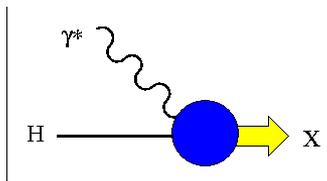


Schema

- riassunto precedente lezione
- teorema di fattorizzazione per ampiezza Compton non forward
 - operatore non locale non diagonale su stati adronici; analisi in twist
 - definizione di **Generalized Parton Distribution** (GPD)
- GPD “interpolano” quantità pertinenti a processi esclusivi (fattori di forma) e semi-inclusivi (PDF)
- operatore non locale non diagonale → GPD ampiezze di probabilità
 - interpretazione probabilistica al twist 2 solo in termini di Light-Cone w.f.
- GPD e localizzazione tridimensionale dei partoni; il momento angolare orbitale dei quark e la “spin crisis”
- possibili misure sperimentali delle GPD

- situazione sperimentale delle SSA per evidenziare l'effetto Collins ed estrarre la trasversita' h_1 ; bersaglio con polarizzazione trasversa efficace
→ contaminazione con contributi di polarizzazione longitudinale a twist 3
- invarianza di gauge del correlatore quark-quark e l'operatore di "gauge link"
→ strutture "time-reversal odd" anche per le PDF
→ nuovi contributi alle SSA: l'effetto Sivers e la funzione di Boer
→ difficoltà di identificazione dell'effetto Collins puro
- info sui diversi contributi da analisi incrociata di SIDIS, e^+e^- e DY (non polarizzato basandosi su universalita' al leading twist (non c'è teorema di fattorizzazione) → funz. di Boer (h_1^\perp) possibile responsabile di asimmetria in DY non polarizzato, che nessun meccanismo QCD spiega
- bersaglio con pura polarizzazione trasversa (analisi in corso)
→ effetto Collins separabile dal resto → richiede dipendenza esplicita da \mathbf{k}_T
→ rottura della fattorizzazione collineare → fattori di forma di Sudakov
- meccanismo per generare partner chiral-odd di h_1 senza dipendenza da \mathbf{k}_T
→ 2-hadron SIDIS e IFF → H_1^\lessgtr , T-odd e chiral-odd a twist 2, genera SSA
→ estrazione di h_1 senza complicazioni dell'effetto Collins; H_1^\lessgtr estraibile da $e^+e^- \rightarrow (\pi\pi)(\pi\pi)X$, dove c'è anche asimmetria per polarizz. longitudinale
→ violazione nonperturbativa di simmetria CP tra i due jets

DIS inclusivo

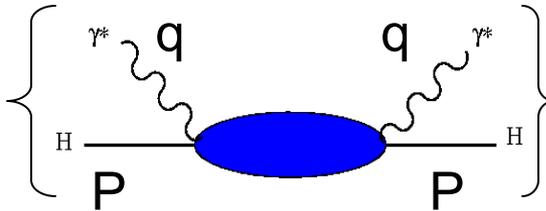


teorema

ottico

$$\sim \text{Im}$$

forward Compton

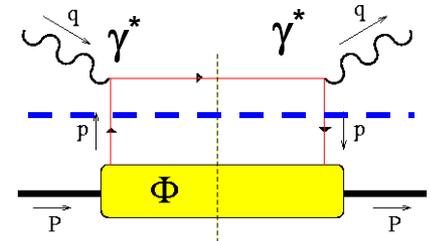


$Q^2 \rightarrow \infty$

x_B fisso

\sim

dominante



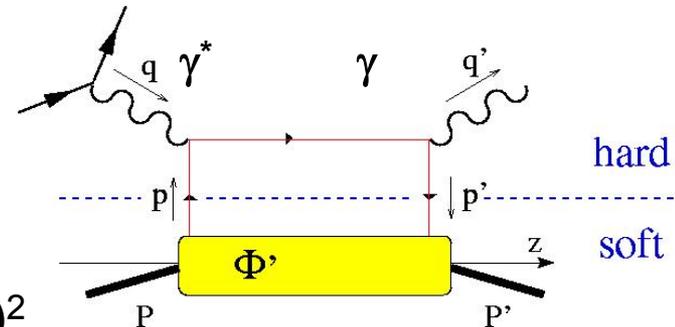
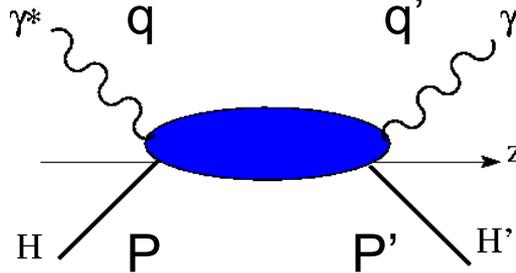
nonforward Compton

$Q^2 \rightarrow \infty$

x_B fisso

\sim

(\rightarrow un partone attivo)



$$t = (P-P')^2$$

piccolo

(gli altri sono spettatori)

Deeply Virtual
Compton Scatt.
(DVCS)

$$\Phi(p, P, S) = \int \frac{d^4 z}{(2\pi)^4} e^{ip \cdot z} \langle PS | \bar{\psi}(0) \psi(z) | PS \rangle$$



$$\Phi'(p, P, P', S', S) = \int \frac{d^4 z}{(2\pi)^4} e^{i\bar{p} \cdot z} \langle P' S' | \bar{\psi} \left(-\frac{z}{2} \right) \psi \left(\frac{z}{2} \right) | PS \rangle$$

$$\bar{p} = \frac{p + p'}{2}$$

(Ji, P.R. D55 ('97) 7114)

teorema fattorizzazione
a twist 2 $\forall \alpha_S$

(Collins e Freund,
P.R.D59 ('99) 074009)

Cinematica del DVCS

$$\bar{P} = \frac{P + P'}{2} = (\bar{P}^+, \bar{P}^-, \mathbf{0}_\perp) \rightarrow \mathbf{P} \parallel \hat{z}$$

skewness $\xi = \frac{(P - P')^+}{(P + P')^+}$

$$\Delta = P' - P = \left(-2\xi\bar{P}^+, \frac{\xi(M^2 + \Delta_\perp^2/4)}{(1 - \xi^2)\bar{P}^+}, \Delta_\perp \right)$$

$$t = \Delta^2 = -\frac{4\xi^2 M^2 + \Delta_\perp^2}{1 - \xi^2}$$



dato $t \rightarrow$ massimo $\xi_t \geq -\frac{4\xi_t^2 M^2}{1 - \xi_t^2}$
possibile

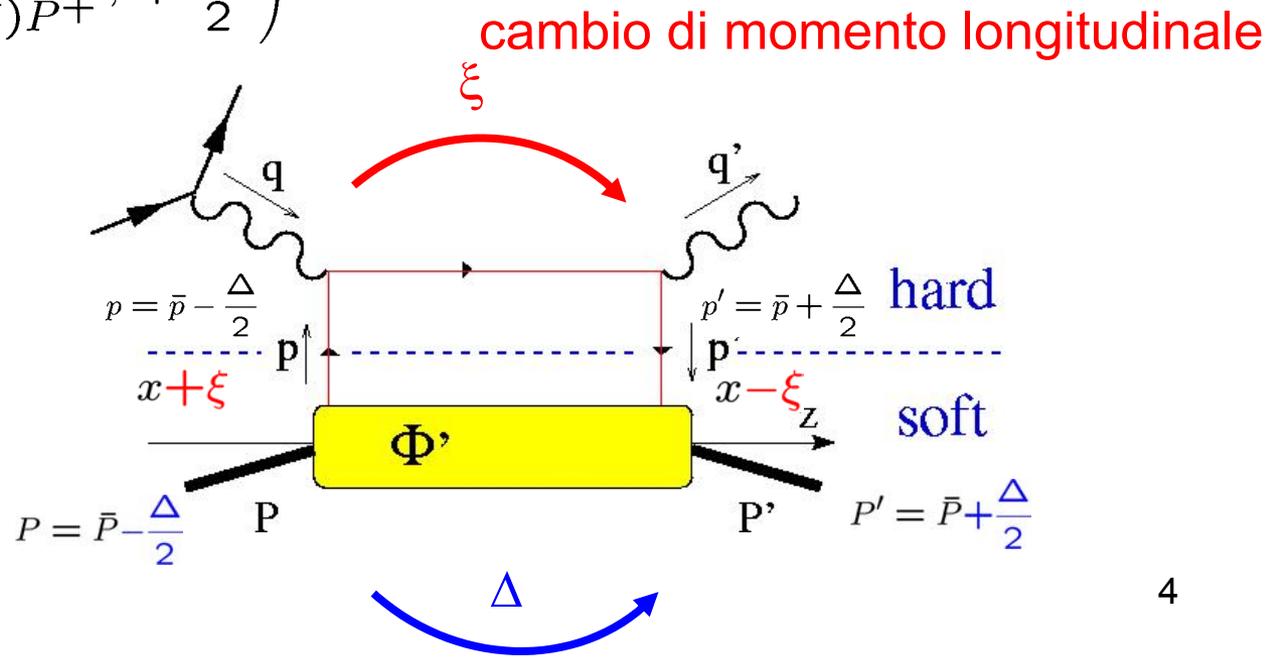
$$P = \left((1 + \xi)\bar{P}^+, \frac{M^2 + \Delta_\perp^2/4}{2(1 + \xi)\bar{P}^+}, -\frac{\Delta_\perp}{2} \right)$$

$$P' = \left((1 - \xi)\bar{P}^+, \frac{M^2 + \Delta_\perp^2/4}{2(1 - \xi)\bar{P}^+}, +\frac{\Delta_\perp}{2} \right)$$

$$\bar{p} = \frac{p + p'}{2} \quad x = \frac{\bar{p}^+}{\bar{P}^+}$$

$$\frac{p^+}{\bar{P}^+} = x + \xi$$

$$\frac{p'^+}{\bar{P}^+} = x - \xi$$



Analisi in twist del correlatore non diagonale

estrazione del contributo al leading twist procede come per PDF:

- decomposizione di Φ' in termini di strutture di Dirac e 4-vettori (P,P',S,S'..)
- proiezione delle componenti LC "good" con operatori γ^+ , $\gamma^+\gamma_5$, $i\sigma^{i+}\gamma_5$ integrando su direzione "-" soppressa

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} \langle P'S' | \bar{\psi} \left(-\frac{z^-}{2} \right) \gamma^+ \psi \left(\frac{z^-}{2} \right) | PS \rangle \Big|_{z^+=z_\perp=0}$$

6

$$= \frac{1}{\bar{P}^+} \bar{u}(P'S') \left\{ \gamma^+ H(x, \xi, t) + \frac{i\sigma^{+\nu} \Delta_\nu}{2M} E(x, \xi, t) \right\} u(PS)$$

10

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} \langle P'S' | \bar{\psi} \left(-\frac{z^-}{2} \right) \gamma^+ \gamma_5 \psi \left(\frac{z^-}{2} \right) | PS \rangle \Big|_{z^+=z_\perp=0}$$

$$= \frac{1}{\bar{P}^+} \bar{u}(P'S') \left\{ \gamma^+ \gamma_5 \tilde{H}(x, \xi, t) + \frac{\gamma_5 \Delta^+}{2M} \tilde{E}(x, \xi, t) \right\} u(PS)$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} \langle P'S' | \bar{\psi} \left(-\frac{z^-}{2} \right) i\sigma^{i+} \gamma_5 \psi \left(\frac{z^-}{2} \right) | PS \rangle \Big|_{z^+=z_\perp=0}$$

$$= \frac{i}{\bar{P}^+} \bar{u}(P'S') \left\{ \sigma^{i+} \gamma_5 H_T(x, \xi, t) + \frac{\epsilon^{i+\alpha\beta} \Delta_\alpha \bar{P}_\beta}{M^2} \tilde{H}_T(x, \xi, t) \right.$$

(Diehl, E.P.J. C19
(01) 485)

$$+ \frac{\epsilon^{i+\alpha\beta} \Delta_\alpha \gamma_\beta}{2M} E_T(x, \xi, t) + \frac{\epsilon^{i+\alpha\beta} \bar{P}_\alpha \gamma_\beta}{M} \tilde{E}_T(x, \xi, t) \left. \right\} u(PS) \quad 5$$

Osservazioni :

- $S' \neq S$: correlatore non diagonale nello spin \rightarrow helicity flip del N possibile
 $\lambda_N \rightarrow x^2 \rightarrow$ n. raddoppia (quadruplica) per GPD che conservano (cambiano)
 $\lambda_N \otimes \lambda_q \rightarrow x^4$ l'elicità del quark, rispetto al n. di PDF corrispondenti

- chiral-even E, \tilde{E} collegate a cambio di elicità nel N ma non nel quark
 \rightarrow possibile solo per effetto del momento angolare orbitale dei quark
 $H_T, \hat{H}_T, E_T, \tilde{E}_T$ sono chiral-odd

- correlatore non diagonale \rightarrow le GPD non sono densità di probabilità, ma ampiezze di interferenza tra due stati differenti \rightarrow non c'è immediata interpretazione probabilistica al leading twist
 (ma è possibile se GPD \equiv overlap di Light-Cone w.f. \rightarrow slide 19)

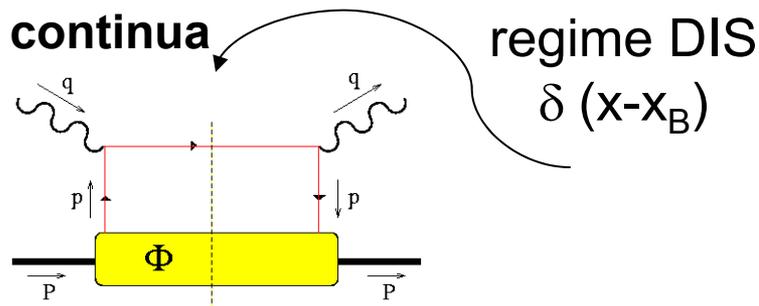
- $P' \neq P$: Δ ($e \Delta_{\perp}$) $\neq 0$ quindi accesso alla dislocazione dei quark nel parametro di impatto \mathbf{b}_{\perp} sul piano \perp al momento longitudinale x

$$\Phi' = \int dz dz' e^{ip \cdot z - ip' \cdot z'} \dots \bar{\psi}(z') \dots \psi(z) \dots \rightarrow e^{i\bar{p} \cdot (z - z') + i\Delta \cdot \frac{z + z'}{2}} \dots$$

PDF, $\Delta = 0$ per invarianza traslazionale \nearrow \longleftarrow

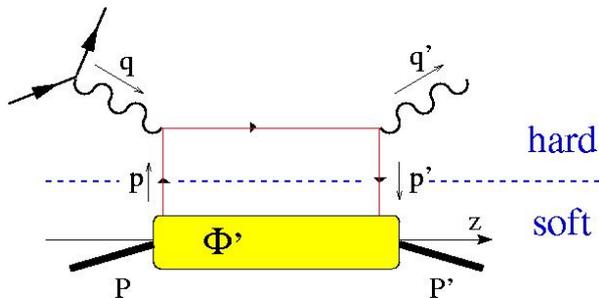
$\Delta \neq 0 \rightarrow \mathbf{b} = (z + z')/2$
 $\Delta_{\perp} \rightarrow \mathbf{b}_{\perp}$
 localizzazione 2d

(L.C.w.f. \rightarrow slide 22)



regime DVCS

$$\begin{aligned}
 q^+ &= P'^+ - P^+ \\
 &= \Delta^+ \equiv -2\xi \bar{P}^+ \\
 -2\xi &= \frac{q^+}{\bar{P}^+} \rightarrow \frac{q^+}{P^+} \\
 &= \frac{2q^+ q^-}{2P^+ q^-} \sim -\frac{Q^2}{2P \cdot q} = -x_B
 \end{aligned}$$



- interpretazione intuitiva sul contenuto partonico delle GPD al leading twist (versione rigorosa con L.C.w.f. → slide 17)

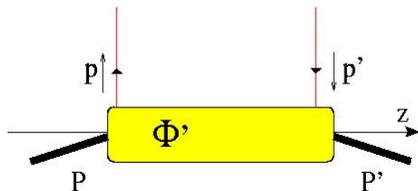


$1 \geq \xi \geq 0$

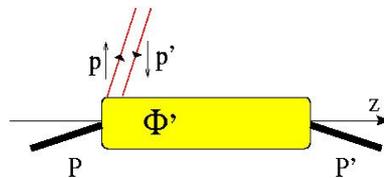
$$\begin{aligned}
 \frac{p^+}{\bar{P}^+} &= x + \xi < 0 \\
 \frac{p'^+}{\bar{P}^+} &= x - \xi < 0
 \end{aligned}$$

$$\begin{aligned}
 \frac{p^+}{\bar{P}^+} &= x + \xi > 0 \\
 \frac{p'^+}{\bar{P}^+} &= x - \xi < 0
 \end{aligned}$$

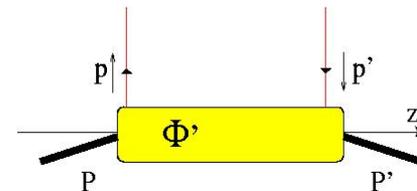
$$\begin{aligned}
 \frac{p^+}{\bar{P}^+} &= x + \xi > 0 \\
 \frac{p'^+}{\bar{P}^+} &= x - \xi > 0
 \end{aligned}$$



GPD di antiquark



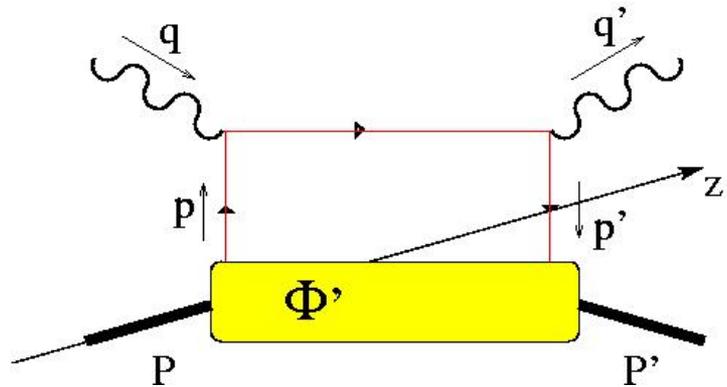
GPD $q\bar{q}$



GPD di quark

continua

- altra possibile parametrizzazione delle GPD (Radyushkin, P.R. D56 ('97) 5524)



$$\begin{array}{ccccccc} -1+\zeta & & 0 & & \zeta & & 1 \\ \hline & & \text{red} & & \text{blue} & & \text{green} \end{array} \xrightarrow{x}$$

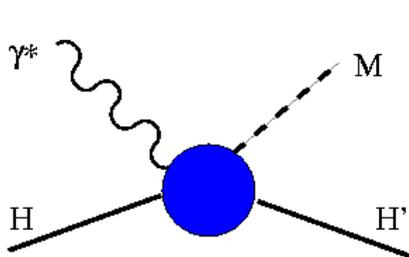
$$X = \frac{p^+}{P^+} \quad \zeta = -\frac{\Delta^+}{P^+}$$

GPD (X, ζ, t)

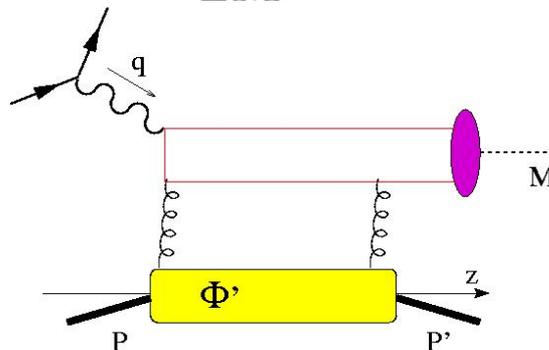
- definizione di GPD per gluone ; proiezioni al leading twist

$$\gamma^+ \rightarrow \gamma^+ H_g(x, \xi, t) + \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} E_g(x, \xi, t)$$

$$\gamma^+ \gamma_5 \rightarrow \gamma^+ \gamma_5 \tilde{H}_g(x, \xi, t) + \frac{\gamma_5 \Delta^+}{2M} \tilde{E}_g(x, \xi, t)$$



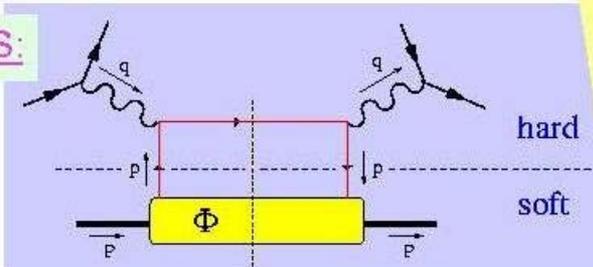
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**Deeply Virtual
Meson Production
(DVMP)**

comportamento "ibrido" delle GPD

DIS:



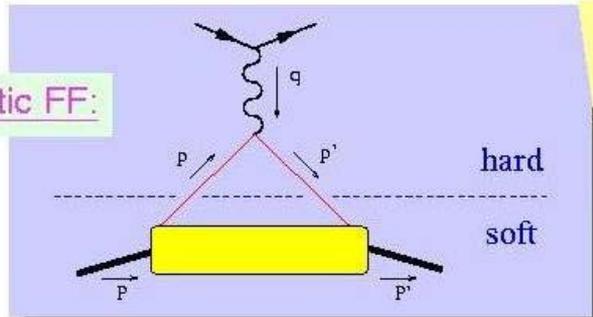
bilocale diagonale

$$P^+ \int \frac{dz^-}{2\pi} e^{ip^+z^-} \langle P, S | \bar{\psi}(\frac{-z^-}{2}) \gamma^+ \psi(\frac{z^-}{2}) | P, S \rangle$$

$$= \bar{u}(P, S) \gamma^+ u(P, S) q(x)$$

locale non diagonale

elastic FF:

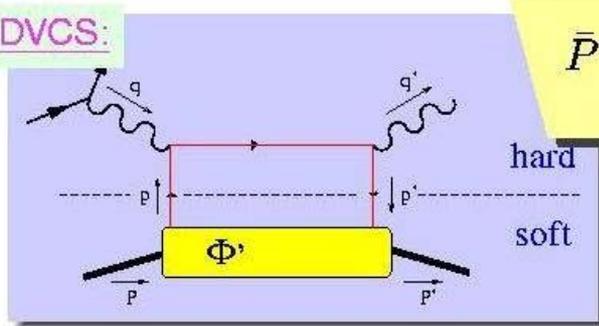


$$\langle P', S' | \bar{\psi}(0) \gamma^+ \psi(0) | P, S \rangle$$

$$= \bar{u}(P', S') \left\{ \gamma^+ F_1(t) + \frac{i\sigma^{+\nu} \Delta_\nu}{2M} F_2(t) \right\} u(P, S)$$

bilocale non diagonale

DVCS:



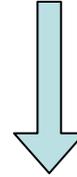
$$\bar{P}^+ \int \frac{dz^-}{2\pi} e^{ip^+z^-} \langle P', S' | \bar{\psi}(\frac{-z^-}{2}) \gamma^+ \psi(\frac{z^-}{2}) | P, S \rangle$$

$$= \bar{u}(P', S') \left\{ \gamma^+ H(x, \xi, t) + \frac{i\sigma^{+\nu} \Delta_\nu}{2M} E(x, \xi, t) \right\} u(P, S)$$

Condizioni al contorno per le GPD :

$$\Phi'(p, P, P', S', S) = \int \frac{d^4 z}{(2\pi)^4} e^{i\bar{p}\cdot z} \langle P' S' | \bar{\psi} \left(-\frac{z}{2}\right) \psi \left(\frac{z}{2}\right) | P S \rangle \quad \text{GPD}$$

$$\left. \begin{array}{l} \xi \rightarrow 0 \\ t \rightarrow 0 \end{array} \right\} \Rightarrow \begin{array}{l} \Delta \rightarrow 0 \\ P'(p') \rightarrow P(p) \end{array}$$



limite forward

$$\Phi(p, P, S) = \int \frac{d^4 z}{(2\pi)^4} e^{ip\cdot z} \langle PS | \bar{\psi}(0) \psi(z) | PS \rangle \quad \text{PDF}$$

tutte le strutture legate a helicity flip del $N \rightarrow 0$; inoltre

5

$$H(x, 0, 0) = f_1(x)$$

$$\tilde{H}(x, 0, 0) = g_1(x)$$

$$H_T(x, 0, 0) = h_1(x)$$

$$\begin{array}{l} \text{gluone} \quad H_g(x, 0, 0) = xG(x) \\ \quad \quad \quad \tilde{H}_g(x, 0, 0) = x\Delta G(x) \end{array}$$

continua

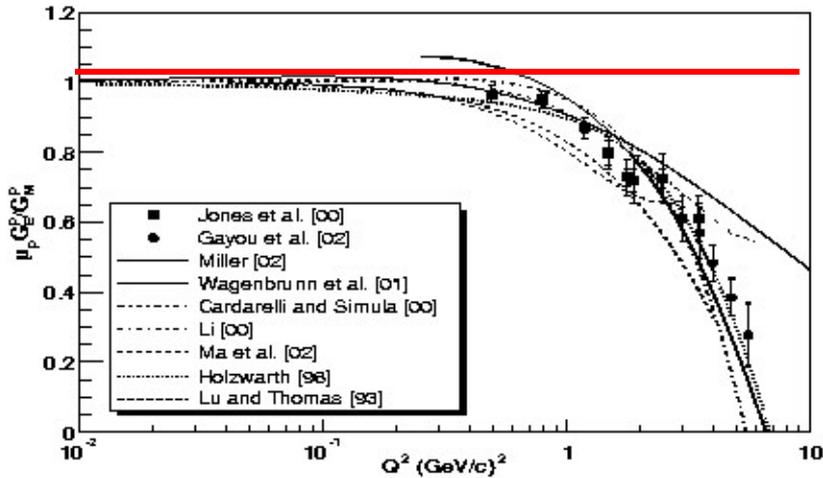
1^o momento di Mellin

$$\begin{aligned} & \int_{-1}^1 dx \quad \bar{P}^+ \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ \cdot z^-} \langle P'S' | \bar{\psi} \left(-\frac{z}{2} \right) \gamma^+ \psi \left(\frac{z}{2} \right) | PS \rangle \Big|_{z^+ = z_\perp = 0} \\ &= \bar{u}(P'S') \left\{ \gamma^+ \int_{-1}^1 dx H(x, \xi, t) + \frac{i\sigma^{+\nu} \Delta_\nu}{2M} \int_{-1}^1 dx E(x, \xi, t) \right\} u(PS) \\ &= \int \frac{dz^-}{2\pi} \bar{P}^+ \delta(\bar{P}^+ z^-) \langle P'S' | \bar{\psi} \left(-\frac{z}{2} \right) \gamma^+ \psi \left(\frac{z}{2} \right) | PS \rangle \Big|_{z^+ = z_\perp = 0} \\ &= \langle P'S' | \bar{\psi}(0) \gamma^+ \psi(0) | PS \rangle \\ &= \bar{u}(P'S') \left\{ \gamma^+ F_1(t) + \frac{i\sigma^{+\nu} \Delta_\nu}{2M} F_2(t) \right\} u(PS) \end{aligned}$$

corrente convettiva
→ fattore di forma
di carica = $M^1 [H(x, \xi, t)]$

corrente magnetica
→ fattore di forma
magnetico (spin flip)
= $M^1 [E(x, \xi, t)]$

continua chiral-even GPD per quark non polarizzato in N : $H(x,\xi,t)$ $E(x,\xi,t)$
 → studio dell'andamento asintotico dei fattori di forma e.m. del N



1

$$\mu_p G_E^p / G_M^p$$

$$Q^2$$

fattori di forma di Sachs

$$G_E(Q^2) = F_1(Q^2) - \frac{\kappa Q^2}{4M^2} F_2(Q^2) \quad G_E(0) = e_N$$

$$G_M(Q^2) = F_1(Q^2) + \kappa F_2(Q^2) \quad G_M(0) = \mu_N$$

relazione di Drell-Yan-West (pQCD):

$$F_1(Q^2) \sim \frac{1}{(Q^2)^{n-1}} \quad n = \text{n. di quark di valenza nel vertice hard}$$

$$\text{scattering elastico } e^-p \quad F_1(Q^2) \sim \frac{1}{(Q^2)^2} \quad \text{inoltre } F_2(Q^2) \sim \frac{F_1(Q^2)}{Q^2}$$

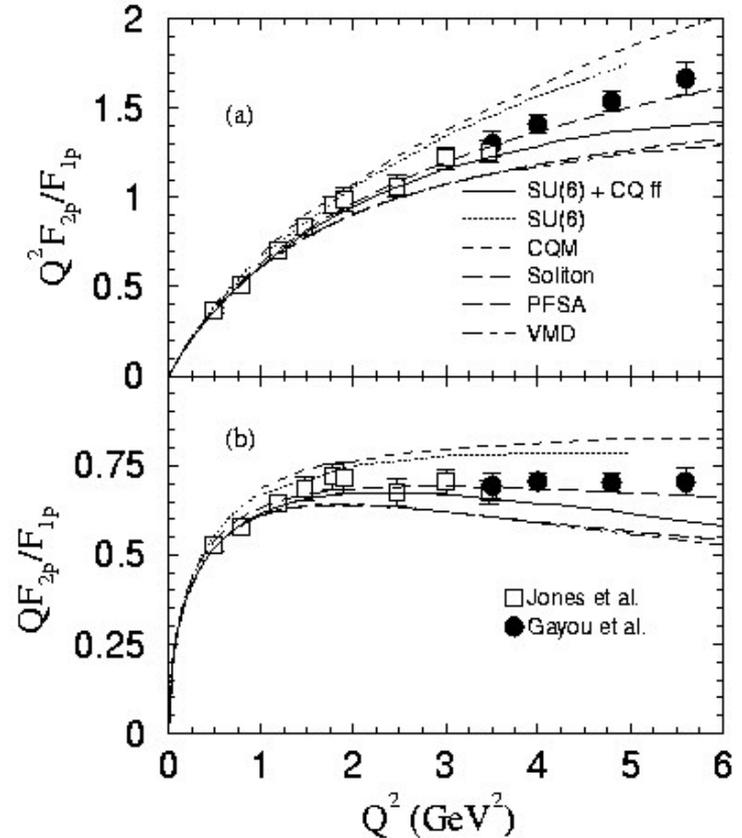
7 mag 04



$$G_E(Q^2) \sim \frac{G_M(Q^2)}{\mu_N} !$$

contraddice i dati !

12



(Gao, Int.J.Mod.Phys. **E12** ('03) 1)

continua

$$\int_{-1}^1 dx \quad \bar{P}^+ \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ \cdot z^-} \langle P'S' | \bar{\psi} \left(-\frac{z}{2} \right) \gamma^+ \gamma_5 \psi \left(\frac{z}{2} \right) | PS \rangle \Big|_{z^+ = z_\perp = 0}$$

$$= \bar{u}(P'S') \left\{ \gamma^+ \gamma_5 \int_{-1}^1 dx \tilde{H}(x, \xi, t) + \frac{\gamma_5 \Delta^+}{2M} \int_{-1}^1 dx \tilde{E}(x, \xi, t) \right\} u(PS)$$

$$= \bar{u}(P'S') \left\{ \gamma^+ \gamma_5 G_A(t) + \frac{\gamma_5 \Delta^+}{2M} G_P(t) \right\} u(PS)$$

corrente assiale
 → fattore di forma assiale
 = $M^1[\tilde{H}(x, \xi, t)]$

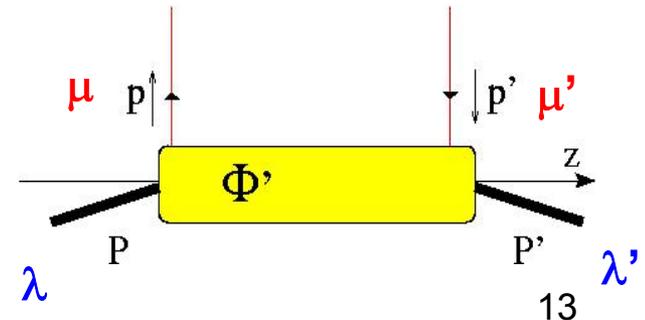
corrente pseudoscalare
 → fattore di forma pseudoscalare
 = $M^1[\tilde{E}(x, \xi, t)]$

chiral-odd GPD

$$\{H_T, \tilde{H}_T, E_T, \tilde{E}_T\} \longrightarrow \{A_{++,+-}, A_{--,+-}, A_{+-,+-}, A_{-+,+-}\}$$

helicity flip di q e N → fattori di forma ?

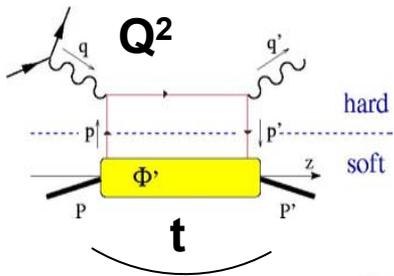
N.B. T-reversal → $\int_{-1}^1 dx \tilde{E}_T(x, \xi, t) = 0$



Mappa delle "interconnessioni" GPD in regime DVCS

small t , large Q^2

$$M^1[\text{GPD}(x, \xi, t)] = \text{FF}(t)$$



deeply virtual Compton scattering

wide angle Compton scattering

form factors

timelike Compton scattering

GPDs

orbital angular momentum
transverse localisation

$p\bar{p}$ annihilation

 $\gamma\gamma \rightarrow \pi\pi, \gamma\gamma \rightarrow KK$

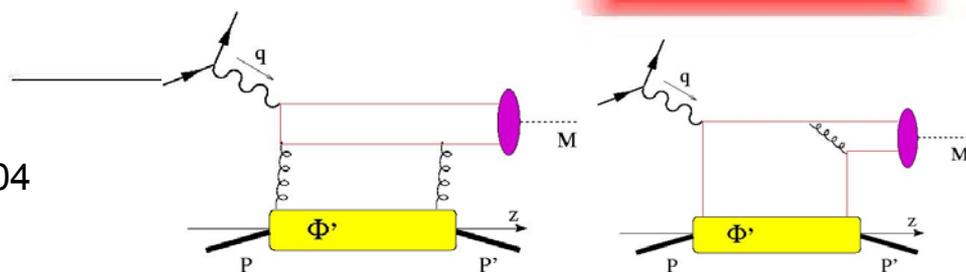
exclusive meson production
deep virtual/large t

deep inelastic scattering
PDFs

trasf. Fourier in \mathbf{b}_\perp

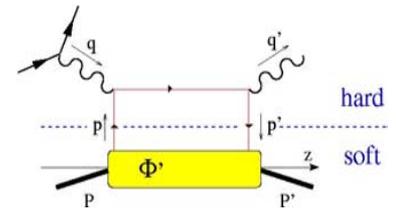
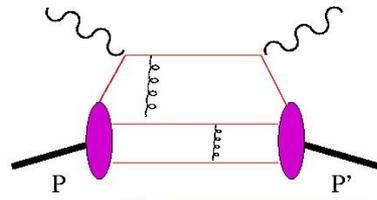
$$M^2[\text{GPD}(x, \xi, t)]$$

$$\lim_{\xi, t \rightarrow 0} \text{GPD}(x, \xi, t) = \text{PDF}(x)$$



regime **Wide Angle Compton Scattering (WACS)**

large t , small Q^2



$R_V(t), R_A(t), R_P(t) \dots$
 $M^{-1}[GPD(x,0,t)]$

deeply virtual Compton scattering

wide angle Compton scattering

form factors

timelike Compton scattering

GPDs

orbital angular momentum
transverse localisation

$p\bar{p}$ annihilation
 $\gamma\gamma \rightarrow \pi\pi, \gamma\gamma \rightarrow KK$

exclusive meson production
deep virtual/large t

deep inelastic scattering
PDFs

crossing con RCS

Rappresentazione di GPD come sovrapposizione di Light-Cone wave functions (LCwf)

richiamo di quantizzazione sul L.C. :

- componente L.C. “good” del campo

$$\phi(z^-, \mathbf{z}_\perp) = P_+ \psi = \int \frac{dk^+ d\mathbf{k}_\perp}{2k^+ (2\pi)^3} \theta(k^+) \sum_\lambda \left\{ b_{\{\alpha\}}(k) P_+ u_{\{\alpha\}}(k) e^{-ik^+ z^- + i\mathbf{k}_\perp \cdot \mathbf{z}_\perp} \right. \\ \left. + d_{\{\alpha\}}^\dagger(k) P_+ v_{\{\alpha\}}(k) e^{ik^+ z^- - i\mathbf{k}_\perp \cdot \mathbf{z}_\perp} \right\}$$

$$\{b_{\{\alpha\}}(k), b_{\{\alpha'\}}^\dagger(k')\} = \{d_{\{\alpha\}}(k), d_{\{\alpha'\}}^\dagger(k')\} = 2k^+ (2\pi)^3 \delta(k'^+ - k^+) \delta(\mathbf{k}'_\perp - \mathbf{k}_\perp) \delta_{\{\alpha\}, \{\alpha'\}}$$

$b^\dagger|0\rangle$ creazione di quark

$d^\dagger|0\rangle$ creazione di antiquark

$a^\dagger|0\rangle$ creazione di gluone

- stato adronico con momento P ,
elicita' λ , n. di partoni M

$$|P, \lambda\rangle = \sum_{M, \{\alpha\}} \int \prod_i^M dx_i \delta\left(1 - \sum_i x_i\right) \int \prod_i^M d\mathbf{p}_{\perp i} \delta\left(\sum_i \mathbf{p}_{\perp i} - \mathbf{P}_\perp\right) \\ \times \Psi_{M, \{\alpha\}}^\lambda(x_i, \mathbf{p}_{\perp i}) \quad |M, \{\alpha\}; p_1 \dots p_M\rangle$$

$$7 \text{ mag } 04 \quad |M, \{\alpha\}; p_1 \dots p_M\rangle \propto \prod_i \frac{b_{\{\alpha_i\}}^\dagger(p_i)}{\sqrt{x_i}} \prod_j \frac{d_{\{\alpha_j\}}^\dagger(p_j)}{\sqrt{x_j}} \prod_l \frac{a_{\{\alpha_l\}}^\dagger(p_l)}{\sqrt{x_l}} |0\rangle$$

- proiezione con γ^+ di operatore bilocale \rightarrow densita` di componenti L.C. "good"

$$\bar{\psi}(z_1) \gamma^+ \psi(z_2) \propto \phi^\dagger(z_1) \phi(z_2)$$

$$= \int \frac{dp'^+ d\mathbf{p}'_\perp}{2p'^+(2\pi)^3} \theta(p'^+) \int \frac{dp^+ d\mathbf{p}_\perp}{2p^+(2\pi)^3} \theta(p^+) \sum_{\{\alpha\}, \{\alpha'\}}$$

$$\left\{ e^{ip'^+ z_1^- - i\mathbf{p}'_\perp \cdot \mathbf{z}_{1\perp} - ip'^+ z_2^- + i\mathbf{p}'_\perp \cdot \mathbf{z}_{2\perp}} \right.$$

$$b^\dagger_{\{\alpha'\}}(p') b_{\{\alpha\}}(p) (P_+ u_{\{\alpha'\}}(p'))^\dagger P_+ u_{\{\alpha\}}(p) \quad \# \text{ quarks}$$

$$+ e^{-ip'^+ z_1^- + i\mathbf{p}'_\perp \cdot \mathbf{z}_{1\perp} + ip'^+ z_2^- - i\mathbf{p}'_\perp \cdot \mathbf{z}_{2\perp}}$$

$$d_{\{\alpha'\}}(p') d^\dagger_{\{\alpha\}}(p) (P_+ v_{\{\alpha'\}}(p'))^\dagger P_+ v_{\{\alpha\}}(p) \quad \# \text{ antiquarks}$$

$$+ e^{-ip'^+ z_1^- + i\mathbf{p}'_\perp \cdot \mathbf{z}_{1\perp} - ip'^+ z_2^- + i\mathbf{p}'_\perp \cdot \mathbf{z}_{2\perp}}$$

$$d_{\{\alpha'\}}(p') b_{\{\alpha\}}(p) (P_+ v_{\{\alpha'\}}(p'))^\dagger P_+ u_{\{\alpha\}}(p) \quad \text{crea } q\bar{q}$$

$$+ e^{ip'^+ z_1^- - i\mathbf{p}'_\perp \cdot \mathbf{z}_{1\perp} + ip'^+ z_2^- - i\mathbf{p}'_\perp \cdot \mathbf{z}_{2\perp}}$$

$$b^\dagger_{\{\alpha'\}}(p') d^\dagger_{\{\alpha\}}(p) (P_+ u_{\{\alpha'\}}(p'))^\dagger P_+ v_{\{\alpha\}}(p) \quad \left. \begin{array}{l} \text{assorbe } q\bar{q} \\ \end{array} \right\}$$

continua

$$p^{(\prime)+} \geq 0 \quad \frac{p^{(\prime)+}}{\bar{P}^+} = x \pm \xi$$

$$\xi > 0$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \bar{\psi}(-z/2) \gamma^+ \psi(z/2)$$

$$\propto \int \frac{dp'^+ d\mathbf{p}'_{\perp}}{2p'^+(2\pi)^3} \theta(p'^+) \int \frac{dp^+ d\mathbf{p}_{\perp}}{2p^+(2\pi)^3} \theta(p^+) \sum_{\{\alpha\}, \{\alpha'\}}$$

$$\left\{ \delta(2x\bar{P}^+ - p'^+ - p^+) \quad x - \xi \quad x + \xi \quad \xi \leq x \leq 1 \right.$$

$$b_{\{\alpha'\}}^{\dagger}(p') b_{\{\alpha\}}(p) \quad (P_+ u_{\{\alpha'\}}(p'))^{\dagger} P_+ u_{\{\alpha\}}(p)$$

$$+ \delta(2x\bar{P}^+ + p'^+ + p^+) \quad -(x - \xi) \quad -(x + \xi) \quad -1 \leq x \leq -\xi$$

$$d_{\{\alpha'\}}(p') d_{\{\alpha\}}^{\dagger}(p) \quad (P_+ v_{\{\alpha'\}}(p'))^{\dagger} P_+ v_{\{\alpha\}}(p)$$

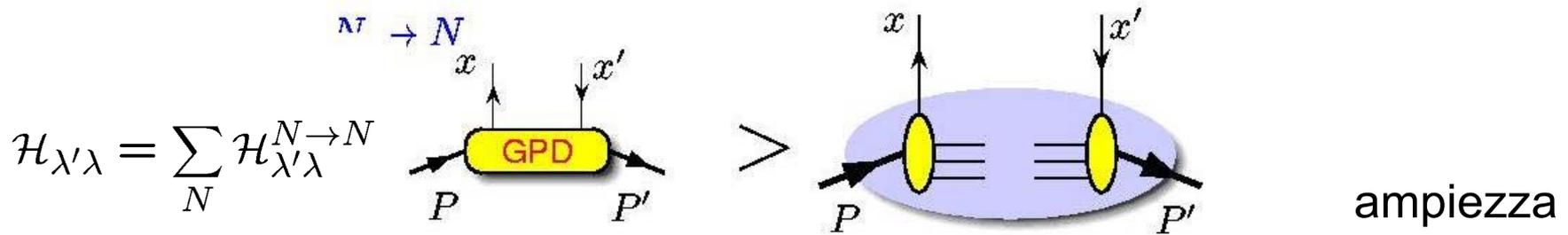
$$+ \delta(2x\bar{P}^+ + p'^+ - p^+) \quad -(x - \xi) \quad (x + \xi) \quad -\xi \leq x \leq \xi$$

$$d_{\{\alpha'\}}(p') b_{\{\alpha\}}(p) \quad (P_+ v_{\{\alpha'\}}(p'))^{\dagger} P_+ u_{\{\alpha\}}(p)$$

$$+ \delta(2x\bar{P}^+ - p'^+ + p^+)$$

$$b_{\{\alpha'\}}^{\dagger}(p') d_{\{\alpha\}}^{\dagger}(p) \quad (P_+ u_{\{\alpha'\}}(p'))^{\dagger} P_+ v_{\{\alpha\}}(p) \left. \right\}$$

decomposizione di GPD in LCwf in zona DGLAP (quark o antiquark)



$$\mathcal{H}_{\lambda'\lambda}^{(N \rightarrow N)}(x, \xi, t) = \sqrt{1-\xi}^{1-N} \sqrt{1+\xi}^{1-N} \int \dots \delta(x-x_j) \Psi_N^{*,\lambda'}(x^{out}, \vec{k}_{\perp}^{out}) \Psi_N^{\lambda}(x^{in}, \vec{k}_{\perp}^{in})$$

$$x_i^{in} = \frac{\bar{x}_i}{1+\xi} \quad \vec{k}_{\perp i}^{in} = \vec{k}_{\perp i} + \frac{\bar{x}_i}{1+\xi} \frac{\vec{\Delta}_{\perp}}{2} \quad \text{spectator}$$

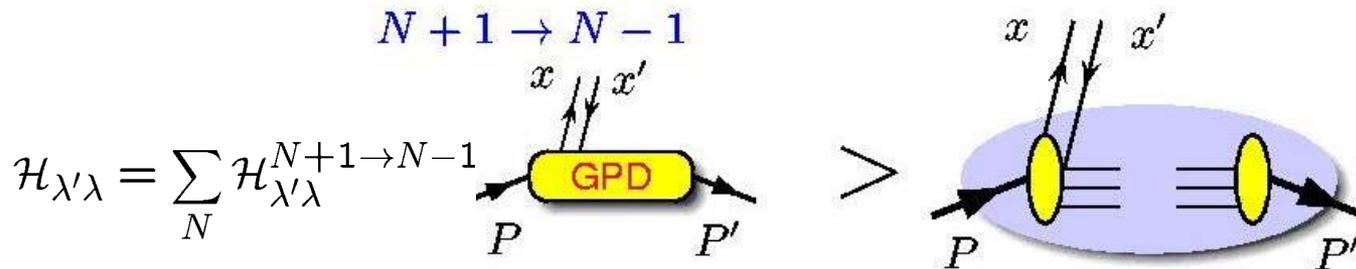
$$x_j^{in} = \frac{\bar{x}_j + \xi}{1+\xi} \quad \vec{k}_{\perp j}^{in} = \vec{k}_{\perp j} - \frac{1-\bar{x}_j}{1+\xi} \frac{\vec{\Delta}_{\perp}}{2} \quad \text{active}$$

$$x_i^{out} = \frac{\bar{x}_i}{1-\xi} \quad \vec{k}_{\perp i}^{out} = \vec{k}_{\perp i} - \frac{\bar{x}_i}{1-\xi} \frac{\vec{\Delta}_{\perp}}{2} \quad \text{spectator}$$

$$x_j^{out} = \frac{\bar{x}_j - \xi}{1-\xi} \quad \vec{k}_{\perp j}^{out} = \vec{k}_{\perp j} + \frac{1-\bar{x}_j}{1-\xi} \frac{\vec{\Delta}_{\perp}}{2} \quad \text{active}$$

7 mag 04 $\bar{x}, \vec{k}_{\perp} \rightarrow \begin{cases} x^{in}, \vec{k}_{\perp}^{in} \\ x^{out}, \vec{k}_{\perp}^{out} \end{cases} \perp \text{boost a}$ adrone iniziale
 adrone finale

decomposizione di GPD in LCwf in zona ERBL (quark-antiquark)

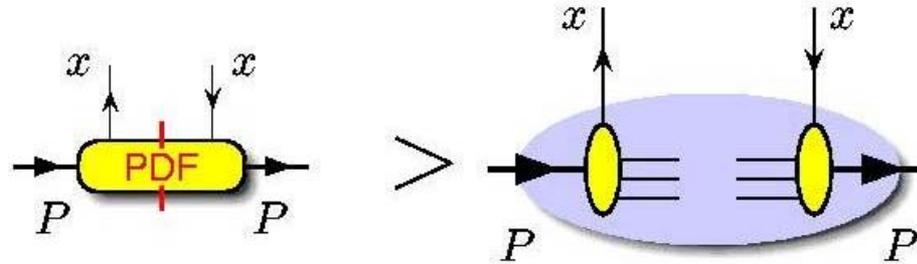


$$\mathcal{H}_{\lambda'\lambda}^{(N+1 \rightarrow N-1)}(x, \xi, t) = \frac{\sqrt{1-\xi}^{2-N}}{\sqrt{1+\xi}^N} \int \dots \delta(x - x_j) \Psi_{N-1}^{*,\lambda'}(x^{out}, \vec{k}_{\perp}^{out}) \Psi_{N+1}^{\lambda}(x^{in}, \vec{k}_{\perp}^{in})$$

| | | |
|--|---|-----------|
| $x_i^{in} = \frac{\bar{x}_i}{1+\xi}$ | $\vec{k}_{\perp i}^{in} = \vec{k}_{\perp i} + \frac{\bar{x}_i}{1+\xi} \frac{\vec{\Delta}_{\perp}}{2}$ | spectator |
| $x_j^{in} = \frac{\bar{x}_j + \xi}{1+\xi}$ | $\vec{k}_{\perp j}^{in} = \vec{k}_{\perp j} - \frac{1-\bar{x}_j}{1+\xi} \frac{\vec{\Delta}_{\perp}}{2}$ | 1' active |
| $x_{j'}^{in} = -\frac{\bar{x}_j - \xi}{1-\xi}$ | $\vec{k}_{\perp j'}^{in} = -\vec{k}_{\perp j} + \frac{1+\bar{x}_j}{1+\xi} \frac{\vec{\Delta}_{\perp}}{2}$ | 2' active |
| $x_i^{out} = \frac{\bar{x}_i}{1-\xi}$ | $\vec{k}_{\perp i}^{out} = \vec{k}_{\perp i} - \frac{\bar{x}_i}{1-\xi} \frac{\vec{\Delta}_{\perp}}{2}$ | spectator |

7 mag 04 ampiezza $\xrightarrow{\hspace{2cm}}$ densita` di probabilita`
 $\xi, t, \Delta_{\perp} \rightarrow 0$

decomposizione di PDF in LCwf in spazio di Fock a N partoni



$$f_1^{(N)}(x) = \int \dots \delta(x - x_j) \Psi_N^*(x, \vec{k}_\perp) \Psi_N(x, \vec{k}_\perp)$$

$$f_1(x) = \sum_N f_1^{(N)}(x)$$

probabilità di trovare un partone con frazione di momento x in
adrone con componente N-partonica in spazio di Fock $\rightarrow \Sigma_N$

Localizzazione trasversa 2d dei partoni

$\xi = 0$

$$H(x, \xi = 0, t = -\Delta_{\perp}^2) = \int d\mathbf{b}_{\perp} e^{-i\Delta_{\perp} \cdot \mathbf{b}_{\perp}} f_1(x, \mathbf{b}_{\perp})$$

densita` di probabilita` nello spazio del parametro di impatto \mathbf{b}_{\perp}

$\xi \neq 0$

(Diehl, E.P. J. C25 ('02) 223)

(Burkardt, P.R. D62 ('00) 071503)

invarianza di Lorentz $\rightarrow t = (P - P')^2 = -\frac{4\xi^2 M^2 + \Delta_{\perp}^2}{1 - \xi^2}$

$1 \geq x \geq \xi > 0$ $= (p' - p)^2 = -\frac{4\xi^2 m^2}{1 - \xi^2} + (1 - \xi^2) D_{\perp}^2$ $D_{\perp} = \frac{p'_{\perp}}{1 - \xi} - \frac{p_{\perp}}{1 + \xi}$

$$\mathcal{I}_{\lambda'\lambda}(x, \xi, \mathbf{b}_{\perp}) = \int \frac{d\mathbf{D}_{\perp}}{(2\pi)^2} e^{-i\mathbf{D}_{\perp} \cdot \mathbf{b}_{\perp}} \mathcal{H}_{\lambda'\lambda}(x, \xi, \mathbf{D}_{\perp})$$

$$= \sum_{M, \{\alpha\}} 1^{-M} \sqrt{1 - \xi^2} \sum_j \int \prod_i^M dx_i \delta\left(1 - \sum_i x_i\right) \prod_i^M d\mathbf{b}_{\perp i} \delta\left(\sum_i x_i \mathbf{b}_{\perp i}\right)$$

$$\delta(x - x_j) \delta(\mathbf{b}_{\perp} - \mathbf{b}_{\perp j}) \psi_{M\{\alpha\}}^{*\lambda'}(x_i^{out}, \mathbf{b}_{\perp i} - \mathbf{b}_{\perp 0}^{out}) \psi_{M\{\alpha\}}^{\lambda}(x_i^{in}, \mathbf{b}_{\perp i} - \mathbf{b}_{\perp 0}^{in})$$

\perp boost

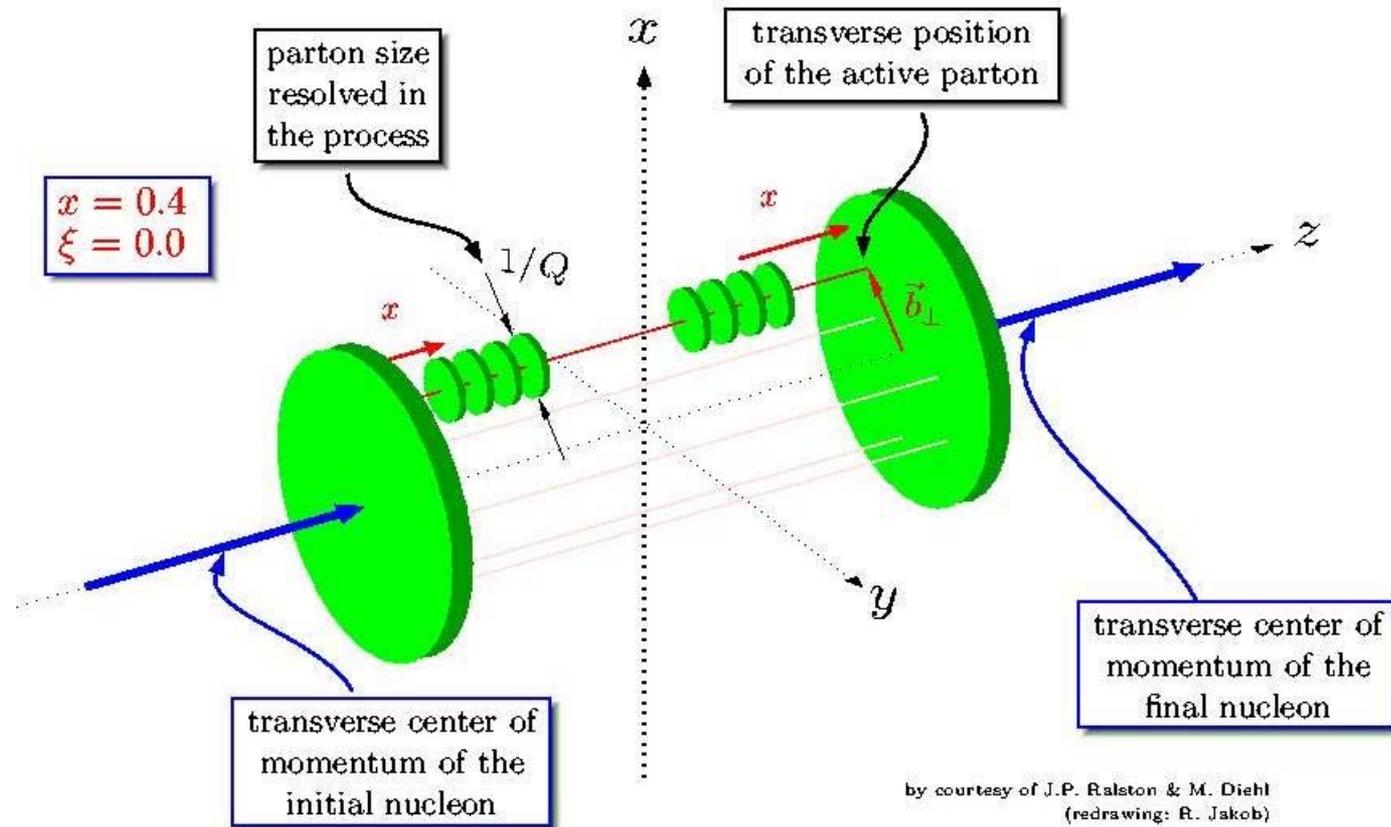
$$x_i^{in} = \frac{x_i}{1 + \xi} \quad x_j^{in} = \frac{x_j + \xi}{1 + \xi}$$

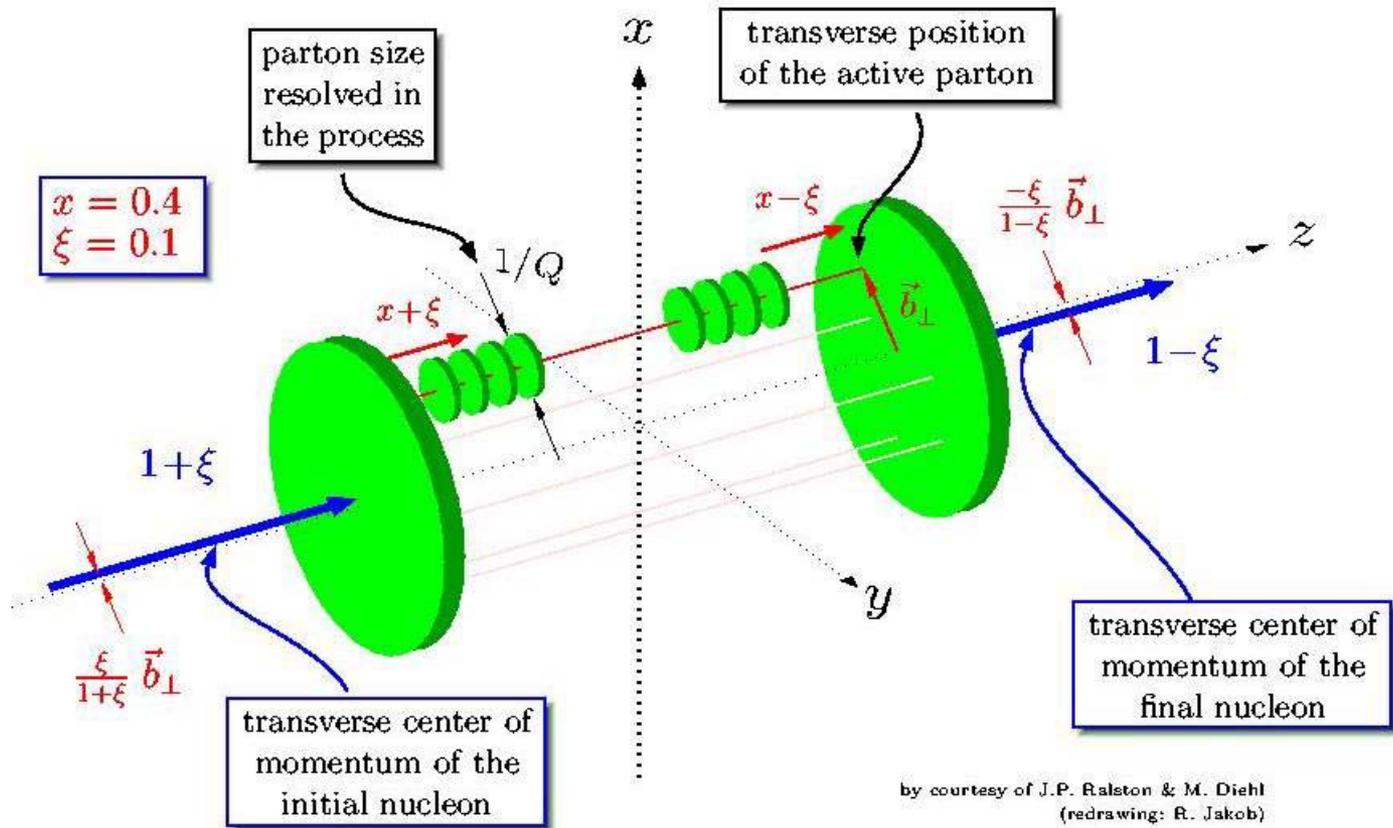
$$x_i^{out} = \frac{x_i}{1 - \xi} \quad x_j^{out} = \frac{x_j - \xi}{1 - \xi}$$

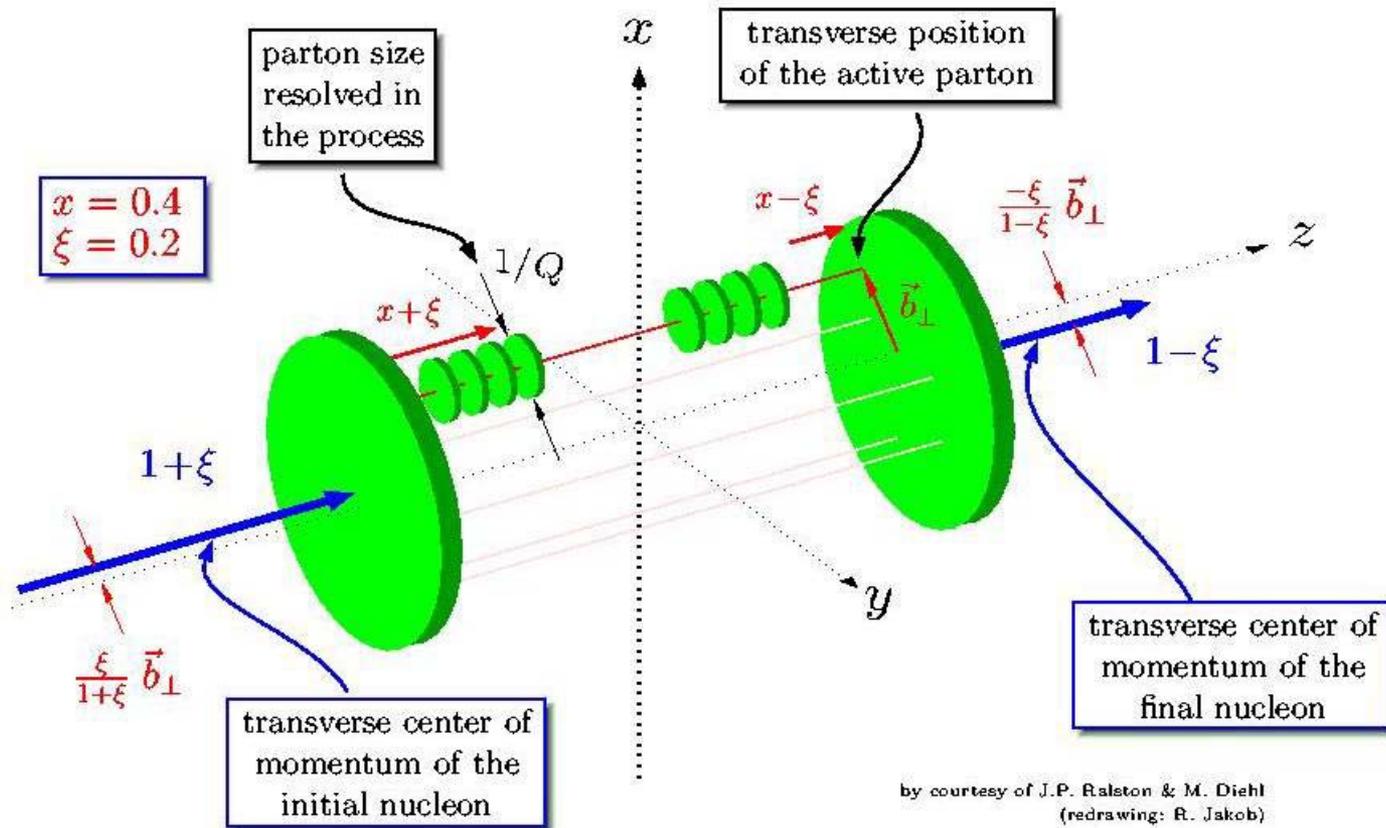
$$\mathbf{b}_{\perp 0}^{in} = \frac{\xi}{1 + \xi} \mathbf{b}_{\perp j}$$

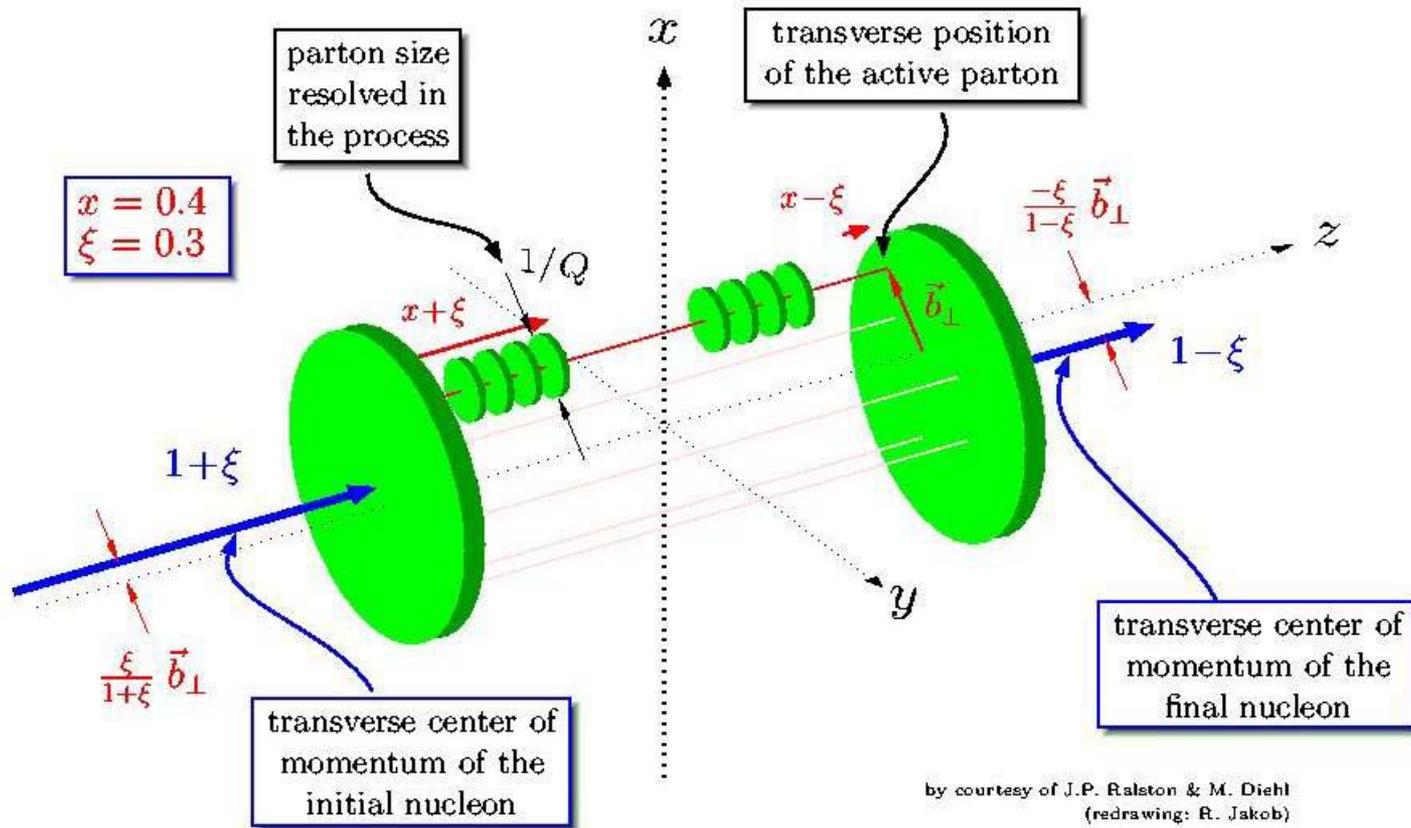
$$\mathbf{b}_{\perp 0}^{out} = -\frac{\xi}{1 - \xi} \mathbf{b}_{\perp j}$$

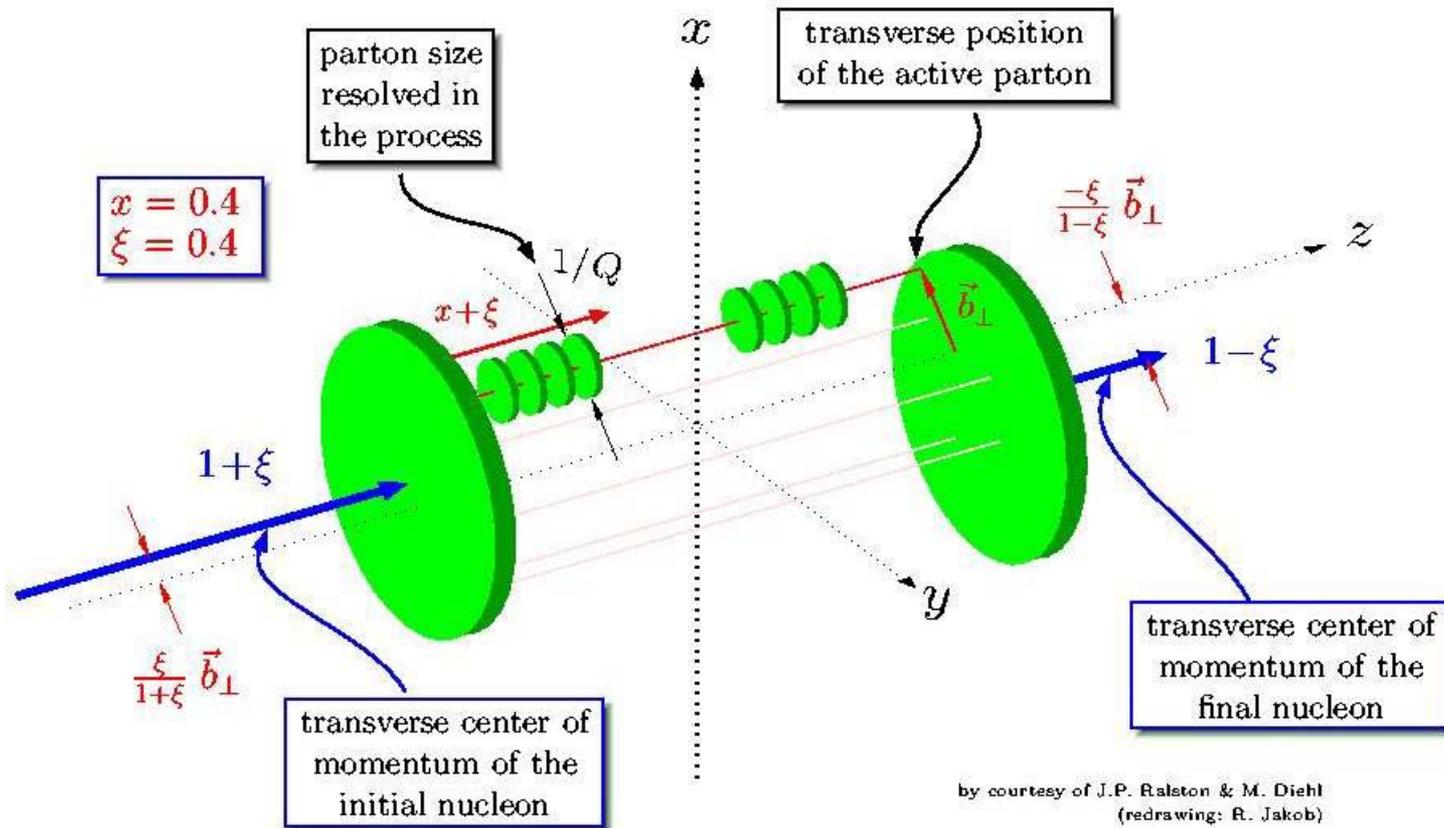
7 mag 04

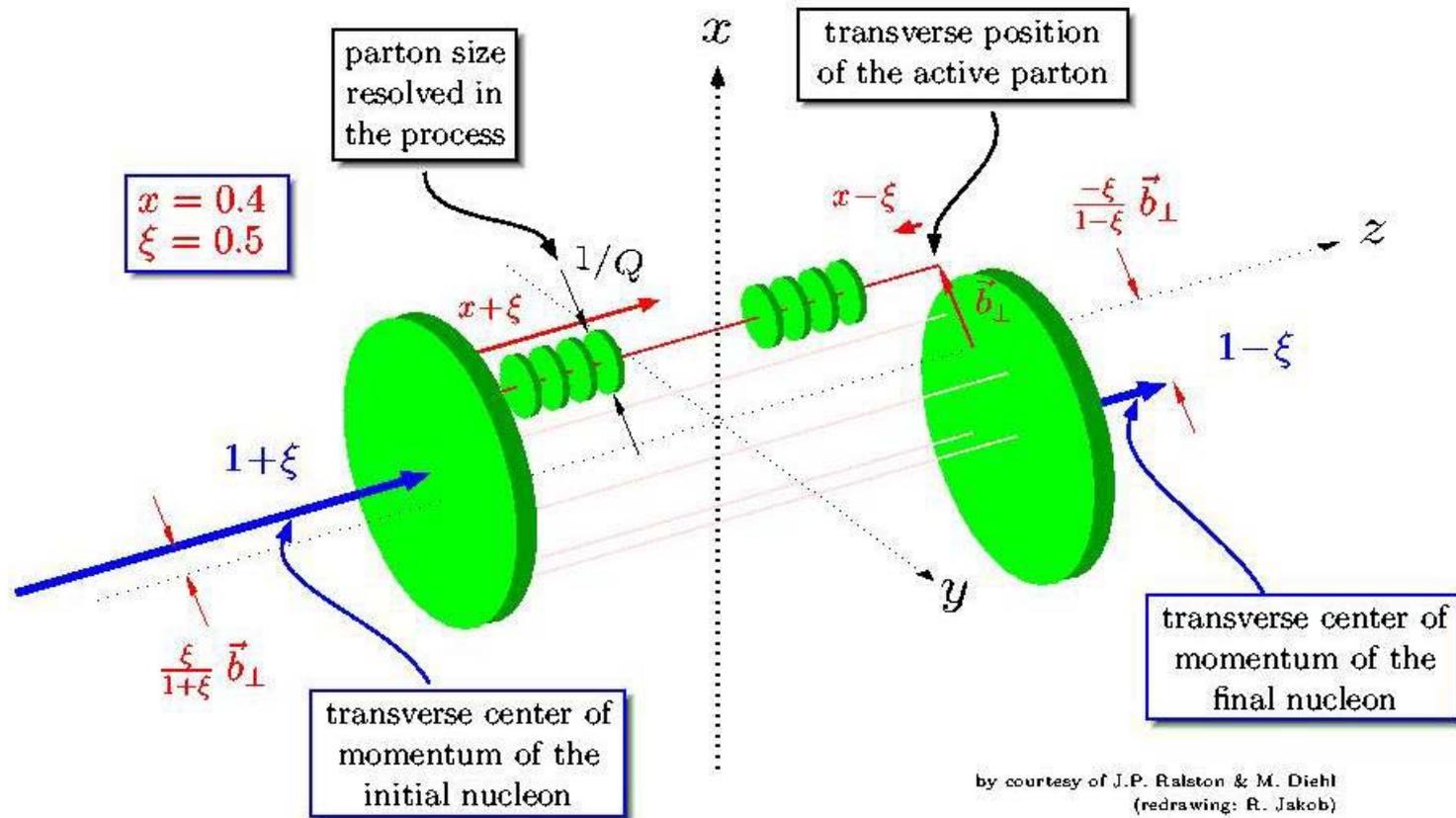


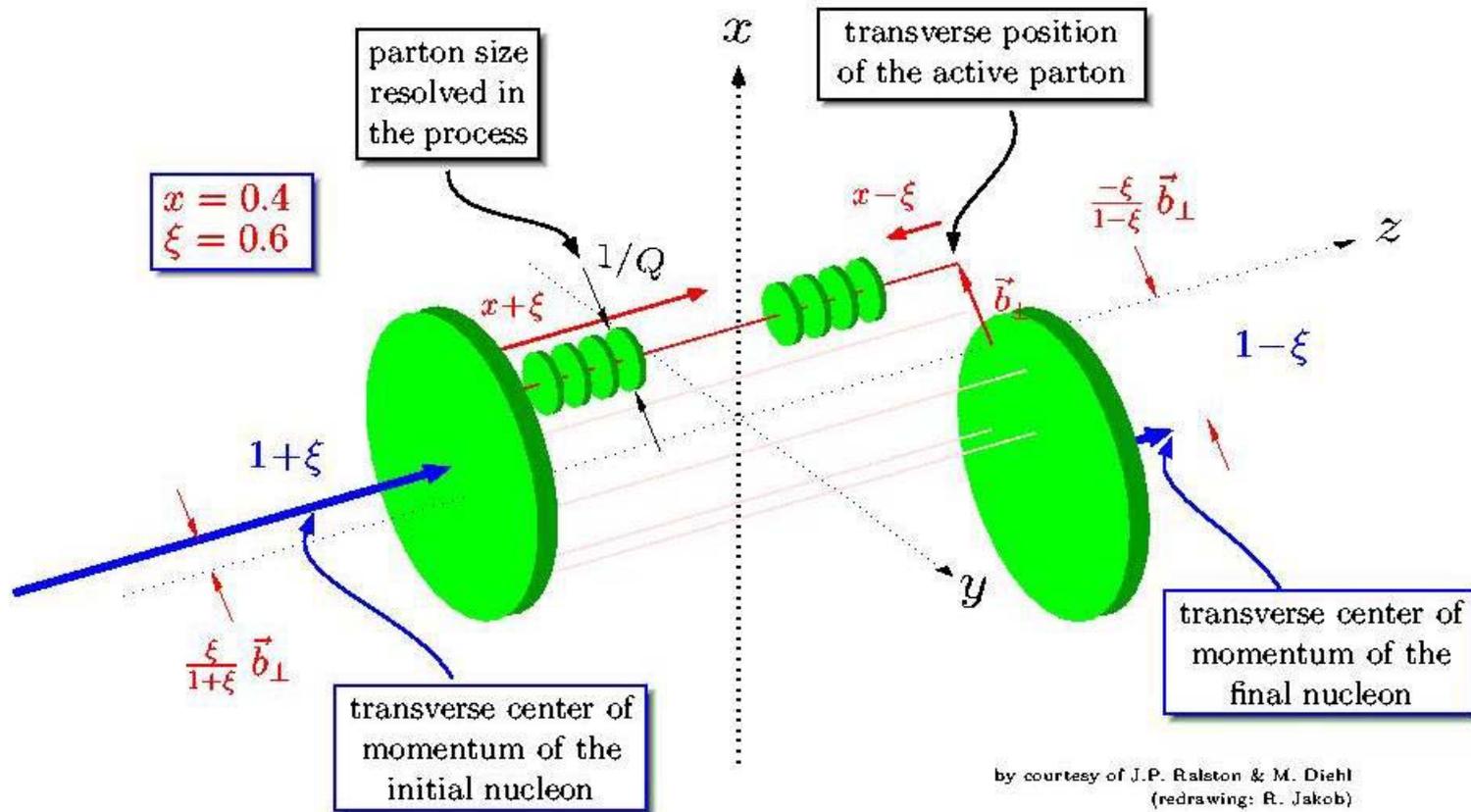












Momento angolare orbitale e regola di somma di spin

esperimento EMC (Cern, 1988) : misura di $g_1(x_B)$ implica che l'elicità dei quark di valenza non satura lo spin $\frac{1}{2}$ del N

Ma $g_1 \rightarrow$ quark spin sul L.C. \neq spin del quark di valenza che da $\frac{1}{2}$ del N

Inoltre c'è il possibile contributo del gluone e dei momenti angolari orbitali
 \Rightarrow serve una definizione gauge invariante degli operatori in gioco

Tensore energia-impulso $T^{\mu\nu} = \frac{\partial \mathcal{L}}{\partial \partial_\mu \psi} \partial_\nu \psi - \delta^{\mu\nu} \mathcal{L} \quad \partial_\mu \equiv \frac{\partial}{\partial x^\mu}$

4-momento $P^\mu = \int d\mathbf{x} T^{0\mu}$ tensore momento angolare $M^{\lambda\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}$
generatore delle trasform. di Lorentz $J^{\mu\nu} = \int d\mathbf{x} M^{0\mu\nu}$

operatore momento angolare $J^i = \frac{1}{2} \epsilon^{ijk} \int d\mathbf{x} M^{0jk}$

N.B. Tutte definizioni gauge invarianti

Decomposizione gauge invariante dell'operatore momento angolare

$$\mathbf{J} = \int d\mathbf{x} \left[\frac{1}{2} \psi^\dagger \boldsymbol{\gamma} \gamma_5 \psi + \psi^\dagger (\mathbf{x} \times (-i\mathbf{D})) \psi + \mathbf{x} \times (\mathbf{E} \times \mathbf{B}) \right]$$

$$= \left[\frac{1}{2} \Delta\Sigma + \mathbf{L}_q + \mathbf{J}_g \right] \Big|_{\mu^2} \equiv [\mathbf{J}_q + \mathbf{J}_g] \Big|_{\mu^2}$$

in $\Delta\Sigma$ per via di

$U_A(1) \rightarrow$ ambiguità

scala di rinormalizzazione

vettore di Poynting
del campo di colore

$$E^i = F^{i0}$$

$$B^i = -\frac{1}{2} \epsilon^{ilm} F_{lm}$$

$$\mathbf{D} = \nabla + ig\mathbf{A}$$

definizione dell'autovalore
momento angolare

$$J_{q/g} \equiv \langle P \frac{1}{2} | J_{q/g}^z | P \frac{1}{2} \rangle$$

eq. di evoluzione

$$\frac{d}{d \log \mu^2} \begin{pmatrix} J_q \\ J_g \end{pmatrix} = 2 \begin{pmatrix} \gamma_{qq} & \gamma_{qg} \\ \gamma_{gq} & \gamma_{gg} \end{pmatrix} \begin{pmatrix} J_q \\ J_g \end{pmatrix} = \frac{\alpha_s(\mu)}{2\pi} \frac{1}{9} \begin{pmatrix} -16 & 3n_f \\ 16 & -3n_f \end{pmatrix} \begin{pmatrix} J_q \\ J_g \end{pmatrix}$$

$$\mu \rightarrow \infty \quad J_q = \frac{1}{2} \frac{3n_f}{16 + 3n_f} \sim J_g = \frac{1}{2} \frac{16}{16 + 3n_f}$$

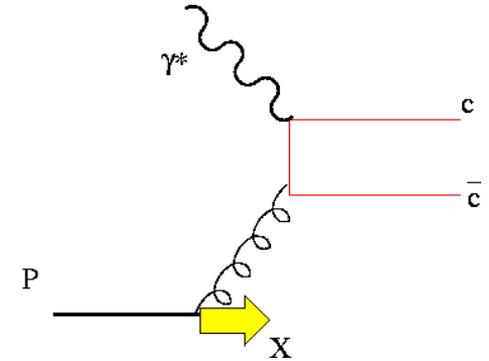
$$J_q \sim 0.25$$

$$\frac{1}{2} \Delta\Sigma \sim 0.15 \Big|_{10 \text{ GeV}} \text{ (EMC)}$$

$$\left. \begin{matrix} J_q \sim 0.25 \\ \frac{1}{2} \Delta\Sigma \sim 0.15 \Big|_{10 \text{ GeV}} \text{ (EMC)} \end{matrix} \right\} L_q \sim 0.1$$

Ma l'elicita` del gluone $\Delta G(x)$ e` estraibile da $e+p \rightarrow D+X$ attraverso meccanismo di photon-gluon fusion (COMPASS)

quindi $J_g|_{\mu^2} = [\Delta G + L_g]|_{\mu^2}$



La decomposizione gauge invariante $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$

e` una regola di somma se stabilisce una corrispondenza tra elementi di matrice di operatori e momenti di PDF calcolati sugli stessi stati adronici

Ad esempio
$$\Delta\Sigma = \langle PS | \bar{q} \gamma^\mu \gamma_5 q | PS \rangle = \int dx (g_1(x) + \bar{g}_1(x))$$

$$\propto \int dz^- e^{ixP^+z^-} \langle PS | \bar{\psi}(0) \gamma^+ \gamma_5 \psi(z^-) | PS \rangle$$

Ora $J_q = \int dx J_q(x)$

operatore di corrente e.m. ; invarianza per Lorentz, parita` e time-reversal

$$\langle P' | \bar{\psi} i D^\mu \psi | P \rangle = \bar{u}(P') \gamma^\mu u(P) A(t) + \bar{u}(P') \frac{i \sigma^{\mu\nu} \Delta_\nu}{2M} u(P) B(t)$$

generalizzazione a "torre" di operatori twist-2

$$\begin{aligned} \langle P' | \bar{\psi} i \vec{D}^{\{\mu_1} \dots \vec{D}^{\mu_{n-1}} \gamma^{\mu_n\}} \psi | P \rangle = & \text{parte intera} \quad \text{simmetrizzazione} \\ & \text{sugli indici} \\ & \left[\frac{n-1}{2} \right] \\ & \bar{u}(P') \gamma^{\{\mu_1} u(P) \sum_i A_{2i}(t) \Delta^{\mu_2} \dots \Delta^{\mu_{2i+1}} \bar{P}^{\mu_{2i+2}} \dots \bar{P}^{\mu_n\}} \\ & + \bar{u}(P') \frac{i \sigma^{\{\mu_1 \alpha} \Delta_\alpha}{2M} u(P) \sum_i B_{2i}(t) \Delta^{\mu_2} \dots \Delta^{\mu_{2i+1}} \bar{P}^{\mu_{2i+2}} \dots \bar{P}^{\mu_n\}} \\ & + \frac{1}{M} \bar{u}(P') u(P) C(t) \text{Mod}(n+1, 2) \Delta^{\{\mu_1} \dots \Delta^{\mu_n\}} \\ & \text{seleziona n pari} \end{aligned}$$

si sceglie vettore light-like

$$n_\mu \quad n^2 = 0 ; n \cdot \bar{P} = 1 ; n \cdot \Delta = -2\xi$$

$$n_{\mu_1} \dots n_{\mu_n} \langle P' | \bar{\psi} i \vec{D}^{\{\mu_1} \dots \vec{D}^{\mu_{n-1}} \gamma^{\mu_n\}} \psi | P \rangle =$$

$$\bar{u}(P') \gamma \cdot n u(P) \left[\sum_i^{\left[\frac{n-1}{2} \right]} A_{2i}(t) (-2\xi)^{2i} + \text{Mod}(n+1, 2) C(t) (-2\xi)^n \right]$$

$$+ \bar{u}(P') \frac{i \sigma^{\mu\alpha} n_\mu \Delta_\alpha}{2M} u(P) \left[\sum_i^{\left[\frac{n-1}{2} \right]} B_{2i}(t) (-2\xi)^{2i} - \text{Mod}(n+1, 2) C(t) (-2\xi)^n \right]$$

$$H_n(\xi, t) = \int dx x^{n-1} H_n(x, \xi, t)$$

$$E_n(\xi, t) = \int dx x^{n-1} E_n(x, \xi, t)$$

definizione originale di GPD

(Ji, P.R.D58 ('98)056003)

polinomialita` = condizione stringente per i modelli, perche` ottenuta da principi generali di invarianza

Applichiamo il risultato all'operatore a twist-2 a due indici
tensore energia-impulso $T^{\mu\nu}$

$$\begin{aligned}
J_q &= \langle P' \frac{1}{2} | J_q^z | P \frac{1}{2} \rangle = \frac{1}{2} \epsilon^{ijk} \int d\mathbf{x} \langle P' \frac{1}{2} | M^{0jk} | P \frac{1}{2} \rangle \\
&= \frac{1}{2} \epsilon^{ijk} \int d\mathbf{x} \langle P' \frac{1}{2} | x^j T^{0k} - x^k T^{0j} | P \frac{1}{2} \rangle = \dots \\
&= \frac{1}{2} [A(0) + B(0)] = \frac{1}{2} \left[\int dx x H(x, 0, 0) + \int dx x E(x, 0, 0) \right] \\
&= \int dx J_q(x)
\end{aligned}$$

quindi

$$J_q(x) = \frac{1}{2} x [f_1(x) + E(x, 0, 0)] \longrightarrow L_q(x) = J_q(x) - \frac{1}{2} g_1(x)$$

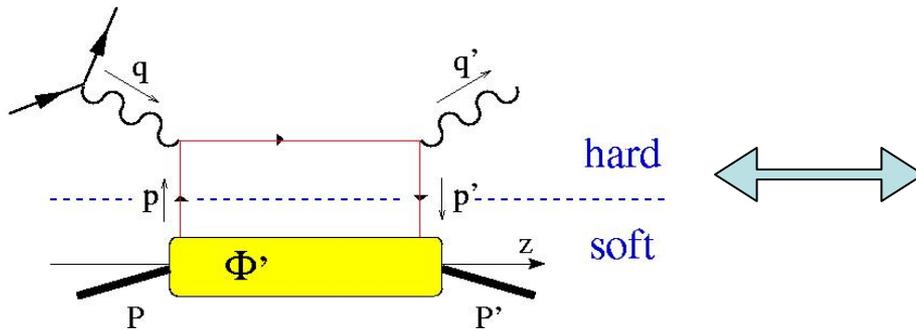
similmente si ottiene

$$J_g(x) = \frac{1}{2} x [G(x) + E_g(x, 0, 0)] \longrightarrow L_g(x) = J_g(x) - \frac{1}{2} \Delta G(x)$$

Regola di somma gauge invariante alla scala μ^2

$$\frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_q + \Delta G + L_g$$

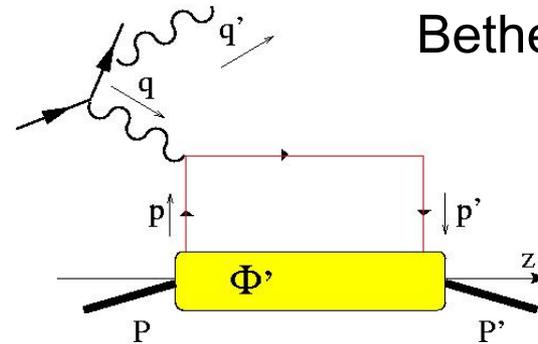
Situazione sperimentale



complesso \rightarrow interferenza

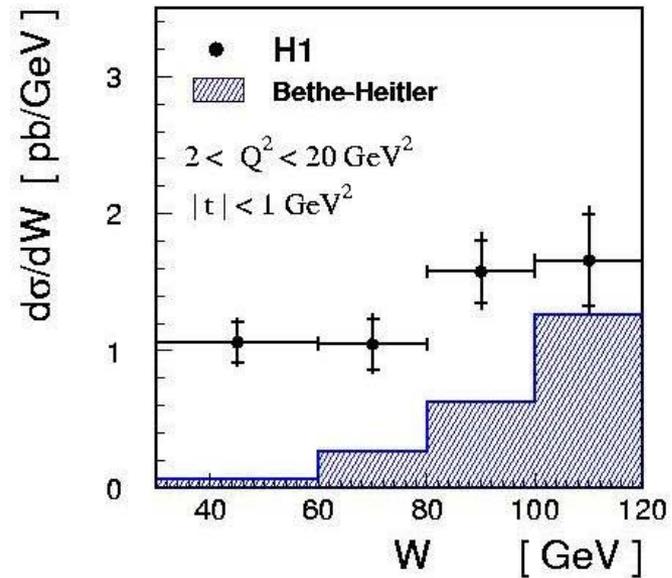
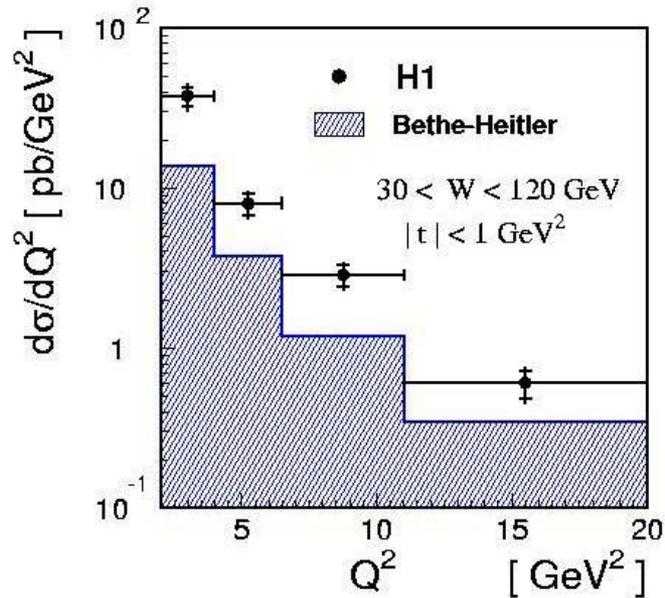
first glimpse on GPDs

Bethe-Heitler



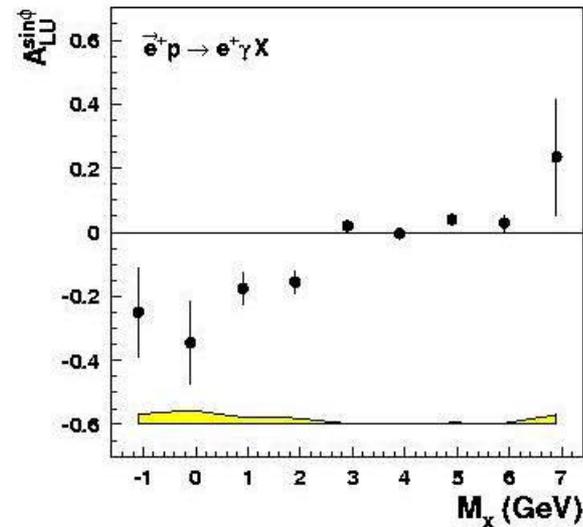
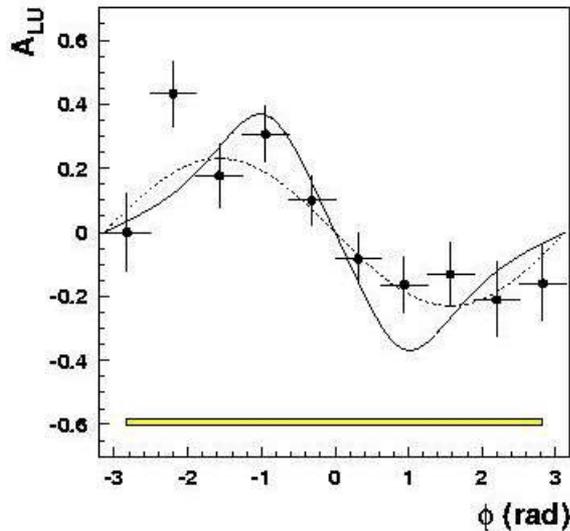
reale \rightarrow background

(H1 coll., P.L.B517 ('01)47)



first glimpse on GPDs

(HERMES coll., P.R.L. **87** ('01)182001)



The DVCS Single Spin Asymmetry from HERMES.

Left: Beam-spin asymmetry A_{LU} for hard electro-production of photons as a function of the azimuthal angle ϕ .

Right: The beam-spin analyzing power $A_{LU}^{\sin\phi}$ for hard electro-production of photons on hydrogen as a function of the missing mass.

Beam spin asymmetry $\vec{e} + p \rightarrow e' + p' + \gamma$

$$7 \text{ mag } 04 \quad \frac{d\sigma(\vec{e}) - d\sigma(\leftarrow e)}{d\sigma(\vec{e}) + d\sigma(\leftarrow e)} \sim \text{Im} [\text{B-H DVCS}^*] \sin \phi$$

charge beam asymmetry

$$\frac{d\sigma(e^+) - d\sigma(e^-)}{d\sigma(e^+) + d\sigma(e^-)} \sim \text{Re} [\text{B-H DVCS}^*] \sim A + B \cos \phi + C \cos 2\phi$$

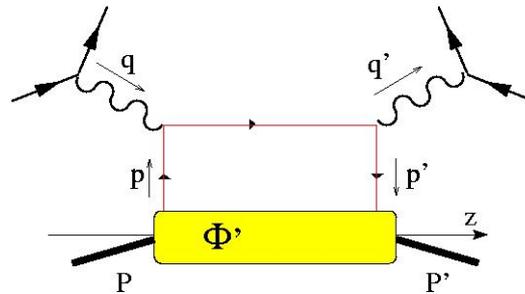
Twist 2

Twist 3

dati in arrivo ... comunque riguardano solo GPD ($x=\xi, \xi, t$)

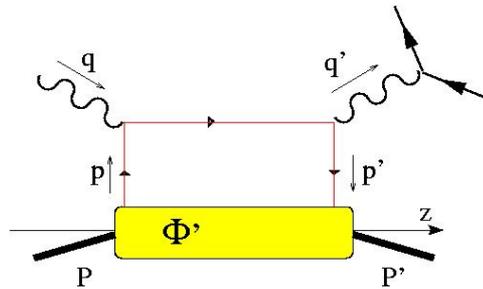
Futuro :

DDVCS

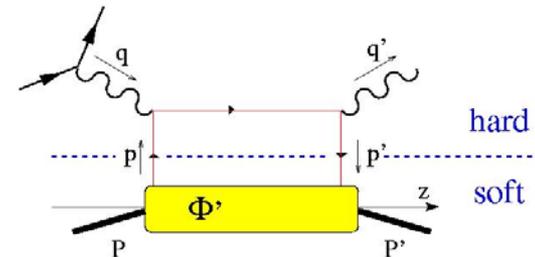


GPD($2\xi'-\xi, \xi, t$)

time-like DVCS



Δ VCS



$P' = \Delta, \dots$