Few-Body Systems Suppl. 0, 1–4 (2006)

Few-
Body
Systems

Nucleon Electromagnetic and Axial Form Factors in Point-Form Relativistic Quantum Mechanics

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Abstract. Results for the proton and neutron electric and magnetic form factors as well as the nucleon axial form factor are presented for constituent quark models, based on either one-gluon-exchange and Goldstone-boson-exchange dynamics. The calculations are performed in a covariant framework using the point-form approach to relativistic quantum mechanics. The only input to the calculations is the nucleon wave function of the corresponding constituent quark model. A comparison is given to results of the instanton-induced constituent quark model treated with the Bethe-Salpeter equation.

It has become fairly obvious that nucleon electromagnetic as well as axial form factors need a relativistic treatment. In order to satisfy all requirements of Lorentz covariance one can either proceed along relativistic field theory or relativistic (Poincaré-invariant) quantum mechanics. The latter approach appears promising whenever one can deal with a fixed number of particles (or a finite number of degrees of freedom). Usually one follows one of the three forms of dynamics, where the generators of the Poincaré group are minimally affected by interactions. Dirac defined them as instant, front, and point forms [1].

The point form is very convenient for practical calculations since only the components of the four-momentum operator $\hat{P}^\mu$ contain interactions. Therefore all Lorentz transformations remain purely kinematic and the theory is manifestly covariant (for more details see ref. [2]). In order to construct the interacting four-momentum operators one follows the Bakamjian-Thomas construction [3], where the interaction is introduced into the mass operator $\hat{M}$ by adding to the free mass operator $\hat{M}_{\rm fr}$ the interacting term $\hat{M}_{int}$. Through multiplication with the free four-velocity operator $\hat{V}^\mu_{\rm fr}$, which is not affected by interactions, one obtains

$$\hat{P}^{\mu} = \hat{M}\hat{V}^{\mu}_{\text{fr}} = (\hat{M}_{\text{fr}} + \hat{M}_{int})\hat{V}^{\mu}_{\text{fr}}. \tag{1}$$

Poincaré invariance implies that $\hat{M}$ commutes with $\hat{V}^{\mu}_{\text{fr}}$ and is a scalar under Lorentz transformations. Therefore eigenstates of the four-momentum operator are simultaneous eigenstates of both the mass and the velocity operators. As a consequence the motion of the system as a whole and the internal motion are separated. The latter is described by a wave function containing only the internal degrees of freedom. It can be found by solving the eigenvalue problem for the mass operator

$$\hat{M}\Psi = M\Psi. \tag{2}$$

The electromagnetic and axial form factors of the nucleons are obtained by sandwiching the electromagnetic and axial current operators between eigenstates $|P, \Sigma\rangle$, where P and Σ are the eigenvalues of the total momentum and the z-component of the total angular momentum (for details see refs. [4]). The calculation boils down to matrix elements of the current operators between free three-particle states $|p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3\rangle$, where p_i are the individual quark four-momenta and σ_i their spin projections. At the present stage we cannot yet deal with the full current operators but have to truncate them to one-body operators by the so-called point-form spectator approximation (PFSA) [2]

$$\begin{aligned}&\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 |\, \hat{J}^{\mu}(0)\, | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3\rangle = \\ &2E_2\delta(\boldsymbol{p}'_2 - \boldsymbol{p}_2)\delta_{\sigma'_2\sigma_2} 2E_3\delta(\boldsymbol{p}'_3 - \boldsymbol{p}_3)\delta_{\sigma'_3\sigma_3} \langle p'_1, \sigma'_1 |\, \hat{j}^{\mu}(0)\, | p_1, \sigma_1\rangle\,,\end{aligned} \tag{3}$$

and similarly for the axial current $\hat{\boldsymbol{A}}^{\mu}(0)$ with $\hat{j}^{\mu}(0)$ replaced by $\hat{\boldsymbol{a}}^{\mu}(0)$. By the small letters we indicate free one-body currents of the constituent quarks. The axial current is denoted as a vector in isospin space. With regard to the PFSA it is important to notice that the impulse $\tilde{q} = p'_1 - p_1$ delivered to the single constituent quark is different from the impulse $q = P' - P$ delivered to the nucleon as a whole. The momentum transfer $\tilde{q}$ can be uniquely determined from q and the two spectator conditions $\boldsymbol{p}'_2 = \boldsymbol{p}_2$ and $\boldsymbol{p}'_3 = \boldsymbol{p}_3$. For the one-body current matrix elements in the above equation we employ the usual expressions for electromagnetic and axial currents of pointlike spin-$\frac{1}{2}$ particles (see refs. [4]).

The results for all electromagnetic and axial form factors from the Goldstone-boson-exchange (GBE) CQM of ref. [5] have already been published in refs. [4]. There it was found that relativistic effects are most important. The direct predictions of the GBE CQM in PFSA come remarkably close to the experimental data in all instances. This observation has recently been confirmed also with regard to the electric radii and magnetic moments not only of the nucleons but all (measured) octet and decuplet ground states [6].

Here we present a comparison of nucleon form factor results from different CQMs and different relativistic approaches. First, we compare the PFSA predictions of the GBE CQM with analogous results from a CQM whose hyperfine interaction is based on one-gluon exchange (OGE), namely a relativized version

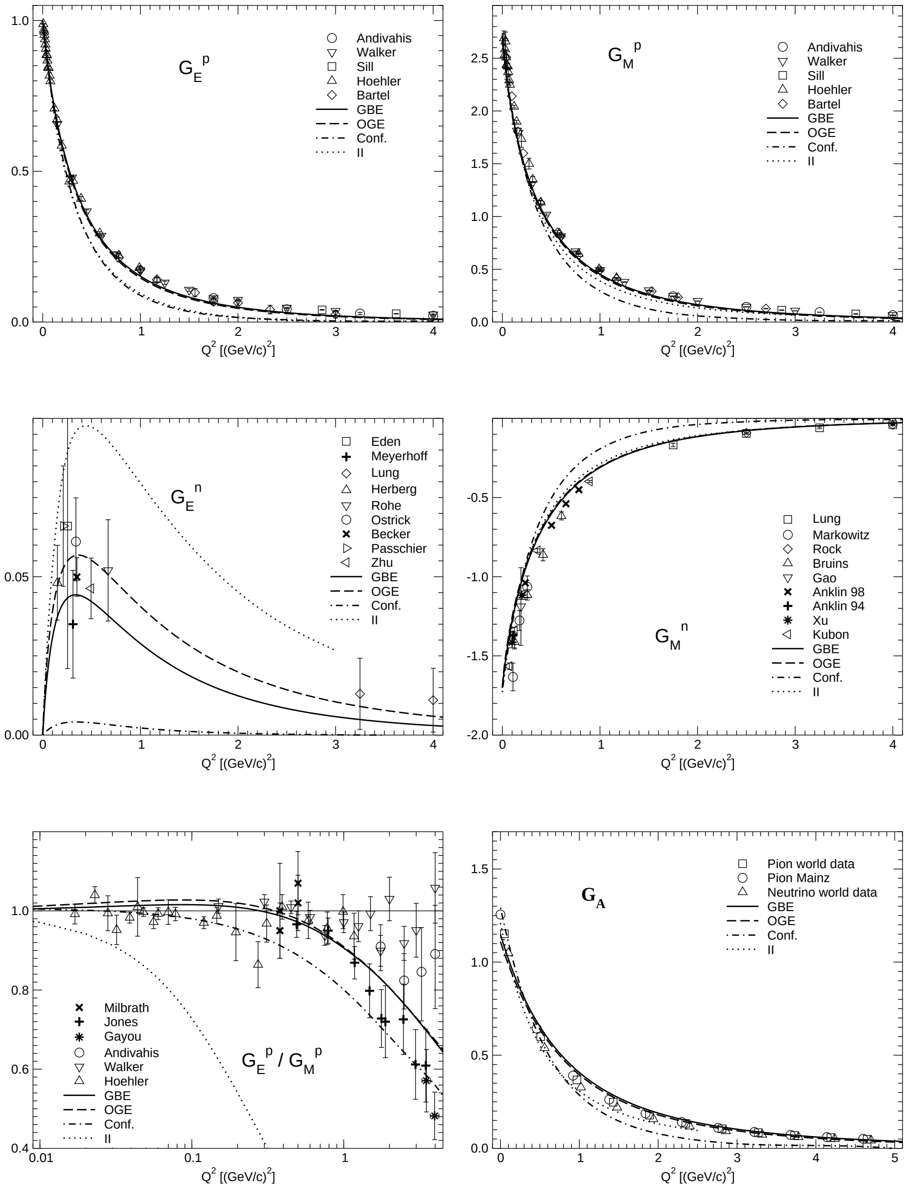

Figure 1. Predictions of different CQMs for the nucleon electromagnetic and axial form factors. The solid and dashed lines represent our PFSA results for the GBE and OGE CQMs, respectively; the dash-dotted line refers to the case with confinement only. The dotted lines show the results of the II CQM within the Bethe-Salpeter approach after ref. [9].

of the Bhaduri-Cohler-Nogami CQM [7] as parametrized in ref. [8]. Then we also provide a comparison to the results of an instanton-induced (II) CQM as obtained by the Bonn group within a Bethe-Salpeter (BS) formalism [9].

As is immediately evident from the results collected in Fig. 1, the overall behaviour of the relativistic predictions appears quite reasonable in all cases. This confirms the previous findings that the inclusion of relativity is most important for the nucleon form factors. For their gross properties dynamical effects are of lesser relevance. Even a three-quark wave function that relies solely on confinement produces the right features, except for the neutron. In this case a realistic wave function is required, with the mixed-symmetry spatial components taken into account. For the proton form factors and also the nucleon axial form factor there is a striking similarity of the results obtained in PFSA and in the BS approach, where the latter also uses a single-particle approximation for the current operators. Only for the subtle details of the neutron electric form factor and the ratio of the proton electric to magnetic form factors the predictions of the II CQM fall short compared to experimental data. A comparison of the CQM results for electric radii and magnetic moments is given in ref. [6], yielding a picture congruent with the one found here.

Acknowledgement. This work was supported by the Austrian Science Fund (Project P14806).

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