P_1\cdot P_2)^2 - M_1^2\,M_2^2} \sim 2s \\
dR &= (2\pi)^4\,\delta(P_1 + P_2 - P_X - k_1 - k_2)\,\frac{d\mathbf{P}_X}{(2\pi)^3 2P_X^0}\,\frac{d\mathbf{k}_1}{(2\pi)^3 2E_1}\,\frac{d\mathbf{k}_2}{(2\pi)^3 2E_2} \\
|\mathcal{M}|^2 &= \frac{e^4}{Q^4}\,L_{\mu\nu}\,H^{\mu\nu}
\end{aligned}$$

$$H^{\mu\nu} = \sum_X \langle P_1 S_1, P_2 S_2 | J^\mu | P_X\rangle\langle P_X | J^\nu | P_1 S_1, P_2 S_2\rangle$$

$$\frac{d\mathbf{k}_1}{E_1}\,\frac{d\mathbf{k}_2}{E_2} = \frac{d^4q\,d\Omega}{2} = \frac{s}{4}\,dx_1 dx_2 d\mathbf{q}_T d\Omega$$

$$q^\mu = k_1^\mu + k_2^\mu$$

$$\frac{d\sigma}{dx_1 dx_2 d\mathbf{q}_T d\Omega} = \frac{\alpha^2}{4Q^4}\,L_{\mu\nu}\,W^{\mu\nu}$$

# Metodo alternativo : Quark Parton Model

approssimazione: $Q^2$ non elevato → bosone di gauge $\gamma$

energia disponibile nel c.m. della reazione elementare :

$(p_1 + p_2)^2 \sim 2\, p_1 \cdot p_2 = x_1\, x_2\, 2\, P_1 \cdot P_2 \sim x_1\, x_2\, (P_1 + P_2)^2 = x_1\, x_2\, s$

processo elementare: $(q\bar{q} \to l\bar{l}) \quad \equiv \quad (e^+e^- \to l\bar{l})$

esempio: produzione di $\mu^+\mu^-$

$$\frac{d\sigma^{el}}{dq^2} = \frac{4\pi\alpha^2}{3Q^2}\, e_f^2\, \delta(x_1 x_2 s - q^2)$$

$$\begin{aligned}\frac{d\sigma}{dq^2} &= \frac{1}{N_c} \bar{\sum}_f \int_0^1 dx_1 dx_2\, \phi_f(x_1)\, \frac{d\sigma^{el}}{dq^2}\, \phi_{\bar{f}}(x_2) \\ &= \frac{4\pi\alpha^2}{9q^4} \bar{\sum}_f e_f^2 \int_0^1 dx_1 dx_2\, \phi_f(x_1)\, \phi_{\bar{f}}(x_2)\, \delta\left(x_1 x_2 \frac{s}{q^2} - 1\right)\end{aligned}$$

$$\bar{\sum}_f \ldots \phi_f(x_1)\, \phi_{\bar{f}}(x_2) \equiv \sum_f \ldots \left[\phi_f(x_1)\, \phi_{\bar{f}}(x_2) + \phi_{\bar{f}}(x_1)\, \phi_f(x_2)\right]$$

Commenti + evidenze sperimentali:

$$q^4 \frac{d\sigma}{dq^2} = \frac{4\pi\alpha^2}{9} \bar{\sum}_f e_f^2 \int_0^1 dx_1 dx_2\, \phi_f(x_1)\, \phi_{\bar{f}}(x_2)\, \delta\left(\frac{x_1 x_2}{\tau} - 1\right)$$

$$\Downarrow$$

$$q^3 \frac{d\sigma}{dq\,dx_1 dx_2} = \frac{8\pi\alpha^2}{9} \bar{\sum}_f e_f^2\, \phi_f(x_1)\, \phi_{\bar{f}}(x_2)\, \delta\left(\frac{x_1 x_2}{\tau} - 1\right)$$

oppure, con $\begin{cases} x_F = x_1 - x_2 \\ \tau = x_1 x_2 \end{cases}$ $\quad J = \begin{vmatrix} \frac{\partial x_F}{\partial x_1} & \frac{\partial x_F}{\partial x_2} \\ \frac{\partial \tau}{\partial x_1} & \frac{\partial \tau}{\partial x_2} \end{vmatrix} = x_1 + x_2$

$$\begin{aligned} M^3 \frac{d\sigma}{dM\,dx_F} &= \int d\tau \frac{d\sigma}{dM\,dx_F\,d\tau} \\ &= \frac{8\pi\alpha^2}{9} \frac{1}{x_1 + x_2} \bar{\sum}_f e_f^2\, \phi_f(x_1)\, \phi_{\bar{f}}(x_2) \int d\tau\, \delta\left(\frac{x_1 x_2}{\tau} - 1\right) \\ &= \frac{8\pi\alpha^2}{9} \frac{x_1 x_2}{x_1 + x_2} \bar{\sum}_f e_f^2\, \phi_f(x_1)\, \phi_{\bar{f}}(x_2) \end{aligned}$$

$$= \frac{\tau}{\sqrt{x_F^2 + 4\tau}} \sim \sqrt{\tau}$$

scaling ∀ s → elementary point-like interaction !

$$x_F = (1-\tau)\,x'_F$$

$$M^3 \frac{d\sigma}{dM\,dx'_F} = \frac{M^3}{\tau}\,\frac{d\sigma}{dM\,dx_F} \sim \frac{\sqrt{\tau}}{\tau} = \frac{1}{\sqrt{\tau}}$$

piccole deviazioni perche\`
pQCD → $\phi_f$ (x, log$Q^2$)

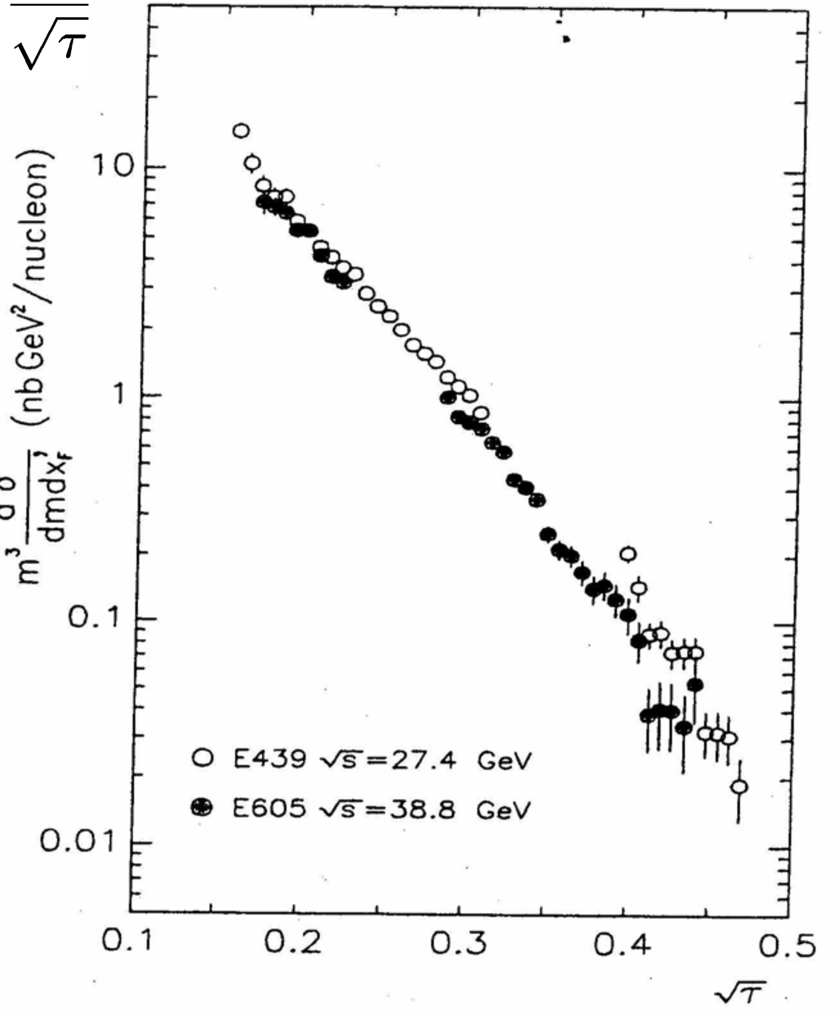


FIG. 12. Scaling form of the dimuon yield versus $\sqrt{\tau}$ comparing this experiment with experiment E439 (Smith *et al.*, Ref. 31, $\sqrt{s} = 27.4$ GeV) for the interval $0 < x'_F < .2$.

exp. E605 - Fermilab
Phys. Rev. D**43** (91) 2815

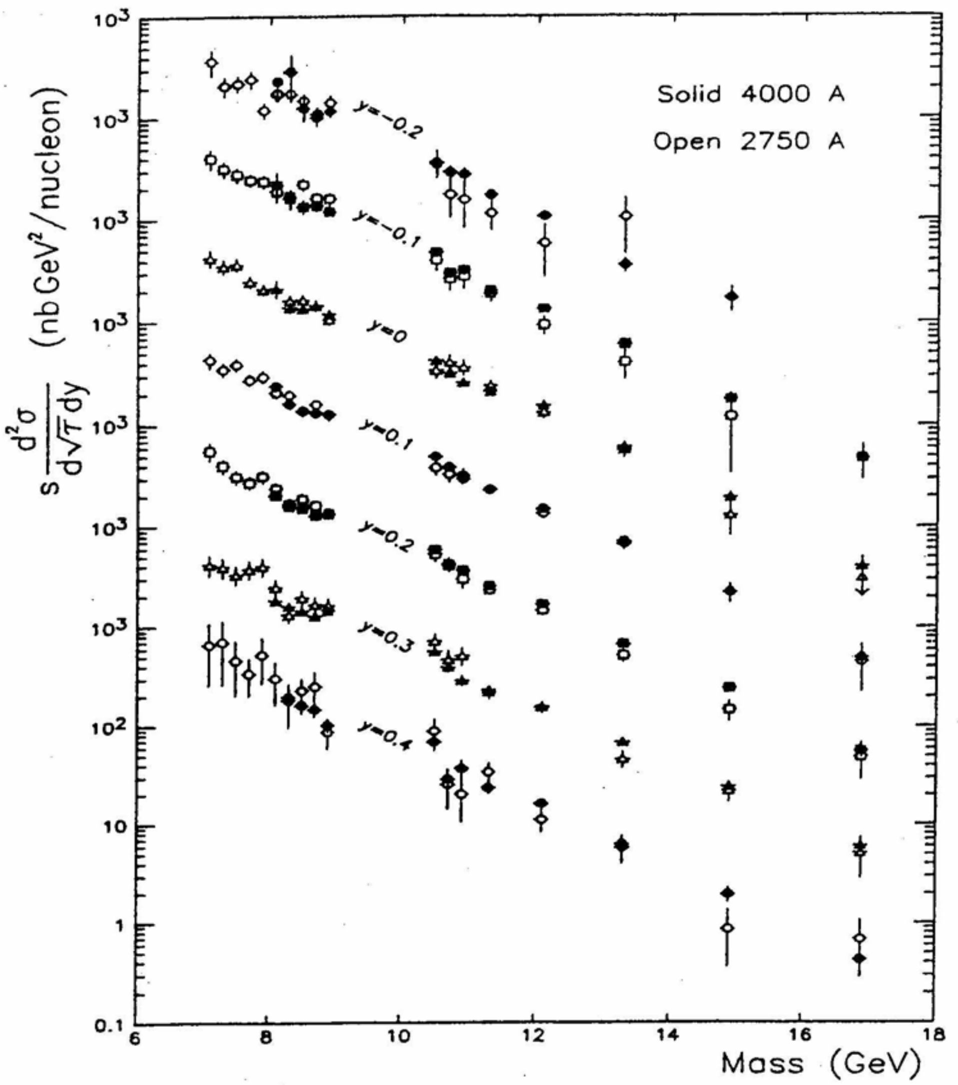


FIG. 10. Scaling form of the dimuon yield, excluding the Υ's, showing the agreement of the two sets in the region of overlap. The average of both sets is given in Table VIII.

$$\tau = \frac{M^2}{s} \rightarrow \frac{d}{d\sqrt{\tau}} = \frac{1}{2}\sqrt{\frac{s}{\tau}}\frac{d}{dM}$$

$$\text{rapidity } y = \frac{1}{2}\log\frac{x_1}{x_2} = \frac{1}{2}\log\left(\frac{x_F}{x_2}+1\right)$$

$$\rightarrow \frac{d}{dy} = \frac{d}{dx_F}\,2x_2\,e^{2y} = 2x_1\frac{d}{dx_F}$$

$$M^3\frac{d\sigma}{dM\,dx_F} = (\tau s)^{3/2}\,2\sqrt{\frac{\tau}{s}}\,\frac{1}{2x_1}\,\frac{d\sigma}{d\sqrt{\tau}dy}$$

$$= x_1 x_2^2\, s\,\frac{d\sigma}{d\sqrt{\tau}dy}$$

$$s\,\frac{d\sigma}{d\sqrt{\tau}dy} \sim \frac{1}{\tau x_2}\sqrt{\tau}$$

$$= \frac{2}{\tau(\sqrt{x_F^2+4\tau}-x_F)}\sqrt{\tau}$$

$$\sim \frac{1}{\tau} \sim \frac{1}{M^2}$$

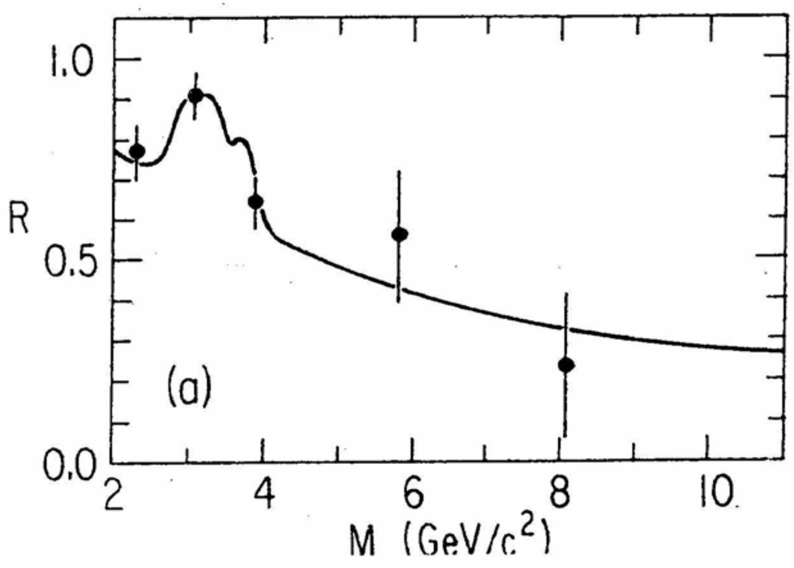
exp. E605 - Fermilab
Phys. Rev. D**43** (91) 2815

# ulteriori evidenze sperimentali

nuclei isoscalari → $n_u = n_d$ (ex. $^{12}$C)

$$1 \xleftarrow{\tau\to 0} \frac{\pi^{+12}C\to\mu^+\mu^-X}{\pi^{-12}C\to\mu^+\mu^-X} \xrightarrow{\tau\to 1} \frac{1}{4}$$

perche`?



Fermilab
Phys. Rev. Lett. **42** (79) 948

$\tau = x_1\, x_2 \to 1$ valence area

$$\frac{\pi^+(u\bar{d})\ C(u_1..u_n d_1..d_m)}{\pi-(d\bar{u})\ C(u_1..u_n d_1..d_m)}$$

$$\sim \frac{e_d^2}{e_u^2} = \frac{1}{4}$$

$\tau = x_1\, x_2 \to 0$ sea area

$$\frac{\pi^+\ C(\text{..sea quarks..})}{\pi-\ C(\text{..sea quarks..})}$$

$$\sim 1$$

meccanismo elementare

$$\boxed{q\bar{q} \to l\bar{l}}$$

continua

nuclei non isoscalari A = α protoni + β neutroni

$$\frac{\pi^+(u\bar{d})\ A(\alpha p+\beta n)\rightarrow\mu^+\mu^- X}{\pi^-(d\bar{u})\ A(\alpha p+\beta n)\rightarrow\mu^+\mu^- X}\sim\frac{\alpha d_p+\beta d_n}{4(\alpha u_p+\beta u_n)}=\frac{\alpha d_p+\beta u_p}{4(\alpha u_p+\beta d_p)}\overset{^1H}{=}\frac{d_p}{4u_p}$$

**isospin symmetry**

$\nearrow$ 0 $\quad x_A\rightarrow 1$

$\searrow$ $\frac{1}{8}$ $\quad x_A\rightarrow\frac{1}{3}$

N non ha antiquark di valenza per annichilazione

$$\rightarrow\ \frac{\pi^- N\rightarrow\mu^+\mu^- X}{NN\rightarrow\mu^+\mu^- X}$$

non scala in τ ma cresce con M = Q

Fermilab
Phys. Rev. Lett. **42** (79) 948

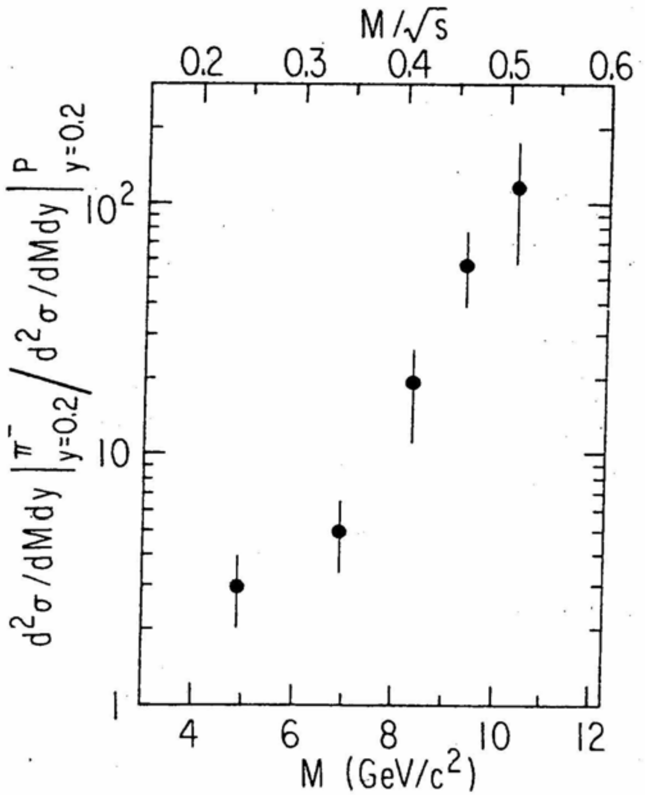


FIG. 1. The ratio of $\pi^-$-induced to proton-induced $\mu$-pair cross section at $y_{\text{c.m.}}=0.2$ as a function of mass. Proton data at 225 GeV/$c$ has been calculated from the scaling observed in 200-, 300-, and 400-GeV/$c$ data of Ref. 4.

continua

$$\frac{d\sigma}{dq^2} = \frac{1}{N_c}\bar{\sum}_f \int_0^1 dx_1 dx_2\, \phi_f(x_1)\, \frac{d\sigma^{el}}{dq^2}\, \phi_{\bar{f}}(x_2)$$

$$= \frac{4\pi\alpha^2}{9q^4}\bar{\sum}_f e_f^2 \int_0^1 dx_1 dx_2\, \phi_f(x_1)\, \phi_{\bar{f}}(x_2)\, \delta\left(x_1 x_2 \frac{s}{q^2} - 1\right)$$

$\frac{1}{N_c}\,\frac{4\pi\alpha^2}{3q^4}\cdot\cdot$

perche` solo $N_c$ modi di creare la coppia conservando il colore nel vertice di annichilazione e ciascuna $\phi_f$ porta $N_c$ colori indipendentemente
$\rightarrow (N_c \times N_c)\ 1 / N_c = N_c \ \rightarrow$ test di $SU_c(3)$

$\gamma^* \rightarrow \mu^+\mu^- X$

M ~ 3 GeV $J/\psi \rightarrow \mu^+\mu^- X$

M ~ 9 ÷ 10.5 GeV $Y \rightarrow \mu^+\mu^- X$

….

M $\gtrsim$ 70 GeV $Z^0$ , $W^\pm$

Al crescere di $Q^2 \equiv M^2$ si eccitano risonanze mesoniche vettoriali :

spettro della famiglia Υ $(b\bar{b})$

differente distribuzione in $\mathbf{p}_T$ della coppia di leptoni $\rightarrow$ nuovo meccanismo ?

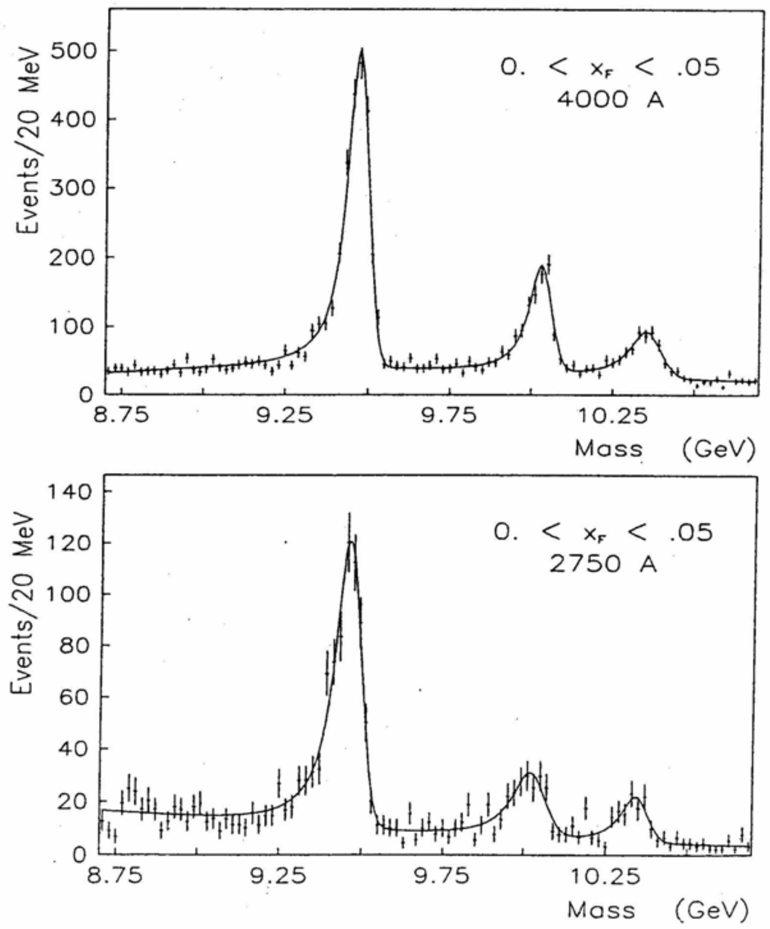


FIG. 16. The fit function used to extract Υ cross sections superimposed on the raw mass spectrum of the two data sets.

exp. E605 - Fermilab
Phys. Rev. D**43** (91) 2815

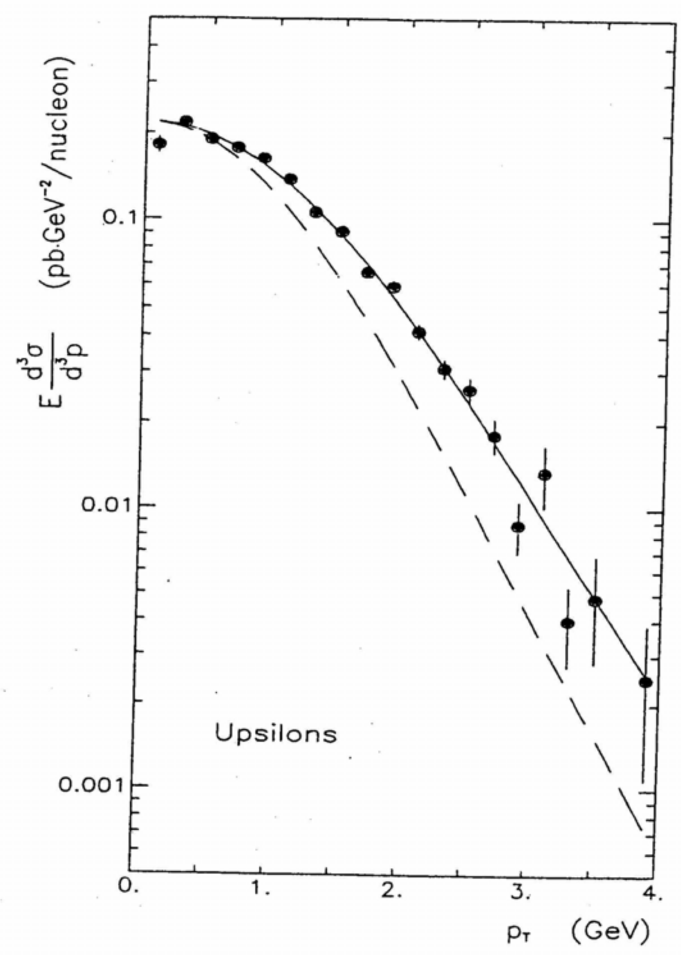


FIG. 19. $p_t$ distribution for the sum of the three Υ's. The solid curve is a fit to the data, the dashed curve gives the shape of the continuum dimuons under the resonances (see text).

# Formule generali per bosoni vettori di gauge elettrodeboli

$$\frac{d\sigma}{dq^2} = \sigma(q^2) \int_0^1 dx_1 dx_2\, \delta(q^2 - x_1 x_2 s)\, D(x_1, x_2) =$$

$\gamma$

$$\sigma = \frac{4\pi\alpha^2}{9q^2} \qquad D = \bar{\sum}_f e_f^2\, \phi_f(x_1)\, \phi_{\bar{f}}(x_2)$$

$Z^0$

$$\sigma = \frac{q^2\pi\alpha^2}{192 N_c \sin^4\theta_W \cos^4\theta_W} \frac{1 + [1 - 4\sin^2\theta_W]^2}{(q^2 - M_Z^2)^2 + M_Z^2\Gamma_Z^2}$$

$$\Gamma_Z = \frac{\alpha M_Z}{24 \sin^2\theta_W \cos^2\theta_W} [1 - 4\sin^2\theta_W + 8\sin^4\theta_W]$$

$$D = \bar{\sum}_f \left\{1 + \left[1 - 4|e_f| \sin^2\theta_W\right]^2\right\} \phi_f(x_1)\, \phi_{\bar{f}}(x_2)$$

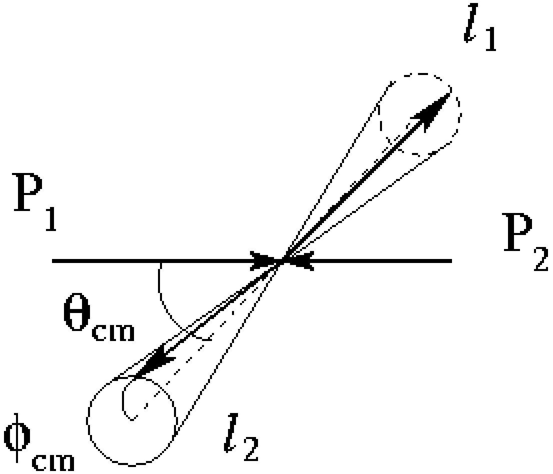
$W^{\pm}$

$$\sigma = \frac{q^2\pi\alpha^2}{12 N_c \sin^4\theta_W} \frac{1}{(q^2 - M_W^2)^2 + M_W^2\Gamma_W^2} \qquad \Gamma_W = \frac{\alpha M_W}{12\sin^2\theta_W}$$

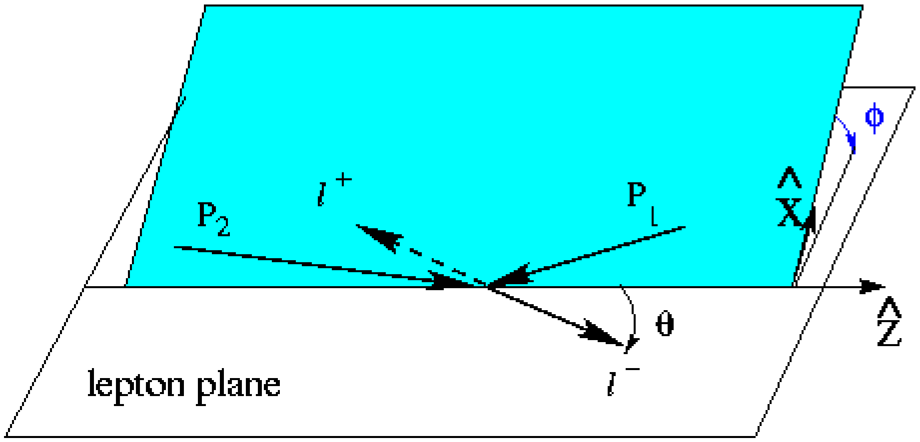
$$D = \cos^2\theta_C\, [\bar{u}_1 d_2 + \bar{c}_1 s_2] + \sin^2\theta_C\, [\bar{u}_1 s_2 + \bar{c}_1 d_2] + 1 \leftrightarrow 2$$

# Distribuzione angolare della coppia leptonica

c.m. degli adroni



c.m. dei leptoni (Collins-Soper frame)



se $p_T$ $(l_1 l_2) \neq 0$ → direzione di annichilazione non nota
Collins-Soper frame: asse z = direzione "media"

$$\frac{d\sigma}{d\Omega} = \frac{1}{N_c} \bar{\sum}_f \int_0^1 dx_1 dx_2\, \phi_f(x_1)\, \frac{d\sigma^{el}(e^+e^- \to \mu^+\mu^-)}{d\Omega}\, e_f^2\, \phi_{\bar{f}}(x_2)\, \delta(x_1 x_2\, s - q^2)$$

$$= \frac{\alpha^2}{12 q^4} (1 + \cos^2\theta) \bar{\sum}_f e_f^2 \int_0^1 dx_1 dx_2\, \phi_f(x_1)\, \phi_{\bar{f}}(x_2)\, \delta\left(x_1 x_2 \frac{s}{q^2} - 1\right)$$

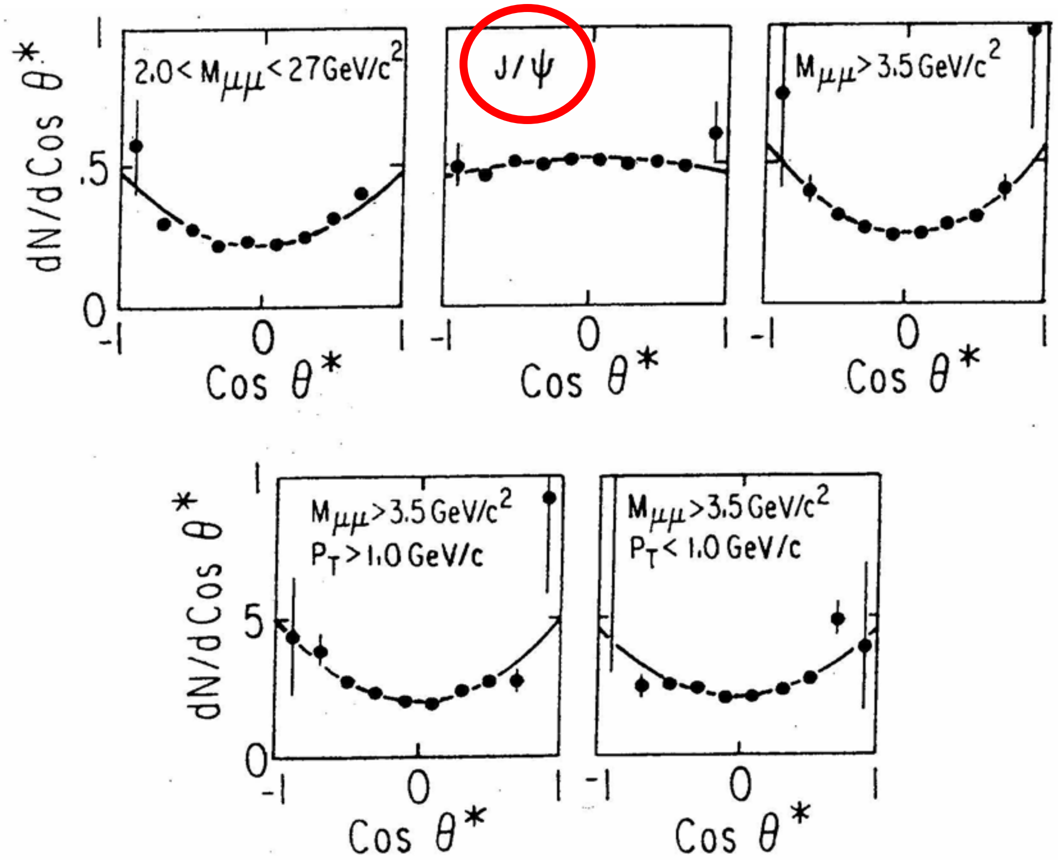


**FIG. 3.** Helicity angular distributions in three different mass intervals. The $M > 3.5$ GeV/$c^2$ interval is also shown divided in two $p_T$ intervals. The Collins-Soper angle ($\theta^*$) is defined in the text.

distribuzione angolare
$\sim (1 + \cos^2 \theta)$
data dal processo elementare
$e^+e^- \to \mu^+\mu^-$

pero` sulla risonanza J/ψ
distribuzione piatta
→ meccanismo diverso

da $\gamma^* \to \mu^+\mu^-$

$J/\psi \to \mu^+\mu^-$ ?

Fermilab
Phys. Rev. Lett. **42** (79) 948

# Formula generale per la sezione d'urto di DY

$$E\,\frac{d\sigma}{dx_F\,d\mathbf{p}_T\,dM\,d\Omega} \;=\; \frac{\alpha^2}{4\pi M s^{3/2}}\left[W_T\,(1+\cos^2\theta_{cm}) + W_L\,\sin^2\theta_{cm}\right.$$

**polarizz. del $\gamma^*$** **trasversa** **longitudinale**

$$\left. +W_{\uparrow}\,\sin 2\theta_{cm}\cos\phi_{cm} + W_{\uparrow\uparrow}\,\sin^2\theta_{cm}\cos 2\phi_{cm}\right]$$

**1 spin flip** **2 spin flip**

**elementi non diagonali della matrice densita` della coppia $H_1$ $H_2$**

$$\int d\phi_{cm}\,'''' = E\,\frac{d\sigma}{dx_F\,dM\,d\mathbf{p}_T\,d\cos\theta_{cm}} \sim 1+\alpha\cos^2\theta_{cm}\ ,\ \alpha = \frac{W_T - W_L}{W_T + W_L}$$

**on-shell (anti)quark**
**con spin 1/2**

$\alpha = 1 \Rightarrow W_L = W_{\uparrow} = W_{\uparrow\uparrow} = 0$

$\alpha \neq 1 \Rightarrow$
- frame differente con $p_T \neq 0$
  **(Collis-Soper frame → $\alpha$ = 0.85)**
- meccanismo differente → $p_T \neq 0$

$p\,p \rightarrow \mu^+\mu^- X$

Fermilab
Phys. Rev. Lett.
**43** (79) 1219

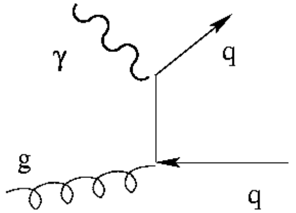
M > 4 GeV (no J/$\psi$)

$$d\sigma \sim 1 + \alpha \cos^2 \theta_{cm}$$

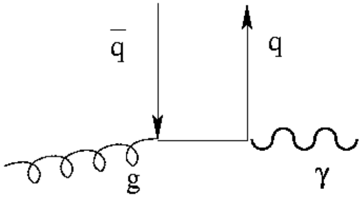
$$\alpha = \frac{W_T - W_L}{W_T + W_L}$$

$\alpha \sim 1 \rightarrow W_T$ dominante

Ma sulla risonanza J/$\psi$ meccanismo puo` essere diverso.
DY appartiene a classe piu` generale di processi A+B $\rightarrow$ C+X dove meccanismo elementare puo` essere piu` complicato:



**QCD Compton**



**γ – g fusion**

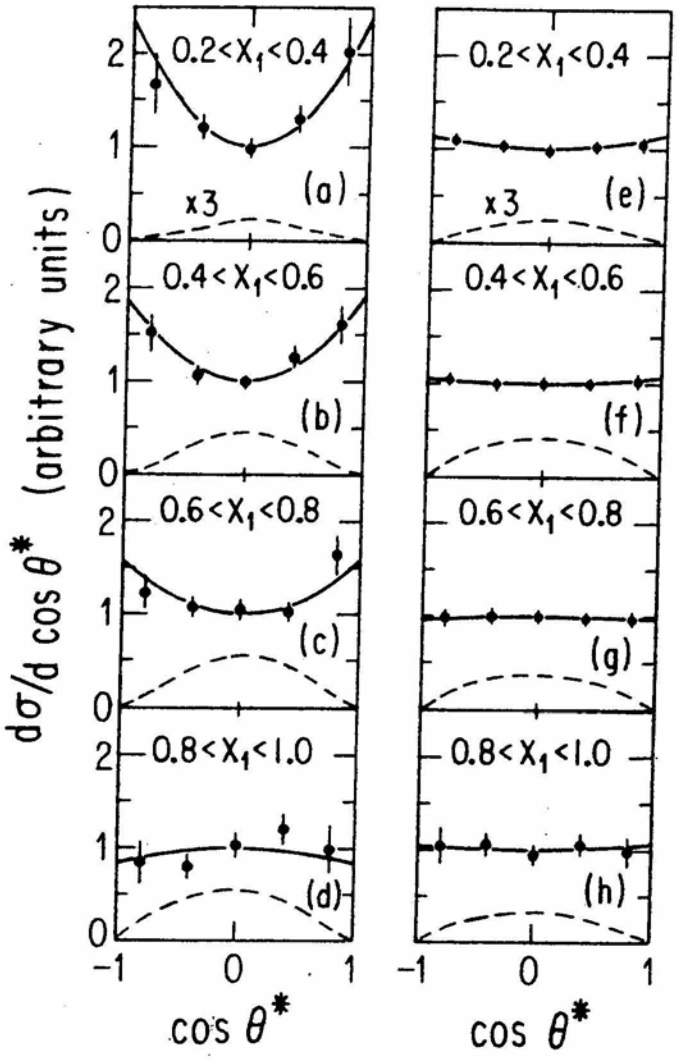


FIG. 1. $d\sigma/d\cos\theta^*$ in the $t$-channel helicity frame for various $x_1$ intervals. (a)−(d) Results for the mass continuum with $M > 4$ GeV; (e)−(h) results for the $J/\psi$ resonance in the same $x_1$ intervals. Data are integrated over $P_T$. The dashed curve shows the variation of detection efficiency with $\cos\theta^*$. The same arbitrary efficiency

presenza di $p_T \neq 0$
correzioni pQCD → q e`off-shell → $W_L \neq 0$

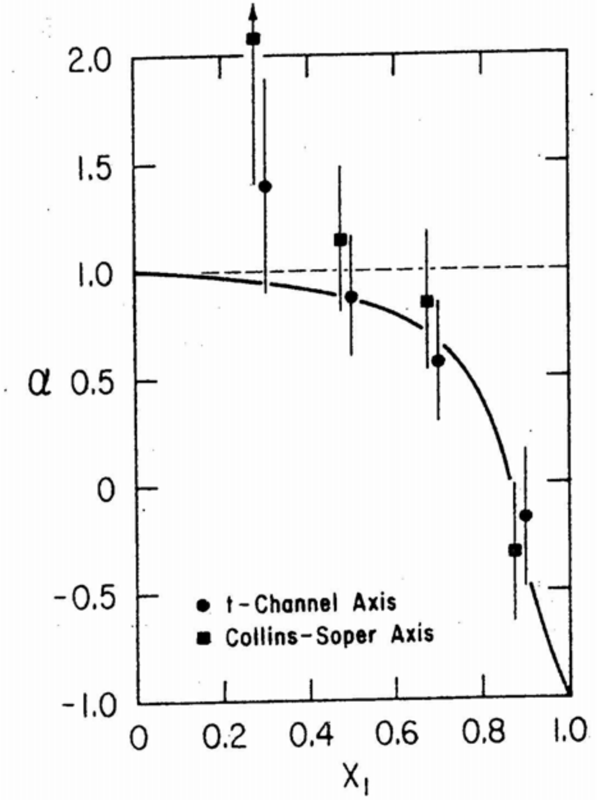


FIG. 2. The dependence of $\alpha$ on $x_1$ for data with $M > 4$ GeV. The dashed line is the expected result for the naive Drell-Yan model. The solid curve is the QCD prediction of Berger and Brodsky (Ref. 8).

Fermilab
Phys. Rev. Lett. **43** (79) 1219

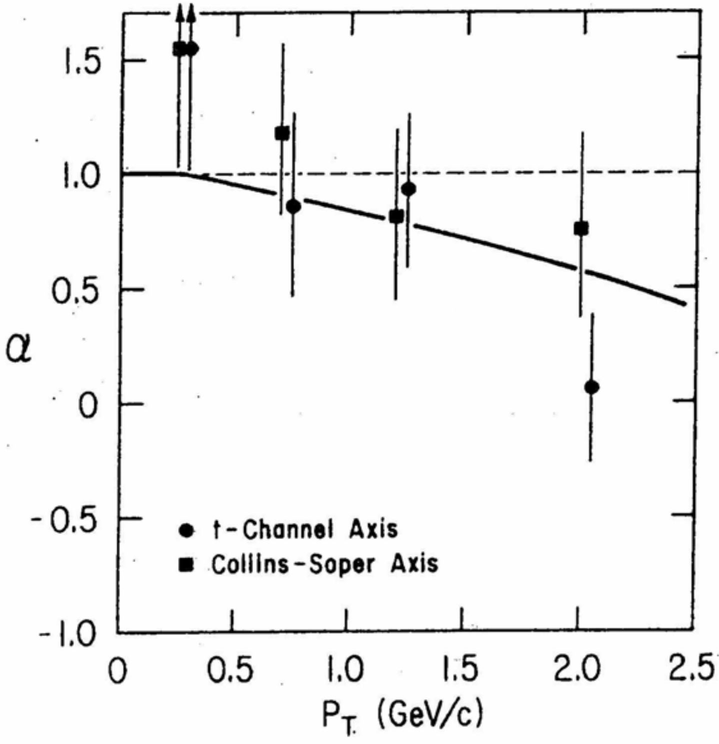
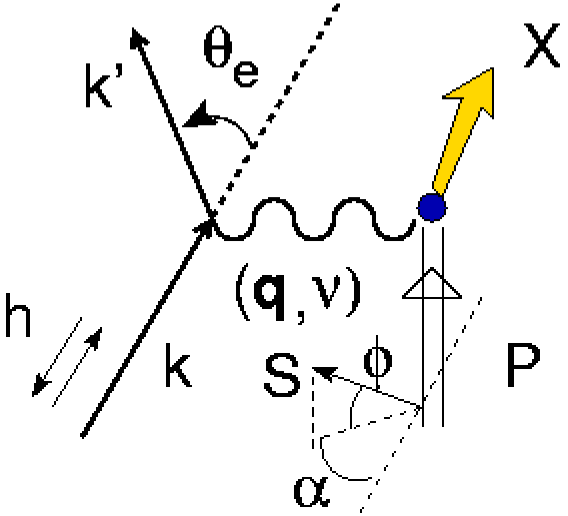


FIG. 3. The dependence of $\alpha$ on $P_T$ for data with $M$ >4 GeV. The smooth curve is the QCD prediction fro Kajantie *et al.* (Ref. 7). These authors consider only $x$ =0 while the data are integrated over $x_F$.

pQCD spiega inoltre
normalizzazione tra
th. ed exp. → K factor
K ∼ 2 !

# DIS inclusivo polarizzato



se S=0 e h ≠ 0 → violazione della parita`
processo debole → corrente *V-A*

se S,h ≠ 0 → 2 4-vettori P,q e 1 4-pseudovettore S indipendenti
struttura del tensore adronico piu` ricca

si sceglie $S^\mu$ tale che $S^2 = -1$ e $S \cdot P = 0$

$$S^\mu = \frac{S \cdot q}{P \cdot q}\left(P^\mu - \frac{M^2}{P \cdot q} q^\mu\right) + S_\perp^\mu = \frac{\lambda}{M}\left(P^\mu - \frac{M^2}{P \cdot q} q^\mu\right) + S_\perp^\mu$$

elicita`
$$\begin{aligned} \lambda &= M\frac{S \cdot q}{P \cdot q} \\ S_\perp \cdot P &= S_\perp \cdot q = 0 \\ S^2 &\sim -(\lambda^2 + \mathbf{S}_\perp^2) = -1 \end{aligned}$$

S = ½ → $W^{\mu\nu}$ e` al piu` lineare in S, perche`
e` matrice 2x2 in spazio di spin

$$W^{\mu\nu} = \sum_{\alpha\alpha'} W^{\mu\nu}_{\alpha\alpha'}\, \rho_{\alpha\alpha'} = \frac{1}{2}\sum_{\alpha\alpha'} W^{\mu\nu}_{\alpha\alpha'}\, (1 + \mathbf{P} \cdot \sigma)_{\alpha\alpha'}$$

**matrice densita` di spin del target**

**vettore di polarizzazione**

$$P_i = \frac{N_+ - N_-}{N_+ + N_-} = <\sigma_i> = \mathrm{Tr}(\rho\sigma_i)$$

# Tensore adronico

- $S^\mu$ coplanar with scattering plane → $\phi = 0$
- hermiticity del tensore
- invarianza per trasformazioni di parita`
- invarianza per trasformazioni di time-reversal
- conservazione della corrente

$$W^{\mu\nu} = W^{\mu\nu}_S + W^{\mu\nu}_A$$

$$W_S^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) W_1 + \frac{\tilde{P}^\mu \tilde{P}^\nu}{M^2} W_2$$

$$\tilde{P}^\mu = P^\mu - \frac{P \cdot q}{q^2}\, q^\mu$$

$$W_A^{\mu\nu} = i\epsilon^{\mu\nu\rho\sigma}\, q_\rho \left[\underset{\underset{\textbf{scalare}}{\uparrow}}{A_1} S_\sigma + \underset{\underset{\textbf{pseudoscalare}}{\uparrow}}{A_2} P_\sigma\right]$$

$$\begin{aligned} W_A^{\mu\nu} \;&=\; i\epsilon^{\mu\nu\rho\sigma}\, q_\rho\, S_\sigma \left[M \underline{G_1(\nu, Q^2)} + \frac{P \cdot q}{M}\, \underline{G_2(\nu, Q^2)}\right] \\ &\quad - i\epsilon^{\mu\nu\rho\sigma}\, q_\rho\, P_\sigma\, \frac{S \cdot q}{M}\, G_2(\nu, Q^2) \end{aligned}$$

Tensore leptonico : $L_{\mu\nu} = L_{\mu\nu}^{S} \pm L_{\mu\nu}^{A}$

$$\begin{aligned} L^S_{\mu\nu} &= 2k_\mu k'_\nu + 2k_\nu k'_\mu - 2k\cdot k'\, g_{\mu\nu} \\ L^A_{\mu\nu} &= 2i\epsilon_{\mu\nu\rho\sigma}\, k^\rho k'^{\,\sigma} \end{aligned}$$

$L_{\mu\nu}^{S}\ W^{\mu\nu}{}_{S} \rightarrow$

$$\frac{d\sigma^0}{dE'd\Omega} = \frac{4\alpha^2}{Q^4} E'^{\,2} \left( 2\sin^2\frac{\theta_e}{2} W_1 + \cos^2\frac{\theta_e}{2} W_2 \right)$$

$$\begin{aligned} \pm L^A_{\mu\nu}\, i\epsilon^{\mu\nu\rho\sigma}\, q_\rho\, S_\sigma &= \mp 2q^2\, S\cdot(k+k') \overset{\text{TRF}}{=} \mp 8EE' \sin^2\frac{\theta_e}{2}\, \widehat{S}\cdot(\mathbf{k}+\mathbf{k}') \\ \pm L^A_{\mu\nu}\, (-i)\epsilon^{\mu\nu\rho\sigma}\, q_\rho\, P_\sigma\, \frac{S\cdot q}{M} &= \pm 2q^2\, P\cdot(k+k')\, S\cdot q \\ &\overset{\text{TRF}}{=} \pm 8EE'(E+E')\, \sin^2\frac{\theta_e}{2}\, \widehat{S}\cdot\mathbf{q} \end{aligned}$$

$$\begin{cases} k &= (E,0,0,E) \\ k' &= (E', E'\sin\theta_e, 0, E'\cos\theta_e) \\ \widehat{S} &= (0, \sin\alpha\cos\phi, \sin\alpha\sin\phi, \cos\alpha) \end{cases}$$

**coplanar → $\phi$ = 0**

$$\begin{aligned} \frac{d\sigma^{h=-} - d\sigma^{h=+}}{dE'd\Omega} &= \frac{2\alpha^2}{Q^4}\frac{E'}{E} 8EE'\sin^2\frac{\theta_e}{2} \left\{ \left( MG_1 + \frac{P\cdot q}{M} G_2 \right) \widehat{S}\cdot(\mathbf{k}+\mathbf{k}') - (E+E')\, G_2\, \widehat{S}\cdot\mathbf{q} \right\} \\ &= \frac{4\alpha^2}{Q^2}\frac{E'}{E} \left\{ \underline{\cos\alpha} \left[ (E + E'\cos\theta_e)\, MG_1 - Q^2 G_2 \right] \right. \\ &\qquad \left. + E'\sin\theta_e\ \underline{\sin\alpha}\, (MG_1 + 2EG_2) \right\} \end{aligned}$$

$\boxed{S \parallel k \rightarrow \alpha = 0}$

$$\frac{d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}}{dE'd\Omega} = \frac{4\alpha^2}{Q^2}\frac{E'}{E}\left[(E + E'\cos\theta_e)\, M G_1 - Q^2 G_2\right]$$

$\boxed{S \perp k \rightarrow \alpha = \pi/2}$

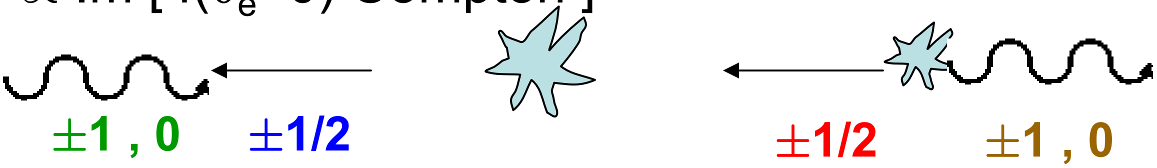
$$\frac{d\sigma^{\uparrow\rightarrow} - d\sigma^{\uparrow\leftarrow}}{dE'd\Omega} = \frac{4\alpha^2}{Q^2}\frac{E'}{E}\left[E'\sin\theta_e\,(M G_1 + 2E\, G_2)\right]$$

Perche` 4 funzioni di struttura $F_1$, $F_2$, $G_1$, $G_2$ ?

sezione d'urto totale per assorbimento di $\gamma^*$ : $\sigma_{tot}\,(\gamma^*\, N)$

teorema ottico : $\sigma_{tot}\,(\gamma^*\, N) \propto \mathrm{Im}\,[\, f(\theta_e{=}0)$ Compton ]

$$f(\theta_e = 0, \vec{q}, \vec{N})$$



iniziale

intermedio

finale

| | | | | | |
|---|---|---|---|---|---|
| 1 | +1 | +1/2 | +3/2 | +1 | +1/2 |
| 2 | +1 | -1/2 | +1/2 | +1 | -1/2 |
| 3 | +1 | -1/2 | +1/2 | 0 | +1/2 |
| 4 | 0 | +1/2 | +1/2 | +1 | -1/2 |
| 5 | 0 | +1/2 | +1/2 | 0 | +1/2 |

legati da time-reversal

→ 4 strutture indipendenti

# Riarrangiamento delle 4 combinazioni indipendenti

$$\begin{array}{rcccccc}
\left[(1,\tfrac{1}{2})\to(1,\tfrac{1}{2})\right] + \left[(1,-\tfrac{1}{2})\to(1,-\tfrac{1}{2})\right] & \equiv & W_T & = & W_1 & = & \sigma^T_{3/2}+\sigma^T_{1/2} \\
(0,\tfrac{1}{2})\to(0,\tfrac{1}{2}) & \equiv & W_L & = & \left(1+\frac{\nu^2}{Q^2}\right)W_2 - W_1 & = & \sigma^L_{1/2} \\
\left[(1,\tfrac{1}{2})\to(1,\tfrac{1}{2})\right] - \left[(1,-\tfrac{1}{2})\to(1,-\tfrac{1}{2})\right] & \equiv & W_{TT} & = & -\nu M G_1 + Q^2 G_2 & = & \sigma^T_{3/2}-\sigma^T_{1/2} \\
(1,-\tfrac{1}{2})\to(0,\tfrac{1}{2}) & \equiv & W_{LT} & = & \sqrt{2Q^2}\,(MG_1+\nu G_2) & = & \sigma^{LT}_{1/2}
\end{array}$$

asimmetrie di elicita`

$$\sigma^{\lambda_{\gamma*}}_{J_z}$$

← elicita` di $\gamma^*$

← intermedio

$$A_1 = \frac{\sigma^T_{1/2}-\sigma^T_{3/2}}{\sigma^T_{1/2}+\sigma^T_{3/2}} = -\frac{W_{TT}}{W_T} = \frac{\nu M G_1 - Q^2 G_2}{W_1} \qquad 1 \geq |A_1|$$

$$A_2 = \frac{W_{LT}}{W_T} = \frac{\sqrt{2Q^2}(MG_1+\nu G_2)}{W_1} \qquad R = \frac{\sigma_L}{\sigma_T} \geq |A_2| = \frac{\sigma_{LT}}{\sigma_T}$$

misura sperimentale accede a

polarizz. lineare di $\gamma^*$

$$\epsilon = \left[1+2\frac{\mathrm{q}^2}{Q^2}\tan^2\frac{\theta_e}{2}\right]^{-1}$$

$$A_{\parallel} = \frac{d\sigma^{\uparrow\uparrow}-d\sigma^{\uparrow\downarrow}}{d\sigma^{\uparrow\uparrow}+d\sigma^{\uparrow\downarrow}} = \frac{E-E'\epsilon}{E(1+\epsilon R)}A_1 + \frac{\epsilon Q}{E(1+\epsilon R)}A_2$$

$$A_{\perp} = \frac{d\sigma^{\uparrow\leftarrow}-d\sigma^{\uparrow\rightarrow}}{d\sigma^{\uparrow\leftarrow}+d\sigma^{\uparrow\rightarrow}} = \frac{E-E'\epsilon}{E(1+\epsilon R)}\sqrt{2\epsilon(1+\epsilon)}\,A_2 - \frac{\epsilon Q}{E(1+\epsilon R)}\sqrt{\frac{(1+\epsilon)^3}{2\epsilon}}\,A_1$$

limite DIS : $\nu, Q^2 \to \infty$ con $x_B$ fisso

scaling :
$$M W_1(\nu, Q^2) \to F_1(x_B) \qquad M^2 \nu G_1(\nu, Q^2) \to \tilde{G}_1(x_B)$$
$$\nu W_2(\nu, Q^2) \to F_2(x_B) \qquad M \nu^2 G_2(\nu, Q^2) \to \tilde{G}_2(x_B)$$

**(vedi espressioni di $A_1$ e $A_2$)**

scaling delle asimmetrie di elicita` :

$$A_1 = \frac{\nu M G_1(\nu, Q^2) - Q^2 G_2(\nu, Q^2)}{W_1(\nu, Q^2)} \longrightarrow \frac{\tilde{G}_1(x_B)}{F_1(x_B)} - 2M^2 \nu x_B \frac{G_2}{F_1(x_B)}$$

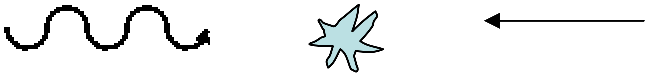
$$= \frac{\tilde{G}_1(x_B)}{F_1(x_B)} - \frac{2 M x_B}{F_1(x_B)} \frac{\tilde{G}_2(x_B)}{\nu} \longrightarrow \frac{\tilde{G}_1(x_B)}{F_1(x_B)}$$

$$A_2 = \frac{\sqrt{2Q^2}\, (M G_1(\nu, Q^2) + \nu\, G_2(\nu, Q^2))}{W_1(\nu, Q^2)} \longrightarrow \sqrt{2} \frac{Q}{\nu} \frac{\tilde{G}_1(x_B) + \tilde{G}_2(x_B)}{F_1(x_B)}$$

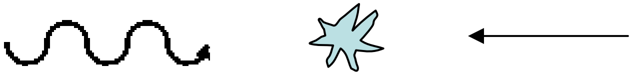
$$= 2\sqrt{\frac{M x_B}{\nu}} \frac{\tilde{G}_1(x_B) + \tilde{G}_2(x_B)}{F_1(x_B)} \longrightarrow 0$$

scrivere $d\sigma^{\uparrow\uparrow}$ e $d\sigma^{\uparrow\Downarrow}$ in termini di $\tilde{G}_1$, $\tilde{G}_2$ ; usare ipotesi di QPM ;
estrarre info sulle funzioni di struttura in termini di densita` partoniche

Metodo alternativo :



$\gamma^*$ +1 $J_z=3/2$ $\sigma^T_{3/2}$ +1/2 $P^\uparrow$
-1 $J_z=1/2$ $\sigma^T_{1/2}$ +1/2



$\gamma^*$ $\pm$ 1 J=1/2 $\pm$ 1/2 $q^{\uparrow\Downarrow}$

perche` $L_z = 0$ (processo collineare)
$\to$ conservazione del momento angolare

Quindi $\gamma^{*\uparrow} q^{\Downarrow} \to q^{\uparrow}$ $\cancel{\gamma^{*\uparrow} q^{\uparrow}}$
$\gamma^{*\Downarrow} q^{\uparrow} \to q^{\Downarrow}$ $\cancel{\gamma^{*\Downarrow} q^{\Downarrow}}$

$$\left.\begin{array}{l}\sigma^T_{3/2} \leftrightarrow \gamma^{*\uparrow} P^{\uparrow} \propto \sum_f e_f^2\, q_f^{\downarrow} \\ \sigma^T_{1/2} \leftrightarrow \gamma^{*\downarrow} P^{\uparrow} \propto \sum_f e_f^2\, q_f^{\uparrow}\end{array}\right\} \longrightarrow A_1 = \frac{\sigma^T_{1/2} - \sigma^T_{3/2}}{\sigma^T_{1/2} + \sigma^T_{3/2}} = \frac{\sum_f e_f^2\,(q_f^{\uparrow} - q_f^{\downarrow})}{\sum_f e_f^2\,(q_f^{\uparrow} + q_f^{\downarrow})} = \frac{g_1(x_B)}{f_1(x_B)}$$

distribuzione di elicita` $g_1(x_B) = \frac{1}{2} \sum_f e_f^2\, [q_f^{\uparrow}(x_B) - q_f^{\downarrow}(x_B)]$

similmente $g_1(x_B) + g_2(x_B) = \frac{1}{2Mx_B} \sum_f e_f^2\, m_f\, [q_f^{\rightarrow}(x_B) - q_f^{\leftarrow}(x_B)]$

interesse in $g_1(x_B)$ e` dovuto al legame con la carica assiale del nucleone

$$\langle PS|\bar{q}\,\gamma^\mu\,\gamma_5\,q|PS\rangle|_{\mu^2} = S^\mu\,\Delta q(\mu^2) \qquad \Delta q = \int_0^1 dx\,(q^\uparrow(x) - q^\downarrow(x))$$

$\mu$ scala di rinormalizzazione sottintesa

1° momento di $g_1$

$$\Gamma_1(Q^2) = \int_0^1 dx\,g_1(x,Q^2) = \frac{1}{2}\sum_f e_f^2 \int_0^1 dx\,(q_f^\uparrow(x) - q_f^\downarrow(x)) = \frac{1}{2}\sum_f e_f^2\,\Delta q_f$$

In QPM per protone : $\Gamma_1^p = \frac{1}{2}\left(\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right)$ (+ correzioni di pQCD)

$$\begin{aligned}
2\langle PS|A^3_\mu|PS\rangle &= g_A S_\mu \equiv (F+D)S_\mu \\
&= (\Delta u - \Delta d)S_\mu \\
2\sqrt{3}\langle PS|A^8_\mu|PS\rangle &= (\Delta u + \Delta d - 2\Delta s)S_\mu \\
&= (3F - D)S_\mu \\
\langle PS|A^0_\mu|PS\rangle|_{\mu^2} &= \Delta\Sigma(\mu^2) S_\mu = (\Delta u + \Delta d + \Delta s)S_\mu
\end{aligned}$$

info su da corrente assiale $A_\mu^a \sim \gamma_\mu\,\gamma_5\,T^a$ in transizioni di Gamow-Teller nell'ottetto barionico

$$\partial^\mu A^0_\mu = \frac{n_f \alpha_s}{2\pi}\,\epsilon^{\mu\nu\rho\sigma}\,F_{\mu\nu}\,F_{\rho\sigma}$$

anomalia triangolare
dipendenza da $\mu^2$

$g_A$ = F+D carica assiale (si misura in
n$\to$ p+e+$\gamma$)

F,D el. Matrice invarianti di $A_\mu^a$ in
SU(3) decadimenti semileptonici

Quindi $\Gamma_1$ ($\Delta$u, $\Delta$d, $\Delta$s) $\rightarrow \Gamma_1$ (F, D, $\Delta\Sigma$) ; inoltre $\begin{cases} \Delta u - \Delta d = F+D \\ \Delta u + \Delta d - 2\,\Delta s = 3F - D \\ \Delta u + \Delta d + \Delta s = \Delta\Sigma \end{cases}$

da fit a decadimenti semileptonici

$\rightarrow$ F= 0.47 $\pm$ 0.004 ; D=0.81$\pm$ 0.003 ; ma no info su $\Delta\Sigma$ !

Ellis-Jaffe ('73) : Hp. SU(3) + $\Delta$s = 0

$$\Gamma_1^p = \int_0^1 dx\, g_1^p(x) = \frac{1}{18}(4\Delta u + \Delta d) + \text{correzioni pQCD} = 0.17 \pm 0.01$$

$$\Delta\Sigma = \Delta u + \Delta d = 3F - D = 0.60 \pm 0.12$$

in QPM funz. d'onda del q in $P^\uparrow$ secondo $SU_f(3) \otimes SU(2) = SU(6)$

$$|P^\uparrow\rangle = \frac{1}{\sqrt{18}}\left(2u^\uparrow u^\uparrow d^\downarrow - u^\uparrow u^\downarrow d^\uparrow - u^\downarrow u^\uparrow d^\uparrow + \text{permutazioni di d ..}\right)$$

probabilita` di : $u^\uparrow = \frac{5}{3}$ ; $u^\downarrow = \frac{1}{3}$ ; $d^\uparrow = \frac{1}{3}$ ; $d^\downarrow = \frac{2}{3}$

Ex. per $u^\uparrow$ : $<P^\uparrow | P^\uparrow> = 1/18\,[\,4 \times 2 + 1 + 1\,] \times 3 = 5/3$

$u^\uparrow u^\uparrow d^\Downarrow$ $u^\uparrow u^\Downarrow d^\uparrow$ $u^\Downarrow u^\uparrow d^\uparrow$ permut. di d

$\rightarrow \Gamma_1^p = 5/18 \sim 0.28$

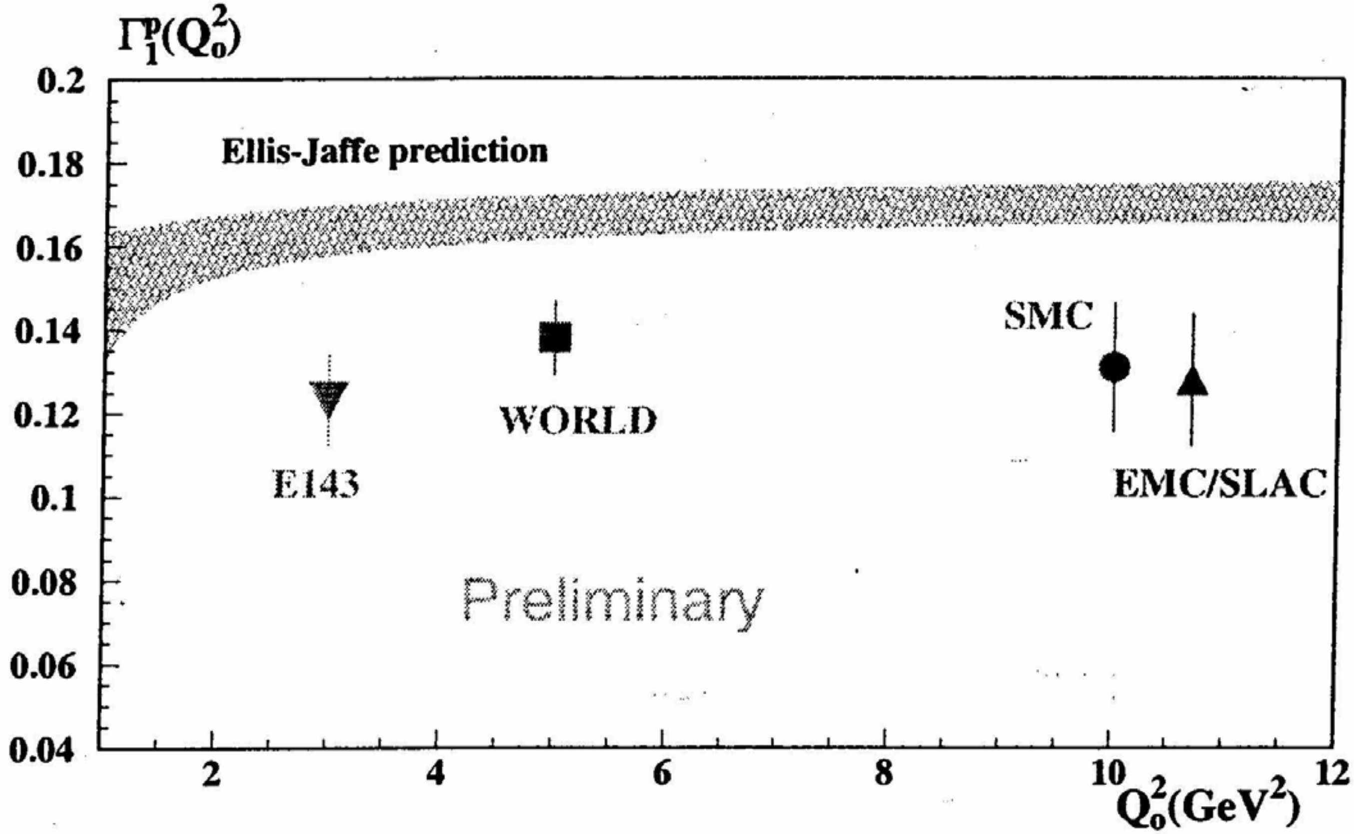
$\Delta\Sigma = 1$

dato sperimentale: $$A_{\|} = \frac{d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}}{d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}} \sim \frac{E - E'\epsilon}{E(1+\epsilon R)} A_1 \sim \frac{E - E'\epsilon}{E(1+\epsilon R)} \frac{g_1(x_B)}{F_1(x_B)}$$

R = $\sigma_L/\sigma_T$ da sez. d'urto non polarizzata

EMC (Cern, ’87) : $\mu^{\uparrow}p^{\uparrow} \rightarrow \mu p$ at $Q^2 = 10.7$ GeV$^2$ $\Gamma_1{}^p = 0.126 \pm 0.010 \pm 0.015$

confermato data altri esperimenti: SMC (Cern), E142 e E143 (SLAC)

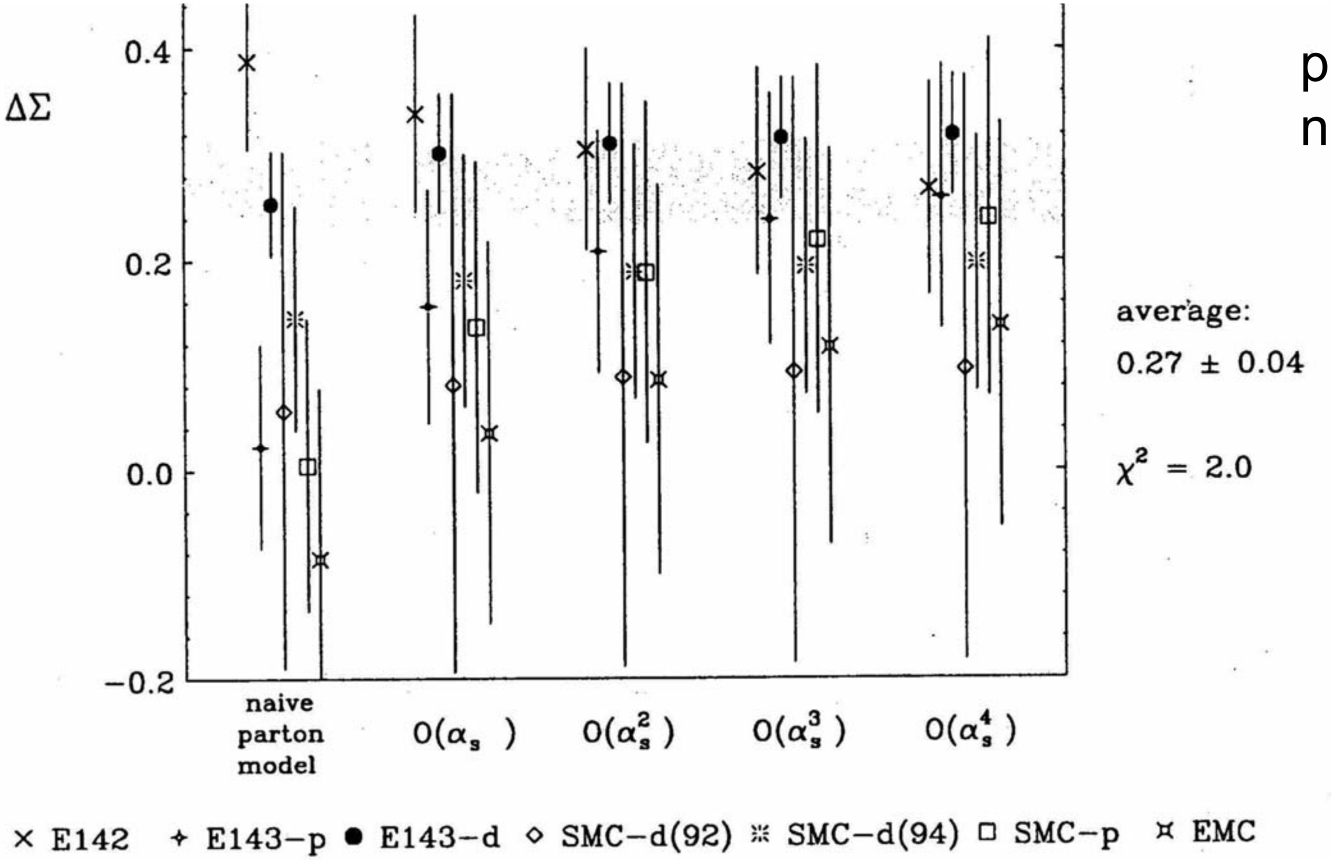


## F,D, $\Gamma_1^p$ ($Q^2$=10.7 GeV$^2$) → $\Delta\Sigma$ ($Q^2$=10.7 GeV$^2$) → $\Delta u$, $\Delta d$, $\Delta s$

$Q^2$ = 10.7 GeV$^2$ $\boxed{\Delta\Sigma = 0.13 \pm 0.19}$

$\Delta u = 0.78 \pm 0.10$ ; $\Delta d = 0.50 \pm 0.10$ ; $\Delta s = -0.20 \pm 0.11$

polarizzazione negativa del mare



all $\Delta\Sigma$ values evolved to $Q^2$=3 GeV$^2$

$Q^2$ = 3 GeV$^2$

$\boxed{\Delta\Sigma = 0.27 \pm 0.04}$

spin crisis

# Altre regole di somma

partoni on-shell con $\mathbf{p}_T = 0 \rightarrow g_2(x_B) = 0$

“ “ + massless → Wandzura-Wilzcek (WW)

$$g_1(x_B) + g_2(x_B) = \int_{x_B}^{1} \frac{dy}{y}\, g_1(y)$$

classe di regole di somma

$$\int_0^1 dx\, x^{J-1} \left[ \frac{J-1}{J}\, g_1(x) + g_2(x) \right] = 0$$

J=1 → regola di somma di Burkhardt-Cottingham $\int_0^1 dx\, g_2(x) = 0$

……..

# regola di somma di Bjorken polarizzata

$$\int_0^1 dx\,[g_1^p(x) - g_1^n(x)] = \frac{1}{6}\,\frac{G_A}{G_V}$$

← assiale
← vettoriale

accoppiamenti deboli
in decadimento β del N

in QPM funz. d'onda del q in P$^\uparrow$ secondo SU$_f$(3) ⊗ SU(2) = SU(6)

$$|P^\uparrow\rangle = \frac{1}{\sqrt{18}}\left(2u^\uparrow u^\uparrow d^\downarrow - u^\uparrow u^\downarrow d^\uparrow - u^\downarrow u^\uparrow d^\uparrow + \text{permutazioni di d ..}\right)$$

probabilita` di : $u^\uparrow = \frac{5}{3}\,;\; u^\downarrow = \frac{1}{3}\,;\; d^\uparrow = \frac{1}{3}\,;\; d^\downarrow = \frac{2}{3}$

$$A_1^p = \frac{\sum_f e_f^2\,(q_f^\uparrow - q_f^\downarrow)}{\sum_f e_f^2\,(q_f^\uparrow - q_f^\downarrow)} = \frac{\frac{4}{9}\left(\frac{5}{3}-\frac{1}{3}\right)+\frac{1}{9}\left(\frac{1}{3}-\frac{2}{3}\right)}{\frac{4}{9}\left(\frac{5}{3}+\frac{1}{3}\right)+\frac{1}{9}\left(\frac{1}{3}+\frac{2}{3}\right)} = \frac{5}{9}$$

$$A_1^n = \text{stesso con } u \leftrightarrow d = \frac{\frac{1}{9}\left(\frac{5}{3}-\frac{1}{3}\right)+\frac{4}{9}\left(\frac{1}{3}-\frac{2}{3}\right)}{\frac{1}{9}\left(\frac{5}{3}+\frac{1}{3}\right)+\frac{4}{9}\left(\frac{1}{3}+\frac{2}{3}\right)} = 0$$

$$\int_0^1 dx\,(g_1^p - g_1^n) = \int_0^1 dx\,(A_1^p\,F_1^p - A_1^n\,F_1^n) = \frac{5}{9}\,\frac{1}{2}\sum_f e_f^2 \int_0^1 dx\,\phi_f^p(x) = \frac{5}{18}$$

$$\Rightarrow \quad \boxed{\frac{G_A}{G_V} \overset{\text{QPM}}{=} \frac{5}{3} \leftrightarrow 1.257 \pm 0.003}$$

riduzione dovuta a $\mathbf{L}_q$ o a
dipendenza da $\mathbf{p}_T$ esplicita